Multi-Layer Wireless Relay Networks

Submitted in partial fulfillment of the requirements
of the degree of

Doctor of Philosophy

by

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Dedicated to my beloved Amma and Appa - Selvi and Elamvazhuthi

Amma,

“I used to be Appa’s boy - who took good care of me
Vacuum came suddenly - when nature took him early
You toiled through those hard days - when I was too small to understand
Did your best - could compensate for the huge loss
Brought me up to this level - made me all that I am worth
Words can never match your deeds - I dedicate this thesis as a token of love”

Selvam
Thesis Approval for Ph.D.

This thesis entitled "Multi-Layer Wireless Relay Networks" by Pannir Selvam Elamvazhuthi is approved for the degree of Doctor of Philosophy.

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I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be cause of disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

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(Pannir Selvam Elamvazhuthi)
Abstract

The concept of multi-layer wireless relay networks is being used in many applications such as wireless sensor networks and multi-hop cellular networks. Though the severe path-loss experienced by a signal travelling through a long-distance wireless link is overcome using multi-hop relays, relay forwarding techniques should be designed carefully due to the accumulation of noise or decoding error. This thesis addresses optimum processing of the signal at the relays, which employ the amplify-and-forward (AF) technique.

Some of the earlier literature that dealt with AF relay forwarding, have obtained a precoder matrix to be used by the relay layer, by minimizing the mean-squared error (MSE) at the destination and hence obtaining low bit-error-rate (BER). It has been possible to design such optimum precoder matrices for two-hop systems, having one layer of relays between the source and the destination. It is difficult to extend it to a multi-layer system. It is also difficult to extend it to accommodate more than one source-destination pair - when closed-form solution of the precoder is needed. Another drawback of the existing schemes is that the relays are presumed to have channel-state-information (CSI) from source to themselves (called backward CSI in this thesis) and from themselves to the destination (forward CSI). In many relay systems, the assumption of forward CSI is not practical.

All the above drawbacks of the existing schemes are overcome by the work in this thesis. First, we propose three ad hoc schemes constructed heuristically. Two of the schemes do not use CSI, while the third uses the local backward CSI (it has no knowledge of CSI of other relays present in its own layer). These proposed schemes are shown to perform better than the existing ad hoc systems. Hence, they can be used when CSI is not available or when only the local backward CSI is present.

We next address the problem of designing optimum precoders. To start with, we highlight the difficulty in obtaining precoders when multiple relay layers are present, while adopting the standard technique from existing literature - minimizing MSE at the destination. Then we
propose a technique, minimizing the MSE at the relays. It breaks down the problem from finding all the precoders together at the destination, to solving one at a time at each of the relay layer stages. This enables us to obtain analytical solutions for the optimal precoders for a multi-layer system. An added advantage is that the precoders thus obtained do not depend on the forward CSI.

In a multi-layer relay network, there are attenuated signals that arrive at a particular layer from distant transmitting layers. Unlike most of the literature available, we make use of these *leaked* signals obtained through the *weak* links and compensate the handicap of not having forward CSI at the relays.

Closed-form solutions of the relay precoder matrices are obtained for both the cases - when the relays can cooperate (i.e., in a particular layer, the relays can exchange information amongst themselves) or not. Using simulations, we show that the BER performance of the proposed scheme with only backward CSI, approaches that of an existing scheme that uses both backward and forward CSI efficiently. This is made possible by using more and more of the leaked signals from distant layers at the relays, enabling the destination receiver to decode the signal with less errors. We also show that when the path loss condition worsens, our scheme outperforms the existing schemes.

Thus, the proposed scheme is a good option to select when there are a large number of relays spread in a wide area, (1) with multiple layers and multiple relays in each of them and/or multiple source-destination pairs, (2) when the relays can cooperate amongst themselves or not, and (3) when the relays do not have forward CSI.

**List of keywords**

MIMO, cooperative communication, channel-state-information, amplify-and-forward, multi-layer relay network, relay precoder, MMSE
Contents

Abstract vii

List of Figures xiii

List of Tables xvii

1 Introduction 1

1.1 State-of-the-art on MIMO/cooperative communication . . . . . . . . . . . . . 4
1.2 Motivation and the scope of the thesis . . . . . . . . . . . . . . . . . . . . . . 5
1.3 Summary of contribution . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8
1.4 Organization of the thesis . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 9

2 System Model and Literature Review 11

2.1 System model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 12
2.2 Literature review . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 15

2.2.1 Ad hoc precoder systems . . . . . . . . . . . . . . . . . . . . . . . . . . 16
2.2.2 Optimal precoder systems . . . . . . . . . . . . . . . . . . . . . . . . . 18

