schemes show an extremely rapid increase of sensitivity to phase noise with increasing M.

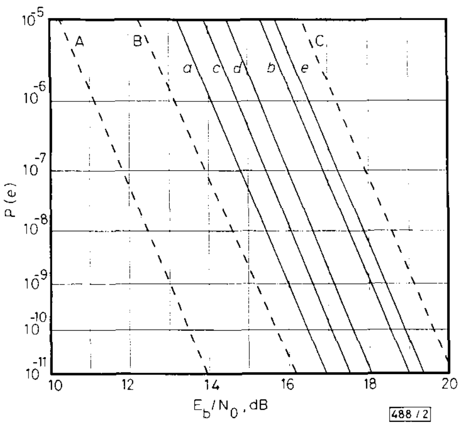

Fig. 2 *Error probability as a function of* E_b/N_0

a 2-POLSK
b 4-POLSK square
c 4-POLSK tetrahedron
d 6-POLSK octahedron
e 8-POLSK cube
A Binary DPSK
B Quaternary DPSK
C Octonary DPSK

Beyond the scope of this letter, there remains the problem of the recovery of the constellation $\{S_j\}$ at the receiver start-up time. This topic is investigated in Reference 6.

Conclusions: We have proposed and exactly analysed four new multilevel modulation schemes based on the state of polarisation of a fully polarised lightwave. A suitable modulator structure has also been described.

The results expressed in terms of $P(e)$ against E_b/N_0 show significant improvements with respect to other standard modulation schemes and make M-POLSK modulation a good candidate for power and bandwidth efficient optical communication.

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References

1 DIETRICH, E., ENNING, B., GROSS, R., and KNUPKE, H.: 'Heterodyne transmission of a 560 Mbit/s optical signal by means of polarisation shift keying', *Electron. Lett.*, 1987, **23**, (8), pp. 421–422
2 CALVANI, R., CAPONI, R., and CISTERNINO, F.: 'Polarisation phase-shift keying: a coherent transmission technique with differential heterodyne detection', *Electron. Lett.*, 1988, **24**, (10), pp. 642–643
3 BETTI, S., CURTI, F., DIANO, B., DE MARCHIS, G., and IANNONE, E.: 'State of polarisation and phase noise independent coherent optical transmission system based on Stokes parameter detection', *Electron. Lett.*, 1988, **24**, (23), pp. 1460–1461
4 BENEDETTO, S., *et al.*: 'Performance evaluation of polarisation shift keying modulation systems'. Conf. optical fiber communication, S. Francisco, CA, January 1990
5 BENEDETTO, S., and POGGIOLINI, P.: 'Theory of polarisation shift keying modulation', submitted to *IEEE Trans. Comm.*
6 BENEDETTO, S., and POGGIOLINI, P.: 'Multilevel polarisation shift keying: theory and applications', to be submitted
7 OKOSHI, T., and KIKUCHI, K.: 'Coherent optical fiber communications' (KTK Scientific Publishers, Tokyo, 1988)
8 BENEDETTO, S., BIGLIERI, E., and CASTELLANI, V.: 'Digital transmission theory' (Prentice-Hall, Englewood Cliffs, NJ, 1987)
9 CALVANI, R., *et al.*: 'High speed polarization modulation through an optical guided-wave two-by-two switch'. OE/FIBERS '89. Conf. 1175, Boston, September 1989

ESTIMATION OF INSTANTANEOUS FREQUENCY USING THE DISCRETE WIGNER DISTRIBUTION

Indexing terms: Signal processing, Frequency modulation, Mathematical techniques

Analytical expressions for the performance of the discrete Wigner distribution (DWD) in estimating the instantaneous frequency of linear frequency modulated signals in additive white noise are derived and verified using simulation. It is shown that the DWD peak provides an optimal estimate at high input signal-to-noise ratios. The applicability of these results to the general case of nonlinear FM signals is discussed.

