Transformer Fundamentals

1 Introduction

The physical basis of the transformer is mutual induction between two circuits linked by a common magnetic field. Transformer is required to pass electrical energy from one circuit to another via pulsating magnetic field, *as efficiently and economically* as possible. This could be achieved using either iron or steel which serves as a good permeable path for the mutual magnetic flux. An elementary linked circuit is shown in Fig 1. The principle of operation of this circuit can be explained as follows:

Let an alternating voltage v_1 be applied to a primary coil of N_1 turns linking a suitable iron core. A current flows in the coil, establishing a flux ϕ_p in the core. This flux induces an emf e_1 in the coil to counterbalance the applied voltage v_1. This e.m.f. is

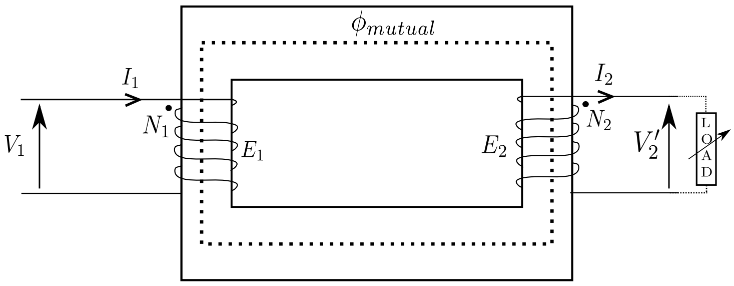

Figure 1: Elementary transformer

$$e_1 = N_1 \frac{d\phi_p}{dt} \tag{1}$$

Assuming sinusoidal time variation of the flux, let $\phi_p = \Phi_m \sin \omega t$. Then,

$$e_1 = N_1 \omega \Phi_m \cos \omega t, \quad \text{where} \quad \omega = 2\pi F$$

The r.m.s. value of this voltage is given by:

$$E_1 = 4.44 F N_1 \Phi_m$$

$\overline{V}_1$ and $\overline{E}_1$ then Now if there is a secondary coil of N_2 turns, wound on the same core, then by mutual induction an emf e_2 is developed therein. The r.m.s. value of this voltage is given by:

$$E_2 = 4.44 F N_2 \Phi'_m$$

where Φ'_m is the maximum value of the (sinusoidal) flux linking the secondary coil (ϕ_s).
If it is assumed that $\phi_p = \phi_s$ then the primary and secondary e.m.f.'s bear the following ratio:

$$\frac{e_1}{e_2} = \frac{\overline{E}_2}{\overline{E}_1} = \frac{N_2}{N_1}$$

Note that in actual practice, $\phi_p \neq \phi_s$ since some of the flux paths linking the primary coil do not link the secondary coil and similarly some of the flux paths linking the secondary coil do not link the primary coil. The fluxes which do not link both the coils are called the "leakage fluxes" of the primary and secondary coil.

In a practical transformer a very large proportion of the primary and secondary flux paths are common and leakage fluxes are comparatively small. Therefore $\phi_p \approx \phi_s = \phi_{mutual}$ and therefore $\Phi_m \approx \Phi'_m$.
If in addition, winding resistances are neglected – being usually small in a practical transformer, then

$$\overline{V}_1 \approx \overline{E}_1$$

Similarly,

$$\overline{V}_2 \approx \overline{E}_2$$

2 Magnetization and associated losses

Although the iron core is highly permeable, it is not possible to generate a magnetic field in it without the application of a small m.m.f.(magneto-motive force), denoted by M_m. Thus even when the secondary winding is open circuited, a small magnetizing current (i_m) is needed to maintain the magnetic flux. The current of the primary circuit on no-load is of the order of 5% of full load current.
Also, the pulsation of flux in the core is productive of core loss, due to hysteresis and eddy currents. These losses are given by:

$$P_h = K_h B_{max}^{1.6} F, \qquad P_e = K_e B_{max}^2 F^2 \qquad \text{and } P_c = P_h + P_e$$