2.3 Limitations in the existing systems . . . . . . . . . . . . . . . . . . . . . . . . 21

2.3.1 Ad hoc precoder systems . . . . . . . . . . . . . . . . . . . . . . . . . . 21
2.3.2 Optimal precoder systems . . . . . . . . . . . . . . . . . . . . . . . . . 22

3 Simple Ad Hoc Schemes 25

3.1 Relay matrix combining scheme . . . . . . . . . . . . . . . . . . . . . . . . . 29

3.1.1 Transmit and receive vectors . . . . . . . . . . . . . . . . . . . . . . . . 30
3.1.2 Maximum-likelihood decoder . . . . . . . . . . . . . . . . . . . . . . . . 33
3.1.3 Receive SNR . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 36
3.2 Relay SNR combining scheme ............................................ 38
  3.2.1 Transmit and receive vectors ..................................... 38
  3.2.2 Maximum-likelihood decoder .................................... 41
  3.2.3 Receive SNR ..................................................... 43
3.3 Relay matrix combining with known channel scheme ............... 44
  3.3.1 Transmit and receive vectors ..................................... 44
  3.3.2 Maximum-likelihood decoder .................................... 48
  3.3.3 Receive SNR ..................................................... 50
3.4 Extended Jing-Hassibi scheme ......................................... 50
  3.4.1 Transmit and receive vectors ..................................... 51
  3.4.2 Maximum-likelihood decoder .................................... 53
  3.4.3 Receive SNR ..................................................... 53
3.5 Modified Jing-Hassibi scheme ........................................... 55
  3.5.1 Transmit and receive vectors ..................................... 56
  3.5.2 Maximum-likelihood decoder .................................... 57
  3.5.3 Receive SNR ..................................................... 58
3.6 Signal-power path loss .................................................. 60
3.7 Optimum power allocation ............................................... 61
3.8 Simulations and results .................................................. 68
3.9 Discussion and observations ............................................. 73

4 Optimum MMSE Schemes ................................................... 75
  4.1 Single source-destination pair ....................................... 76
    4.1.1 Precoder matrix at $L_1$ ..................................... 79
    4.1.2 Precoder matrix at $L_k$, $k \in [2, K]$ ....................... 84
    4.1.3 MMSER precoders independent of forward CSI ............... 87
    4.1.4 Enhancement of MMSED schemes ............................... 87
    4.1.5 Decoder at the destination .................................... 94
    4.1.6 Simulations and results ....................................... 98
    4.1.7 Discussion and observations .................................. 107
  4.2 Multiple source-destination pairs ................................... 108
    4.2.1 Cooperative relays ......................................... 109
4.2.2 Noncooperative relays ........................................ 110
4.2.3 Selection of $C$, the combining matrix ..................... 111
4.2.4 Decoder at the destination .................................. 112
4.2.5 Simulations and results .................................... 112
4.2.6 Discussion and observations ................................. 115

5 Summary and Conclusion ........................................... 117
  5.1 Summary ....................................................... 117
  5.2 Open problems ............................................... 120
  5.3 Conclusion .................................................... 122

Appendices ............................................................. 122

A Proof of Claim 1: Cardinality of Link Sets ...................... 123
B Proof of Claim 2: Scaling Factor of RMCS ...................... 124
C Proof of Claim 3: Receive SNR of RMCS ...................... 126
D Proof of Claim 4: Receive SNR of RSCS ......................... 129
E Proof of Claim 5: Scaling Factor of RMCKCS .................. 132
F Proof of Claim 6: Receive SNR of RMCKCS .................... 135
G Proof of Claim 7: Scaling Factor of EJHS ....................... 138
H Proof of Claim 8: Optimum Precoder at $L_1$ for MMSER Scheme When the Relays Cooperate ......................... 140
I Proof of Claim 9: Shortcut Method to Derive Optimum Precoder ......................................................... 141
J Proof of Claim 10: Simplified Optimum Precoder at $L_1$ for MMSER Scheme When the Relays Cooperate ............... 142
K Proof of Claim 11: Diagonal Element of the Optimum Precoder at $L_1$ for MMSER Scheme When the Relays Do Not Cooperate ......................................................... 144
L  Proof of Claim 12: Optimum Precoder at $L_k$ for MMSER Scheme When the Relays Cooperate  146

M  Proof of Claim 13: Optimum Precoder at $L_k$ for MMSER Scheme When the Relays Do Not Cooperate  147

N  Expressions of the Correlation Matrices $R_{sr_k}$ and $R_{r_k}$  150
   N.1 Correlation matrix $R_{sr_k} \in \mathbb{C}^{N \times (k-n)}$, $k \in [1, K]$  150
   N.2 Auto correlation matrix, $R_{r_k} \in \mathbb{C}^{(k-n)N \times (k-n)N}$, $k \in [1, K]$  151