Introduction: Measuring the instantaneous frequency of a signal is important for many applications. Time–frequency distributions, such as the Wigner distribution (WD) have been used to describe the instantaneous frequency of time-varying signals.[1] Boashash[2] developed the concept of instantaneous frequency (IF) and discussed various estimation methods based on time–frequency distributions. The use of the peak of the time–frequency distribution to estimate the IF was determined to be the most computationally attractive of these methods. Since the WD provides a concentrated time–frequency representation, its application to the estimation of the IF is of interest. Estimating the IF from the WD peak has limitations which are determined by the nature of the time-varying signal and the input signal-to-noise ratio (SNR). Experimental results comparing the performance of the WD estimator to other methods for the estimation of the instantaneous frequency and amplitude of an FM signal in additive white noise have been reported by Harris and Salem.[3] In this letter we analytically examine the performance of the instantaneous frequency estimator for a frequency modulated signal in additive white Gaussian noise (WGN) using the peak of the discrete-time WD (DWD). Exact expressions for the variance of the estimate of IF for signals with quadratic phase functions are derived and validated using computer simulation.

The class of signals considered is assumed to be given in the complex form, by

$$s(n) = v(n)e^{j\phi(n)} + z(n) \tag{1}$$

where $z(n)$ is complex white Gaussian noise with zero-mean and variance σ_z^2. The DWD of $s(n)$ is given by

$$W_s(n, f) = 2\sum_{k=-\infty}^{\infty} s(n+k)s^*(n-k)e^{-j4\pi kf} \tag{2}$$

The basis for applying the DWD to the estimation of the instantaneous frequency of a signal is the first moment property.[4] The first-order moment with respect to frequency of the DWD provides what is considered an acceptable definition of the instantaneous frequency of a time-varying signal. The DWD can therefore be used to recover the IF since it's first moment, with respect to frequency, provides an unbiased estimate and is independent of any amplitude of frequency modulation that is present in the signal. The presence of noise, however, leads to serious degradation of the first moment estimate because of the absence of any averaging in its definition. Thus the first moment may have a high statistical variance even at high values of input SNR. Since the WD provides a highly concentrated distribution of signal energy in time–frequency for the type of signals being considered, a natural alternative is the use of the peak detection of the DWD to estimate the IF. For the general case of the signal given by eqn. 1, the peak estimator is biased, depending on the amount of amplitude and frequency modulation present in the signal. That is, only for signals of constant amplitude and quadratic phase functions, or linear FM signals, is an unbiased estimate of IF obtained using this approach. For signals that meet this condition, it will be shown here that the DWD peak provides an optimal estimate of the IF at high values of SNR and degrades only slowly as the SNR decreases down to a threshold value. The threshold SNR for the breakdown of the DWD peak estimator of IF will be evaluated. The effect of nonlinear FM signals on the performance of the estimator is discussed and windowing is suggested to improve performance.

Performance of the estimator: In practice the DWD (eqn. 2) is implemented over a finite data record of N samples, using a standard DFT.[4] Hence the DWD can be looked upon as the frequency-scaled DFT of the kernel sequence with a scaling factor of two. For the signal given by eqn. 1, assume for the present a constant amplitude $v(n) = A$ which yields an input SNR $= A^2/\sigma_z^2$, we obtain for the DWD kernel

$$s(n+k)s^*(n-k) = A^2 e^{j[\phi(n+k)-\phi(n-k)]} + Ae^{j\phi(n+k)}z^*(n-k) + Ae^{-j\phi(n-k)}z(n+k) + z(n+k)z^*(n-k) \quad (3)$$