where P_h, P_e and P_c are hysteresis, eddy current and core losses respectively, K_h and K_e are constants which depend on the magnetic material, and B_{max} is the maximum flux density in the core. These losses will remain almost constant if the supply voltage and frequency are held constant. The continuous loss of energy in the core requires a continuous supply from the electrical source to which the primary is connected. Therefore, there must be a current component i_c which accounts for these losses. It should be noted that magnetizing current (i_m) and core loss component of current(i_c) are in phase quadrature. The resultant of these two currents is the no-load current i_o. Generally the magnitude of this current is very small compared to that of the rated current of the transformer (may be of the order of 5% of the rated). This current makes a phase angle ζ_o of the order of ($\cos^{-1}(0.2)$) with the applied voltage.

3 Ideal transformer

If a load of finite impedance is connected across the second coil, a current i_2 will flow through it. This tends to alter the mmf and thereby the flux in the core. But this is prevented by an immediate and automatic adjustment of the primary current i_1, thereby maintaining the flux ϕ at the original value. This value of flux is required to produce the emf of self induction e_1. Any reduction of the flux would cause a reduction of e_1, leaving a voltage difference between v_1 and e_1 which would be sufficient to increase the primary current and thereby re-establish the flux. Thus any current which flows in the secondary causes its counterpart to flow in the primary so that the flux ϕ (and therefore the mmf - M_m) shall always be maintained at a value such that the voltage applied v_1 to the primary terminals shall be balanced by the induced emf e_1 (neglecting voltage drops due to resistance and leakage flux effects). Thus if current flows in the secondary (i_2), then $i_1 = i_o + \frac{N_2}{N_1} i_2$ so that effective mmf in the core remains at M_m. In phasor notation:

$$\overline{I}_1 = \overline{I}_o + \frac{N_2}{N_1}\overline{I}_2$$

$\overline{I}_o$ is quite small compared to the rated current and is usually neglected if transformer is loaded. Thus:

$$\overline{I}_1 \approx \tfrac{N_2}{N_1}\overline{I}_2$$

It is therefore, evident that energy is conveyed from the primary to secondary by the flux: the primary stores the energy in the magnetic field, and an extraction of some of this for the secondary load is made up by the addition of energy from the primary, which consequently takes an increased current.

Thus by making the assumptions :

- Winding resistances are small
- Magnetising current is small
- Core losses are small
- Leakage fluxes are small

we can infer that (for an "ideal transformer"):

$$\frac{\overline{I}_1}{\overline{I}_2} = \frac{N_2}{N_1} = \frac{\overline{E}_2}{\overline{E}_1} = \frac{\overline{V}_2}{\overline{V}_1} \tag{2}$$

3.1 Equivalent Circuit of a practical Transformer

The practical transformer has coils of finite resistance. Though this resistance is actually distributed uniformly, it can be conceived as concentrated. Also, all the flux produced by the primary current cannot be confined into a desired path completely as an electric current. Though a greater proportion links both the coils(known as mutual flux), a small proportion called the leakage flux links one or other winding, but not both. It does not contribute to the transfer of energy from primary to secondary. On account of the leakage flux, both the windings have a voltage drop which is due to 'leakage reactance'. The transformer shown in Fig. 1 can be resolved into an equivalent circuit as shown in Fig. 2 in which the resistance and leakage reactance of primary and secondary respectively are represented by lumped R_1, X_1, R_2 and X_2. This equivalent circuit can be further simplified by referring all quantities in the secondary side of the transformer to primary side and is shown in Fig. 3. These referred quantities are given by:

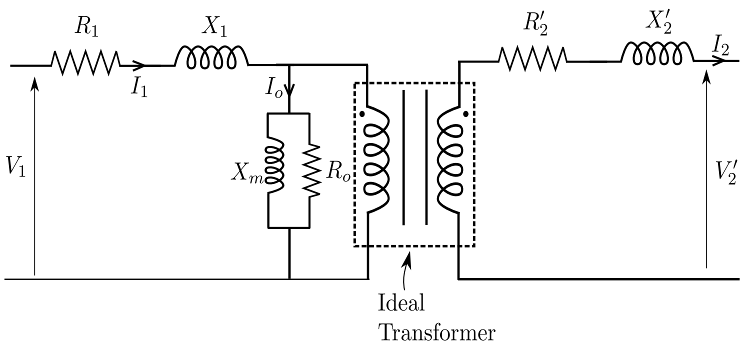

Figure 2: Equivalent circuit of a transformer

$$R_2' = R_2(\tfrac{N_1}{N_2})^2 \qquad X_2' = X_2(\tfrac{N_1}{N_2})^2 \qquad I_2' = I_2(\tfrac{N_2}{N_1}) \qquad V_2' = V_2(\tfrac{N_1}{N_2})$$

Generally the voltage drops I_1R_1 and I_1X_1 are small and magnitude of $\overline{E}_1$ is approximately equal to that of $\overline{V}_1$. Under this condition, the shunt branch (comprising X_m and R_o) can be connected across the supply terminals. This approximate equivalent circuit (shown in Fig. 4) simplifies the computation of currents and other performance characteristics of a practical transformer.

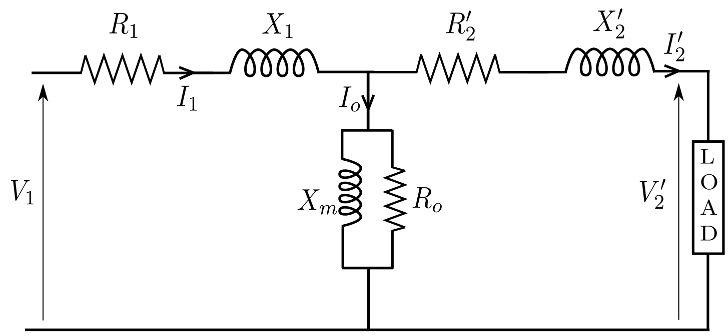

Figure 3: Reduced equivalent circuit of a transformer

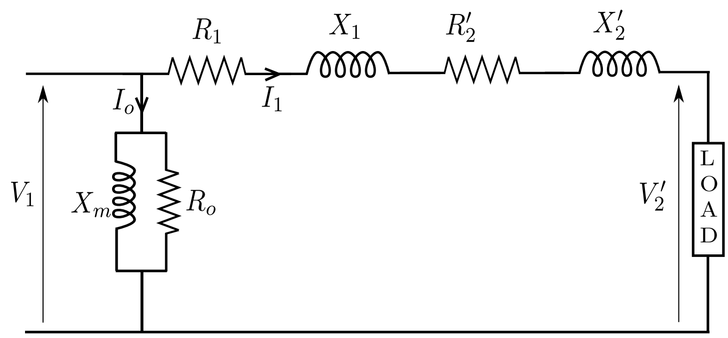

Figure 4: Simplified equivalent circuit of a transformer

4 Efficiency

Efficiency of the transformer is defined as:

$$\eta = \frac{\text{output power}}{\text{input power}}$$

Interms of losses,

$$\eta = \frac{\text{output power}}{\text{output power+ iron losses+copper losses}}$$

Let ‘S’ be the rated VA of the transformer, ‘x’ is the fraction of full load the transformer is supplying, and ζ is the load power factor angle. Under this condition the output power of the transformer is $= x.S.\cos\zeta$. If P_c is the copper loss (loss in winding resistance) at rated current, the corresponding loss while supplying the fraction of load is $= x^2.P_c$. With transformers of normal design, the flux in the core varies only a few percent between no-load to full load. Consequently it is permissible to regard the core loss (iron loss) as constant, regardless of load. Let this loss be P_i. Therefore equation becomes :

$$\eta = \frac{x.S.\cos\zeta}{x.S.\cos\zeta + P_i + P_c.x^2}$$

5 Regulation

From Fig. 4 it can be seen that if the input voltage is held constant, the voltage at the secondary terminals varies with load. Regulation is defined as the change in magnitude of secondary (terminal) voltage, when the load is thrown off with primary voltage held constant. Since, the change in secondary voltage depends only on the load current, the equivalent circuit is further simplified and is shown in Fig. 5. The phasor diagrams for lagging, unity and leading powerfactor loads are shown in Fig. 6 to