O  Proof of Claim 14: Precoder of E-MMSE-D-L  154

P  Proof of Claim 15: SNR of E-MMSE-K and L  158
   P.1 E-MMSE-K  159
   P.2 E-MMSE-L  159

Q  Proof of Claim 16: Optimum Precoder at $L_k$ for MMSER Scheme When the Relays Do Not Cooperate for $M > 1$ Case  160

Bibliography  163

Publications  168
# List of Figures

1.1 Spatial multiplexing in a $2 \times 2$ MIMO system. ........................................ 2
1.2 A group of mobile phones in a multi-hop cellular system. Firm and dashed lines indicate low and high path loss links respectively. ........................................ 6

2.1 A group of radio nodes located in a planar disc of unit area. Square shaped nodes shown on the left, $S_1, \ldots, S_M$ are the sources and the crosses on the right, $D_1, \ldots, D_M$ are the corresponding destinations. Remaining nodes can be used as relays. ........................................ 11
2.2 A multi-layer relay network. Here $S_m$, $L_k$, $R_{ki}$, $D_m$, and $\rho_{ij}$ represent the $m$th source, $k$th layer, $i$th relay in $L_k$, the $m$th destination, and the length of the links from $L_i$ to $L_j$ respectively. *In this figure, the suffix is avoided in $\rho$ for simplicity.* 13
2.3 System model with one layer of relays and one source-destination pair. .......... 16
2.4 System model with one layer of relays and multiple source-destination pairs. ... 19

3.1 System model with two layers of relays and one source-destination pair. ........ 25
3.2 Various phases in RMCS/RSCS/RMCKCS. .................................................. 29
3.3 Various phases in EJHS. ................................................................. 51
3.4 Various phases in MJHS. ................................................................. 55
3.5 A simplified system model showing *lengths* and distances of various links with their channel variances. Here, $\beta$ is the path loss exponent and this model is used only for simulations. .................................................. 61
3.6 Search for the optimum power point ($\hat{p}_0$, $\hat{p}_1$, $\hat{p}_2$) in the hashed surface. 62
3.7 Receive SNR for RMCS-2 and RMCS-3 for $\beta = 2$. ................................. 63
3.8 Receive SNR for RSCS-2 and RSCS-3 for $\beta = 2$. ................................. 63
3.9 Receive SNR for RMCKCS-2 and RMCKCS-3 for $\beta = 2$. ........................ 64
3.10 Receive SNR for EJHS and MJHS for $\beta = 2$. ............................................. 64
3.11 Plot of optimum power allocations for RMCS-3, RSCS-3, RMCKCS-3, and MJHS for $\beta = 2$. .......................................................... 65
3.12 Plot of optimum power allocations for RMCS-3, RSCS-3, RMCKCS-3, and MJHS for $\beta = 3$. .......................................................... 66
3.13 Plot of optimum power allocations for RMCS-3, RSCS-3, RMCKCS-3, and MJHS for $\beta = 4$. .......................................................... 67
3.14 Comparison of JHS when unitary and orthogonal matrices are used. ............. 69
3.15 Comparison of BER of the schemes when $\beta = 4$. ........................................ 70
3.16 Comparison of BER of the schemes when $\beta = 3$. ........................................ 71
3.17 Comparison of BER of the schemes when $\beta = 2$. ........................................ 72

4.1 A multi-layer relay network. Here, $S_1$, $L_k$, $R_{ki}$, $D_1$, and $\rho$ represent the source, $k$th layer, $i$th relay in $L_k$, the destination, and the length of the corresponding link respectively. .................................................. 76
4.2 Transmission schemes showing link sets in $\mathcal{L}_3$ (dashed lines) for MMSER-3 and $\mathcal{L}_4$ (firm lines) for MMSER-4 with four relay layers, i.e., $K = 4$. It also shows how transmission and reception progress in various phases. Here, $L_0$ and $L_5$ represent the source and the destination respectively. ......................... 78
4.3 System model with two layers of relays and one source-destination pair for MMSEED. ................................................................. 88
4.4 Enhanced MMSEED scheme. Firm arrows show the links that are used when $S_1$ transmits and the dashed arrows show the links that are used when the relays transmit. ................................................................. 90
4.5 Plots of SNR and BER of MMSER-1 for varying number of layers ($K$) with total power $P = 1$ Watt. As $K$ is increased, due to noise getting amplified in this AF system, performance worsens unlike MMSER-$\mu$, $\mu \in [2, K + 1]$ where it is compensated by leaked signals. .................................................. 99
4.6 BER plots of MMSER-2 for varying number of layers ($K$) with total power $P = 1$ Watt. Unlike MMSER-1, the BER performance improves when $K$ is increased, as MMSER-2 uses leaked signals. ................................. 100
4.7 E-MMSEED-KK and E-MMSEED-L showing same BER performance for varying $K_0$. Total power used in the simulations is $P = 1$ Watt. ................................. 100
4.8 Search for optimum power allocation for MMSED. Shows that when power is equally distributed to $S_1$ and layers, BER performance of E-MMSED-KK is the best and when 50% of power is allocated to $S_1$, E-MMSED-L attains best BER performance. Total power used in the simulations is $P = 1$ Watt.