For signals with a quadratic phase function, the first term is a constant frequency sinusoid of amplitude A^2. The noise in the kernel consists of the last three terms in eqn. 3 (which arise from the input noise components as well as from signal × noise terms). Since the input noise sequence is assumed white and Gaussian, the noise terms in the kernel are uncorrelated and hence white. The resultant noise is not Gaussian however due to the introduction of the product term. The resultant noise power in the kernel is then the sum of the variances of the three uncorrelated noise components

$$NP_{kernel} = 2A^2\sigma_z^2 + \sigma_z^4 \quad (4)$$

There are only $N/2$ independent samples in the (conjugate symmetric) kernel, so the variance of the noise in the DWD spectrum is scaled by the factor $N/2$ rather than the expected factor N over the input SNR. Hence the SNR in the DWD spectrum is

$$SNR_{DWD} = \frac{A^4 \times (N/2)}{2A^2\sigma_z^2 + \sigma_z^4} \quad (5)$$

The problem is now reduced to that of estimating a constant frequency sinusoid in white noise using the peak of the DFT magnitude. Using Reference 5 it is seen that the variance of the DFT estimate at high SNRs is given by

$$\mathrm{var}_{DFT}(\hat{f}) = \frac{6}{(2\pi)^2(SNR)(N^2-1)} \quad (6)$$

where SNR in eqn. 6 stands for the SNR in the DFT and is given by NA^2/σ_z^2. The frequency scaling by a factor of two inherent in the DWD means that the frequency estimate obtained from its peak must be corrected by the same factor. This leads to a reduction of variance by a factor of four for the DWD. Substituting the SNR of the DWD spectrum, as given by eqn. 5, into eqn. 6 and scaling by four, we obtain the variance of the DWD peak estimator of IF of a linear FM signal in white Gaussian noise, which is

$$\mathrm{var}_{DWD}(\hat{f}) = \frac{6(2A^2\sigma_z^2 + \sigma_z^4)}{(2\pi)^2 2A^4 N(N^2-1)} \quad (7)$$

At high input SNRs ($A^2 \gg \sigma_z^2$) the above expression reduces to

$$\mathrm{var}_{DWD}(\hat{f}) = \frac{6\sigma_z^2}{(2\pi)^2 A^2 N(N^2-1)} \quad (8)$$

which is the same as the variance of the maximum-likelihood estimate of the frequency of a stationary sinusoid in white, Gaussian noise.[5] The DWD peak is therefore an optimal estimator of IF for linear FM signals in WGN at high SNRs. As the SNR decreases a threshold effect is expected to occur as in the case of the DFT estimate of a stationary sinusoid in noise. The threshold effect is due the phenomenon (known as outliers) of frequency estimates falling outside the main lobe in the DWD spectrum and on one of the minor maxima leading to a sharp increase in the mean squared error (MSE). The probability of occurrence of an outlier is related to the SNR in the spectrum, and it is known from the DFT case that the threshold SNR below which the MSE starts to increase abruptly is about 15 dB.[5] The SNR in the DWD is related to the input SNR by eqn. 5. Hence the threshold SNR for the DWD peak estimator is given by

$$\mathrm{SNR}_{DWD} = \frac{A^4 \times (N/2)}{2A^2\sigma_z^2 + \sigma_z^4} = 15\,\mathrm{dB} \quad (9)$$

which at high input SNRs ($A^2 \gg \sigma_z^2$) reduces to the condition

$$\frac{A^2 N}{\sigma_z^2} = 21\,\mathrm{dB} \quad (10)$$

Some reduction in the probability of an outlier is obtained by directly using the real valued DWD rather than its magnitude to estimate the peak value. Computer simulations indicate that this reduction is slight (between 0·5–0·75) resulting in a nearly insignificant effect on the MSE.

In the case of nonlinear FM signals, the DWD peak estimator is typically biased with the amount of bias depending on the extent of the nonlinearity (or more accurately, on the deviation of the IF law from skew-symmetry within the data window[6]). The bias arises in the attempt to apply a least-square's fit of a constant frequency sinusoid to the kernel which is no longer narrowband. For an arbitrary nonlinear FM signal, the bias can be controlled by suitably windowing the input signal before generating the DWD. For a slowly varying FM signal, the window length can be chosen so that the IF law is practically linear within the window. In this case the performance results derived in this section can be directly applied. This approach reduces the bias at the expense of an increase in the variance of the estimate due to the increased main lobe width in the DWD. Alternative window types such as, for example, the Gaussian window can be employed in order to minimise the trade-off between bias and variance of the estimator.