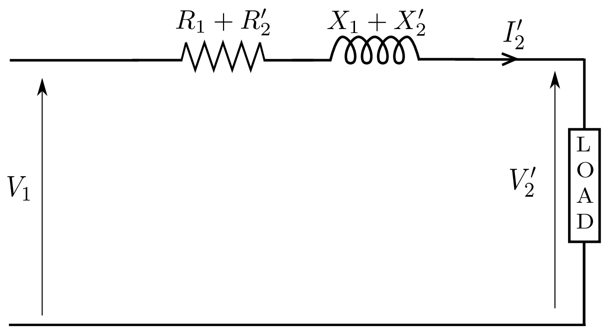

Figure 5: Equivalent circuit to determine regulation

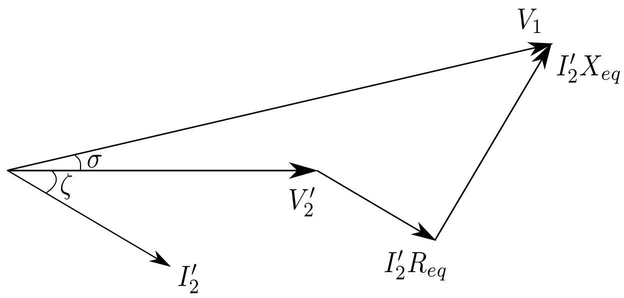

Figure 6: Phasor diagram for lagging power factor

Fig. 8. It can be proved that angle σ is very small and can be neglected. In that case, the expression for regulation is given by

$$\%\text{regulation} = \frac{I_2'.R_{eq}.\cos\zeta \pm I_2'.X_{eq}.\sin\zeta}{V_2'} \times 100 \qquad \text{...`+' for lagging pf and `$-$' for leading pf} \quad (3)$$

where, I_2'=load current, $R_{eq} = R_1 + R_2'$, $X_{eq} = X_1 + X_2'$,

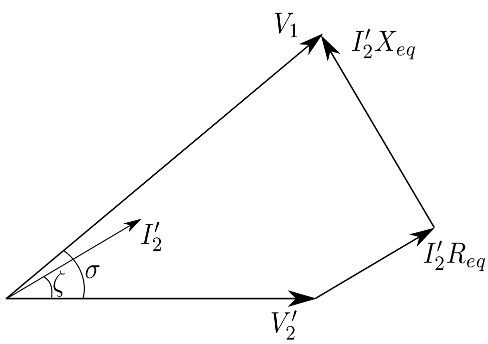

Figure 7: Phasor diagram for leading power factor

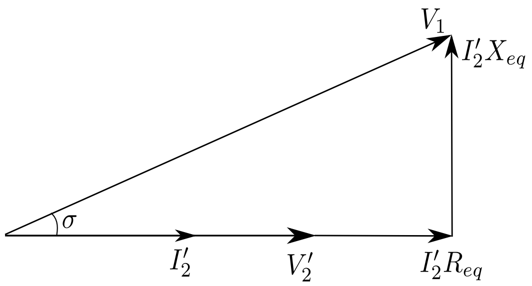

Figure 8: Phasor diagram for unity power factor

Note to TAs/RAs: Open the cover of the transformer and show the students HV and LV terminals, conductors used for LV and HV winding. Also show them E & I laminations, and ferrite core. Also, conduct the no-load test on high frequency ferrite core transformer at

• 50 Hz (using single phase ac source). Ask the students to observe the deflection on the ammeter as you increase the voltage. Observe the current waveform on the power analyzer. Note down the magnitude of applied voltage.

• about 10 kHz (use the signal generator) and observe the waveforms on the storage oscilloscope. Note down the magnitude of applied voltage.

[1]

[1]This experiment/chapter is prepared by *Makarand M Kane* (Research Scholar, 2015 batch).