4.9 Cooperative relays performance comparison. Performances of MMSER-1 and MMSED-K are the same when $N = 1$, though MMSED-K uses global CSI as it is compensated in the decoder at $D_1$ for MMSER-1. For $N = 2$, the performance of MMSED-K is better than MMSER-1 and MMSER-2 schemes. Total power used in the simulations is $P = 2$ Watts.

4.10 Noncooperative relays performance comparison. BER plots of MMSER-3 when the precoder is obtained using ‘scalar optimization’ and ‘zeroising non diagonal’ are shown. We use $N = 2$ and $K = 3$ in the simulations. Total power used in the simulations is $P = 1$ Watt.

4.11 BER plots of EJHS, E-MMSED, and MMSER-$\mu$, $\mu \in [1-4]$. Performance of MMSER-$\mu$ is better than E-MMSED-KK when $\mu \geq 2$ and it approaches that of E-MMSED-L when $\mu$ is increased. Total power used in the simulations is $P = 1$ Watt.

4.12 BER plots of EJHS, E-MMSED, and MMSER-$\mu$, $\mu \in [1-5]$. Performance of MMSER-$\mu$ is better than E-MMSED when $\mu \geq 2$ and it approaches that of E-MMSED-L when $\mu$ is increased. For BER=$10^{-3}$, the SNR advantage of E-MMSED-L over MMSER is reduced from 7 dB (for MMSER-2) to 1 dB (for MMSER-5). Total power used in the simulations is $P = 1$ Watt.

4.13 BER plots of E-MMSED-L and MMSER-5 for various values of $\beta$ with $K = 4$ and $d = 5$. For high path loss conditions, MMSER-5 performs better than E-MMSED-L. Total power used in simulations is $P = 1$ Watt.

4.14 BER plots of MMSER-5 for multiple source-destination pairs. Performance of MMSER-5 becomes better when the number of source-destination pairs reduces. Total power used in the simulations is $P = 1$ Watt and the number of relays is $N = 4$.

4.15 BER plots of MMSER-4 and MMSER-5 for multiple source-destination pairs. Parameters used are $K = 4$, $M = 2$, and $N = 3$ with a total power used in the simulations is $P = 1$ Watt.
4.16 BER plots of MMSER-2 and E-MMSED-K for multiple source-destination pairs. Here, $K = 1$, $M = 2$, and $N = 4$ with a total power used in the simulations is $P = 2$ Watts.
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Transmitted vectors and scaling factors - RMCS</td>
<td>33</td>
</tr>
<tr>
<td>3.2</td>
<td>Transmitted vectors and scaling factors - RSCS</td>
<td>41</td>
</tr>
<tr>
<td>3.3</td>
<td>Transmitted vectors and scaling factors - RMCKCS</td>
<td>48</td>
</tr>
<tr>
<td>3.4</td>
<td>Transmitted vectors and scaling factors - EJHS</td>
<td>52</td>
</tr>
<tr>
<td>3.5</td>
<td>Transmitted vectors and scaling factors - MJHS</td>
<td>57</td>
</tr>
<tr>
<td>4.1</td>
<td>Transmitted and received signals - MMSER</td>
<td>79</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Communication has been a challenge to human beings, since the day they started to spread across geography from their original location of birth. The migration was required probably because they wanted to improve their life styles or for want of food - but remained social animals. This lead to their urge to communicate using some means with their relatives, who may be staying beyond their easy reach geographically.

When this need for long distance communication was necessitated, initial primitive forms of communication devices and later radios were invented. Radio technology has come a long way from electrostatic telegraphy [1], through vacuum tube technology, to presently where it exploits integrated on-chip artificial-magnetic-conductor antennas [2]. Also as electronic device technology is taking leaps and bounds, complex signal processing algorithms could be implemented in a tiny integrated circuitry and make the communications receiver more sophisticated.