Computer simulations: Computer simulations were used to verify the performance of IF estimation using the DWD peak. The MSE at various input SNRs for a linear FM signal in WGN was estimated by taking the average of the squared errors between the actual and estimated frequencies, obtained over between 300 to 3000 trials. A 64-sample data window was used and a large, zero-padded FFT computed in order to minimise errors in the IF estimate due to frequency quantisation (in practice an efficient interpolation scheme can be substituted). Fig. 1 compares the simulation results with the theoretical MSE given by eqn. 7. It is seen that at SNRs above

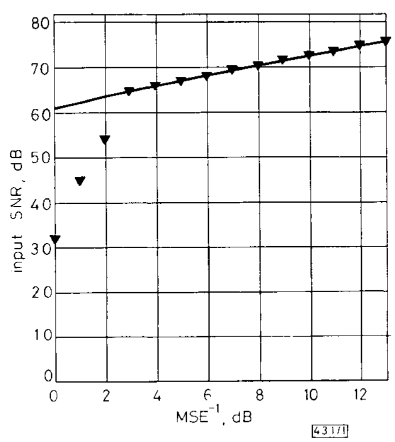

Fig. 1 *Performance of DWD peak estimator for linear FM signal in WGN*

$N = 64$

——— eqn. 7

▼▼▼ simulation results

the threshold the experimentally obtained values closely match the theoretical values of MSE. The threshold SNR is correctly predicted at 3 dB for the given value of N.

Summary: Analytical expressions for the performance of the DWD peak in the estimation of IF of a linear FM signal in WGN have been presented. The estimator is optimal at high SNRs above a threshold value. These results enable one to properly choose estimator parameters such as data window length and type under given input signal conditions.

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References

1 COHEN, L.: 'Time-frequency distributions—A review', *Proc. IEEE*, 1989, **77**, (7), pp. 941–981
2 BOASHASH, B., JONES, G., and O'SHEA, P.: 'Instantaneous frequency of signals: concepts, estimation techniques and applications', to appear in Advanced Algorithms and Architectures for Signal Processing IV, *Proc. SPIE*, 1990, **1152**
3 HARRIS, F. J., and SALEM, A. A.: 'Performance comparison of Wigner-Ville based techniques to standard FM-discriminators for estimating instantaneous frequency of a rapidly slewing FM sinusoid in the presence of noise', Advanced Algorithms and Architectures for Signal Processing III, *Proc. SPIE*, 1989, **975**, pp. 232–244
4 CLAASEN, T. A. C. M., and MECKLENBRAUKER, W. G. F.: 'The Wigner distribution—A tool for time-frequency signal analysis. Part II: discrete time signals', *Philips J. Res.*, 1980, **35**, pp. 276–300
5 RIFE, D. C., and BOORSTYN, R. R.: 'Single-tone parameter estimation from discrete-time observations', *IEEE Trans.*, 1974, **IT-20**, (5), pp. 591–598
6 FLANDRIN, P., and ESCUDIE, B.: 'An interpretation of the pseudo-Wigner distribution', *Signal Process.*, 1984, **6**, pp. 27–36

SINGLE SIDED SWITCHING NETWORKS

Indexing terms: Switching and switching circuits, Networks, Algorithms

A novel and simple architecture for realising single-sided, rearrangeably-nonblocking, N-port switching networks (N is a power of 2), that uses $N/2 \log (N/2)$ elements, together with an efficient routing algorithm with time complexity $O(N \log (N))$ is presented. The networks also exhibit a useful measure of fault tolerance.

