Nonlinear Op-amp Circuits

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Overview of op-amp operating regions

Linear Region

- Occurs when the op-amp output is stable i.e. does not tend to saturate
- Low gain ($A_V < A_O$) and stable phase characteristics
- Negative feedback exceeds positive feedback

Nonlinear Region

- The output of the op-amp is unstable i.e. tends to saturate
- Positive feedback exceeds negative feedback
- Virtual short (ground) concept **does not** hold

Test for Linearity - I

How do we test if a circuit is behaving in the linear region? Theoretically, we say that if an input $v_i(t)$ produces an output $v_o(t)$ such that, $v_o(t) = av_i(t)$ where a is any real number, the circuit is linear.

Practically, a signal can have mutliple frequency components (e.g. a square wave has infinite no. of frequency components, all integral multiples of a base frequency f_0. A 100 Hz square wave has frequency components at 100 Hz, 200 Hz, 400 Hz etc.)

Thus, a circuit is said to be nonlinear if it **takes away** or **adds some** frequencies from/to the input signal. This gives rise to frequency distortions. Have you noticed that some circuits change the waveform shape completely?

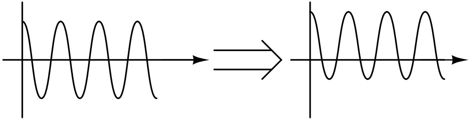
Test for Linearity - II

The best test for linearity is to give a sine wave an an input. Why?

The sine wave is the 'purest' wave i.e. it has only one frequency component. If the system output is anything but a sine wave of the same frequency, it is surely nonlinear.

It is interesting to note that any periodic waveform can be obtained by superimposing sine waves of different amplitudes, and frequencies being integral multiples of a 'base frequency'. It is for this reason that mathematician Joseph Fourier considered a sine wave as a basis for his famous series!

Now consider the following input-output set. Is the system linear?



Examples of Nonlinear Op-amp Circuits

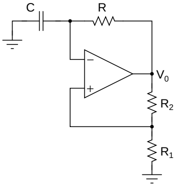
- Comparators and Schmitt Triggers- where outputs swing to saturation depending on amplitudes of an input signal.
- Multivibrators- where outputs swing to saturation based on a timing constraint verification (usually an $R - C$ or $R - L$ time constant)
- Oscillators- where the circuit generates a sustained oscillation to produce (usually) a sine wave. **Caution:** Most people think that oscillators are linear circuits. In fact, they are not, since an output is produced by giving no input. They are also highly unstable.
- Precision rectifiers- where an input sine wave in conditioned to produce a half- or full-wave rectification. There are added frequency components to the wave, hence nonlinear.

Astable Multivibrator - I

Here the circuit switches between $\pm V_{sat}$, having no stable state, hence *astable*.

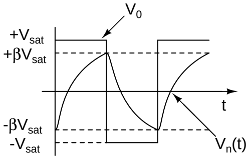
The voltage at the noninverting input V_p is

$$V_p = \frac{R_1}{R_1 + R_2} V_0 = \beta V_0$$



V_0 will switch state when the inverting input V_n crosses V_p. Assume that initially the capacitor is charged to $-\beta V_{sat}$ i.e. $V_n = -\beta V_{sat}$. At this point, V_0 switches to $+V_{sat}$ and $V_p = +\beta V_{sat}$. Now, the capacitor tries to charge up to the source voltage $+V_{sat}$. The capacitor voltage $V_n(t)$ is thus

$$V_n(t) = V_{sat}(1 - e^{-\frac{t}{RC}}) - \beta V_{sat} e^{-\frac{t}{RC}}$$



The final value of V_n will be $+\beta V_{sat}$, since at this voltage, the output will switch again to $-V_{sat}$. At this point,

$$\beta V_{sat} = V_{sat}(1 - e^{-\frac{t}{RC}}) - \beta V_{sat} e^{-\frac{t}{RC}}$$

$$\therefore t = RC \ln\left(\frac{1+\beta}{1-\beta}\right) \qquad (1)$$

Astable Multivibrator - II

The time expression derived in equation (1) shows half the time period, i.e. only for the capacitor charging. The total time period is then

$$T = 2RC \ln \left(\frac{1+\beta}{1-\beta} \right)$$

Putting back $\beta = \frac{R_1}{R_1+R_2}$, we get the time period as

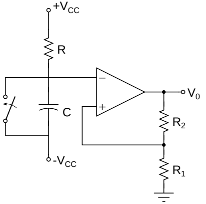
$$T = 2RC \ln \left(1 + \frac{2R_1}{R_2} \right) \tag{2}$$

Monostable Multivibrator

Here, the circuit has a single stable state and switches to an unstable state for some time, depending on the $R - C$ network.

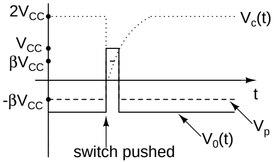
The voltage at the noninverting input V_p is

$$V_p = \frac{R_1}{R_1 + R_2} V_0 = \beta V_0$$



In the steady state when the switch is OFF, the voltage at the inverting input V_n will be $+2V_{CC}$, i.e. capacitor stays fully charged. Hence, $V_0 = -V_{sat}$. This is the **stable state output.** Now if the switch made is OFF and then ON, the capacitor will try to charge up to $+2V_{CC}$. When the capacitor voltage $V_c(t) < V_p$, the output will be $+V_{sat}$. V_0 will swing back to the stable output when $V_c(t)$ crosses $+\beta V_{CC}$ (Assume $|V_{CC}| \approx |V_{sat}|$). Now,

$$V_c(t) = 2V_{CC}(1 - e^{-\frac{t}{RC}}) - 2V_{CC}e^{-\frac{t}{RC}}$$



Thus, the time for which the output stays high (unstable) is given by

$$T = RC \ln \left(\frac{4}{2 - \beta} \right) \tag{3}$$

Food for Thought

- Are op-amp based differentiators and integrators linear or nonlinear circuits?
- Try to predict the behaviour of a monostable multivibrator if the switch is kept in ON position for a long time.
- Suppose we design an astable multivibrator for a time period of 10ms. Keeping component values unchanged, the supply voltage is varied from $\pm$15V to $\pm$12V. How will the time period change?
- We wish to design an auto-controlled monostable multivibrator. Can you think of devices which can be used in place of the switch? How will they be controlled?