Along with the growth of technology, luxuries become necessities and expectations from the communication system are much more. Scientists and communication engineers are also working over time and bringing highly sophisticated communication networks. For example, long-term-evolution (LTE) cellular phones operate at peak date rates of 300 Mega bit-per-second (Mbps) in downlink\(^1\) (DL) and 75 Mbps in uplink\(^2\) (UL) for a 20 MHz bandwidth and allows flexible bandwidth operation of upto 20 MHz [3]. In LTE-Advanced the data rates can go in excess of 1 Giga bit-per-second (Gbps) and 500 Mbps for DL and UL respectively for 100 MHz bandwidth [4]. Modern communication systems, like LTE, have to achieve best performance in spite of many challenges they face.

---

1 Downlink - Transmission from base station to the end-user mobile phone in a cellular network
2 Uplink - Transmission from mobile phone to the base station
Fading\(^3\) is one of the main challenges for these systems and diversity is a technique that effectively mitigates it. Diversity is to use replicas of the same signal and extract transmitted information by combining them in a particular fashion. As the probability that all the replicas fading away is smaller when the number of replicas is high, diversity helps to mitigate fading.

Diversity could be exploited in many dimensions, e.g. if the same data is transmitted in many frequencies, the probability that all of them would fade away is less and hence frequency diversity is used here. Similarly, the same information can be sent repeatedly in time and time diversity can be achieved. Space can be also used to get diversity and is called spatial diversity. Multiple-input multiple-output (MIMO) is the technology used to achieve spatial diversity, wherein multiple antennas are used at the transmitter and receiver. With multiple antennas at the receiver, receive diversity can take care of fading by combining multiple received signals from various antennas.

The other challenge is the need to accommodate many users or to get high data rates for few users in a communication system. Time-division multiplexing, Frequency-division multiplexing, and code-division multiplexing are some of the techniques used to allocate resources to users and allow communication in independent dimensions. Another form of multiplexing is obtained using MIMO and is called spatial multiplexing.

Consider a MIMO system with two antennas in the transmitter and two in receiver and let the channel coefficients be as shown in Figure 1.1. Assume that the transmitter transmits from its antennas 1 and 2 at the same time in the same frequency band. The receiver receives two

---

\(^3\)Fading - Drop of the received signal strength due to destructive interference of multi-path signals
signals in the antennas 1 and 2

\[ y_1 = h_{11}x_1 + h_{21}x_2 \quad \text{and} \quad y_2 = h_{12}x_1 + h_{22}x_2 \]

respectively, assuming that the system is noise free. The above equations can be written in matrix form as

\[ \mathbf{y} = \mathbf{Hx}, \quad (1.1) \]

where

\[ \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix}, \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \]

Let us also assume that the channel-state-information\(^4\) (CSI) is obtained at the receiver using prior pilot signals. Now, we can extract the spatially multiplexed transmitted signals \(x_1\) and \(x_2\) from (1.1), if \(\mathbf{H}\) has full rank [5, 6]. That is, equation (1.1) has a unique solution for \(\{x_1, x_2\}\) only if \(\mathbf{H}\) has independent row/column vectors. Hence, MIMO works in an environment where the channel coefficients are independent. To meet this condition, the antennas are to be placed some distance apart from each other.

Tse and Viswanath [7] explain under what conditions the spatial multiplexing gain or the degree-of-freedom (DoF) gain of the multiple antennas can be exploited. With \(L_t\) and \(L_r\) as the transmit and receive antenna array lengths respectively normalized by the wavelength of operation \(\lambda\), the DoF is shown to be \(\min\{2L_t, 2L_r\}\). This is applicable only if the signal is assumed to be arriving at the receiver along a continuum set of paths, uniformly from all directions. With equidistant placement of array elements and a separation between them \(d = \lambda/2\), we get the DoF as \(\min\{n_t, n_r\}\), where \(n_t\) and \(n_r\) are the number of antenna elements in the transmitter and receiver of the MIMO system respectively.

In the example shown in Fig. 1.1, spatial multiplexing gain = 2, as we have assumed that the distance between the antenna elements \(d = \lambda/2\) and the signal is arriving along a continuum set of paths uniformly from all directions. Hence two independent streams of data can be transmitted from the transmitter in the same frequency band at the same time, and they can be extracted out at the receiver.

Due to the separation requirement of the antennas, we cannot put more antennas in a small device like a mobile phone, which operates at around 1 GHz frequency with wavelengths of the order of 30 centimetres. Thus, it is difficult to keep the antennas away from each other to

\(^4\)The knowledge of channel coefficients \(h_{ij}, i, j \in \{1, 2\}\)
obtain uncorrelated receptions and take benefit of the multiplicity of antennas. This shortcoming necessitated the development of cooperative communication, which is a technique to simulate MIMO with many single-antenna radio nodes and create a virtual array of antennas.