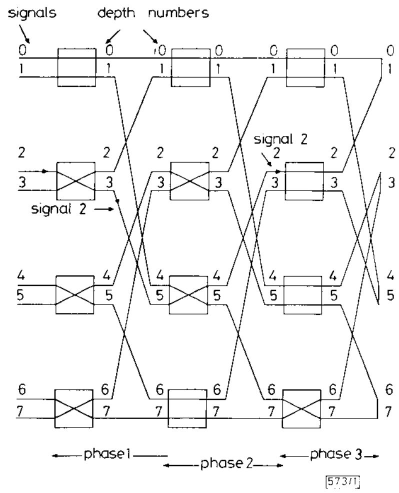
Introduction: Matrix switches provide a dynamic architecture invaluable for controlling the connectivity of networks. They find numerous applications within computer and communication centres in rearranging the configuration of processors, terminals and communications entities, and also as a dynamic connectivity tools within parallel processors.

Since crosspoint architecture requires $O(N^2)$ switch elements, recent work has concentrated on multistage networks using fewer components consisting of exchange elements (also called beta-elements, or swoppers). Most authors have worked with double sided (DS) networks[1] which interconnect only between ports on opposite sides. Here we are concerned with single sided (SS) rearrangeably nonblocking, N-port networks which provide dynamic connectivity for any of the $(N-1) \times (N-3) \times (N-5) \times \ldots \times 3 \times 1 = (N-1)!!$ possible interconnection patterns of pairs of ports.

An SS architecture has been proposed by Osatake and Ogawa[2] resembling a folded DS network. Hill[3] recently described an unusual looking and ingenious SS architecture for optical networks for which there is, unfortunately, no known routing algorithm. Both these designs use $(N/2) \log (N/2)$ elements (all logarithms are base 2). The networks described here exhibit simplicity, universality, modularity and fault tolerance. We describe a fast, routing algorithm with time complexity $O(N \log (N))$.

Theory: Since there are $(N-1)!!$ possible, distinct connection patterns, no universal, SS, N-port network can be constructed with less than $\log [(N-1)!!]$ independent, binary, switching elements. This number is asymptotically close to $(N/2) \log (N/2)$.

A typical single sided network is illustrated in Fig. 1. Representations of this kind will be called routing diagrams. Signals pass through the network from left to right, then through a 'short circuit' at the right before returning from right to left.



Let $N = 2^n$, each of the N signals (numbered 0 to $N-1$) enters the network at a port (numbered the same as the signal) then passes through n 'phases', 0 to $n-1$. Each phase consists of a bank of swoppers followed by a wiring permutation. It is convenient to use phase and depth number for the horizontal and vertical position in the routing diagram, where the depth number (0 to $N-1$) of a given signal at any phase is just the vertical position, measured in wires counted from the top of the routing diagram. It is written in binary.

The architecture illustrated in Fig. 1 was carefully chosen to generate critical relationships (*a*) between the wiring permutation and the depth number, and (*b*) between the swopper state and depth numbers. In particular, (*a*) the wiring permutations cause a right-rotation of the bits of the (binary) depth number, and (*b*) the effect of swoppers in the swopped state is to complement the least significant bits of the depth number of both signals passing through it, while a swopper in the non-swopped state has no effect on depth number.

Theorems: First we briefly describe two algorithms (algorithm-1 and algorithm-0) by means of the two theorems upon which they are based. These are in effect extensions of the 'looping algorithm' (of, for example, Huang[4] and Opferman and Tsao-Wu[5]).

Both algorithms are described by means of a topological structure we will call an (N, E, e)-net: a network consisting of N vertices numbered 0 through $N-1$ ($N = 2^k$) and two sets $\{E\}$ and $\{e\}$, each consisting of $N/2$ edges, each edge joining two vertices and each vertex connecting two edges, one edge from each set. No vertex is connected by two edges from the same set.

Theorem 1: For any (N, E, e)-net, it is possible to assign a binary digit (0 or 1) to each vertex such that (*a*) all vertices joined by a common edge in $\{E\}$ have opposite parity, and (*b*) vertices joined by a common edge in $\{e\}$ also have opposite parity.