When multiple radio nodes are present, the effectiveness of cooperative communication can be increased by arranging these nodes into a layered architecture to relay information. Though the path loss experienced by the multi-hop signal is not as severe as that of a direct signal received over a long distance, it would be difficult to decipher the information at the destination due to the high noise content or the accumulated decoder error. Hence relay forwarding technique needs to be carefully designed, which is the focus of this thesis.

In the next section, we will see a brief overview of some of the important work in literature on MIMO and cooperative communication.

1.1 State-of-the-art on MIMO/cooperative communication

Sendonaris et al. [8, 9] considered a system with one destination and two sources cooperating with each other to achieve better performance than that of non-cooperating sources. Distributed space-time coding (DSTC) proposed by Laneman and Wornell [10] used a space-time code (STC) at relays which does not require orthogonal channels to be allocated to various transmitting units. These units have single antenna each and the DSTC system achieved higher spectral efficiency than repetition-based schemes. Jing and Hassibi [11] used a system with a layer of relays between source and destination and proved that the relays make DSTC at the destination without having the knowledge of the transmitted signal. These authors have extended their work to include multiple antenna nodes in their system in [12, 13]. Sirkeci and Scaglione [14] showed how with randomized strategies, they could decentralize the transmission of STC while eliminating the need for internode communication or a central control unit and achieve diversity/coding gains.

The choice of the right STC in a distributed fashion depends on the requirement. Orthogonal space-time block coding [15] can be selected to maximize the diversity gain and minimize the receiver complexity. Codes [16–18] that maximize both diversity gain and transmission rate, but with a rather high receiver complexity are also available to choose from. Bell Laboratories layered space-time (BLAST) codes [19] or codes with trace-orthogonal design (TOD) [20] can also be selected. BLAST codes maximize the rate, sacrificing part of the diversity gain, but with
intermediate receiver complexity and the TOD has a flexible way to trade complexity, bit rate, and bit-error-rate (BER).

Borade et al. [21] used an amplify-and-forward (AF) strategy at the relays and analyzed the diversity-multiplexing trade-off when the number of layers increases in a multi-layer scenario. These authors used a fixed scaling factor at the relays while forwarding. Decode-and-forward (DF) (e.g. [10]), coded cooperation (e.g. [22]), compress-and-forward (e.g. [23,24]), and partial-decode-and-forward (e.g. [25]) are some of the other strategies used by the relays to forward the received signal. Out of these techniques, AF is the simplest which can be easily implemented in a relay, as it does not require decoding and requiring to carry out tasks that do not require processing power.

In this thesis, we will be discussing DSTC under the ad hoc schemes in Chapter-3 and AF is the strategy used in both the ad hoc schemes in Chapter-3 and optimum schemes in Chapter-4. Now, we will discuss why this research work has been undertaken and what is the scope, in the subsequent paragraphs.

1.2 Motivation and the scope of the thesis

Let us discuss an application where a multi-layer wireless relay network concept can be applied. We will use this and motivate as to why this research work is undertaken.

Consider a multi-hop cellular network [26], where a distant mobile phone could be assisted by other mobile phones to access the base station by relaying. Also they can help each other and thus form a multi-hop cooperative system. Let us assume that these mobile phones are clustered as shown in Fig. 1.2 to form layers of relays $L_i$, to convey information from left to right. Here, firm arrows indicate channel links from one layer to the next or a consecutive layer, while the dashed arrows indicate channel links from one layer to non-consecutive layers (we will call these links shown dashed as weak links). So, a signal which travels through a weak link would experience more power loss than those which travel through firm links. We will call these signals, which travel through weak links as leaked$^5$ signals. Also here, relays with same channel strength of their links are assumed to be grouped into the same layer.

We will call, at $L_k$, the knowledge of the CSI from $L_{k-j}$ to $L_k$ to be backward CSI and $L_k$ to $L_{k+j}$ as forward CSI, $j > 0$. Knowledge of both CSI at $L_k$ is said to be global CSI. The relays

$^5$Throughout this thesis, we will use this term leaked for denoting signals that travel through weak links
Figure 1.2: A group of mobile phones in a multi-hop cellular system. Firm and dashed lines indicate low and high path loss links respectively.

can use this knowledge while forwarding their received signals to improve the performance of the communication system. Knowledge of backward or forward CSI also means that all the relays in a particular layer have the information of the CSI of other relays as well. In case it has the knowledge of only its own, then it is called local CSI.

Global CSI is assumed to be present at the relays in most of the available literature [27–33]. Obtaining forward CSI requires either sending the estimated channels from the receiving nodes over reliable feedback channels, or direct estimation of these channels from the backward transmission (in a time-division duplex system). While designing AF precoder matrices to be used by the relay layers, these authors have considered optimization techniques like minimizing mean-squared error (MSE), maximizing receive power at the destination or maximizing the transmission rate. The relay layers use these matrices and forward the received signals to obtain reliable communication. Existing work in literature, which discuss optimum design of AF precoders have:

- closed-form solution of the precoders, only when there is a single layer of relays between the source and the destination, and resort to an iterative technique when there are two layers
- assumed that global CSI is available with the relays
• to solve a $2M$th order polynomial even for single layer of relays with $M$ source-destination pairs and hence for $M > 2$ it becomes difficult to obtain the precoder

• not considered the leaked signals that arrive from non-consecutive layers

All the above concerns are addressed in this thesis.

First, we develop simple ideas on relay forwarding and incorporate them into a three-hop relay network, and propose ad hoc schemes. All the three ad hoc schemes proposed use leaked signals and two of them do not need CSI. These two can be employed when it is not viable to obtain CSI at the relays. The third scheme uses *local backward* $^6$ CSI. A basic ad hoc scheme used in a two-hop network available in literature, is then enhanced to work in the three-hop network for comparison with the proposed schemes.

In the optimum schemes chapter, we propose a scheme which minimizes MSE at the relays. The concept of minimizing MSE at the relays, breaks down the problem of finding closed-form solution to relay precoders, when multiple layers are present by finding them at every layer stage, instead of considering all of them together at the destination. So, it obtains a closed-form solution even when there are multiple source-destination pairs. An added advantage is that the precoder obtained does not depend on forward CSI, which is not obtainable for the relays in many situations. Also, it is shown using simulations that the BER performance of the proposed scheme with only backward CSI, approaches that of the existing schemes which use global CSI, for nominal path loss conditions with path loss exponent is 2. When the path loss condition worsens with path loss exponent 3 or 4, it is shown that our scheme outperforms the existing schemes. This is made possible by using more and more of the leaked signals at the relays, which has lead to an increase in the signal content at the destination.

Thus, when there are a large number of relays spread in a wide area, the proposed scheme is a good option to select when: (1) there are multiple layers with multiple relays in each of them and/or multiple source-destination pairs, (2) the relays can cooperate amongst themselves or not, (3) the relays do not have forward CSI, and (4) when the path loss condition worsens.

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$^6$This scheme requires only the knowledge of the channel coefficients from previous layers to itself and none of the others in its own layer. This is a less stringent requirement than having backward CSI, which is defined on page 5.
1.3 Summary of contribution

Our contributions in this thesis include the following:

• When there are two layers of relays present between the source and the destination, we propose three ad hoc schemes that employ intuitive relay combining methods and exploit leaked signals. We also enhance the existing basic scheme into two newer versions for comparison with the proposed schemes.

• We show that all the three proposed ad hoc schemes perform better than the enhanced versions of the existing scheme, when the value of the path loss exponent is two, and two of them are better even in the worst case scenario when the value of the path loss exponent is four.

• For a multi-layer wireless relay network, we propose an optimum relaying strategy, which breaks down the complex MMSE problem at the destination into an MMSE problem at every relay layer stage and makes it solvable.

• We show that the optimum relay precoders thus obtained do not depend on forward CSI, which is not obtainable at relays in many situations.

• We obtain closed form solutions for the optimum relay precoders for all the following cases:
  
  – Relays are cooperative or noncooperative within a layer
  
  – Multiple layers of relays are present between the source and the destination
  
  – Multiple source-destination pairs are present
  
  – Multiple leaked signals are available at the relays, which can be combined optimally in the construction of the precoder

• We enhance the existing optimum strategies (though these enhancements may not be optimal) proposed in [27], [28], and [29] to work in this multi-layer network for meaningful comparison with the proposed optimum scheme.

• We show using simulations that combining more number of leaked signals improves the performance of the proposed scheme and makes it approach that of an existing scheme. It also out performs the existing scheme when the path loss condition worsens.
• When there are multiple source-destination pairs with one layer of relays, we show that the proposed scheme with no forward CSI is better than the MMSED scheme which uses global CSI, for lower values of SNR.

1.4 Organization of the thesis

The thesis has five chapters including this chapter and is organized as follows: In this chapter we enumerated the motivation for this research work and defined the scope. In Chapter 2, the system model to be used throughout this thesis is developed and briefly an overview of the literature that is relevant to the work undertaken is given. Thereafter, in Chapter 3, simple ad hoc systems are proposed and analyzed, while Chapter 4 focuses on optimum schemes. Finally, Chapter 5 concludes, summarizes all the contributions made in this thesis, and discusses open problems.

In the next Chapter, as mentioned, we will discuss the system model that is used in this thesis and give a brief on the existing literature which use ad hoc as well as optimum relay precoders.
Chapter 2

System Model and Literature Review

In this Chapter, we explain the framework of the system model used in this thesis and show how it is more general in nature than the system models used in existing literature. We also give a brief overview of the existing literature that are relevant to the research work undertaken in this thesis. At the end, we highlight the limitations and drawbacks in the existing systems and explain briefly how they are addressed in this thesis.

![Diagram of radio nodes](image)

Figure 2.1: A group of radio nodes located in a planar disc of unit area. Square shaped nodes shown on the left, $S_1, \cdots, S_M$ are the sources and the crosses on the right, $D_1, \cdots, D_M$ are the corresponding destinations. Remaining nodes can be used as relays.
2.1 System model

Pottie and Kaiser [34] showed how a distributed and layered signal processing architecture can overcome the energy and bandwidth constraints in wireless sensor network (WSN) applications. Jing and Hassibi [11] proved that a spatially distributed network of these sensors can emulate a MIMO communication system and proved that a DSTC can be created by these relays without the knowledge of the source transmitted signal. In this network of sensors, the authors in [11] considered a single source-destination pair and one layer of relays between them, whereas this thesis considers a similar network as the system model but with multiple source-destination pairs and multiple layers of relays.

Let us assume that a group of radio nodes are located randomly and uniformly identical independently distributed (i.i.d.) in a planar disc of unit area as shown in Fig. 2.1. Connectivity of these nodes is a prerequisite for the layered architecture assumed in this thesis. Gupta and Kumar [35] proved that if these nodes are to transmit at a power level so as to cover an area of \( \pi r^2 = (\log n + c(n))/n \), then the resulting network is asymptotically connected with probability one if and only if \( c(n) \to +\infty \). Here, \( c(n) \) is an arbitrary function and \( r \) is the radius of coverage for each of the \( n \) transmitting radio nodes.

Now, we assume that the network is asymptotically connected and \( M \) of them want to communicate with \( M \) others. We will call them source-destination pairs and denote by 2-tuples in the set \( \{(S_1, D_1), \cdots, (S_M, D_M)\} \). The remaining nodes can be used as relays to convey information from \( S_i \) to \( D_i, \ i \in [1, M] \). Assume that there are \( K N \) of them and that a network is formed, as shown in Fig. 2.2, with \( K + 1 \) layers. Layering of the relays is carried out using the information of the channel variance, wherein relays with same channel variance from the source layer are grouped together.

The layers are denoted as \( L_k, \ k \in [0, K + 1] \), as shown in Fig. 2.2, where \( L_0 \) and \( L_{K+1} \) refer to the source and the destination layers respectively and the remaining layers are those of relays. \( R_{kn} \) denotes the \( n \)th relay in the \( k \)th layer and hence, the first letter in the subscript denotes the layer the relay belongs to and the second letter denotes the relay number in that layer. The letters \( \ell \) and \( h \) are reserved to represent the links and the channel coefficients respectively, with the subscript denoting transmitter and receiver. Therefore, the channel coefficients of the links, \( \ell_{0,m,j,i} \) from \( S_m \to R_{ji} \), \( \ell_{j,i,k,n} \) from \( R_{ji} \to R_{kn} \), and \( \ell_{k,n,K+1,m} \) from \( R_{kn} \to D_m \) are denoted as \( h_{0,m,j,i} \), \( h_{j,i,k,n} \), and \( h_{k,n,K+1,m} \) respectively. Here \( m \in [1, M], \ j, k \in [1, K], \) and
Figure 2.2: A multi-layer relay network. Here $S_m$, $L_k$, $R_{ki}$, $D_m$, and $\rho_{ij}$ represent the $m$th source, $k$th layer, $i$th relay in $L_k$, the $m$th destination, and the length of the links from $L_i$ to $L_j$ respectively. In this figure, the suffix is avoided in $\rho$ for simplicity.

$i, n \in [1, N]$. Let $H_{0,i} \in \mathbb{C}^{N \times M}$, $H_{i,j} \in \mathbb{C}^{N \times N}$, and $H_{j,K+1} \in \mathbb{C}^{M \times N}$ represent the matrices of channel coefficients from $L_0$ to $L_i$, $L_i$ to $L_j$, and $L_j$ to $L_{K+1}$ respectively. It is also assumed that the relays are half-duplex, and the channels are Rayleigh fading and quasi static. The coherence interval\footnote{Time duration when the channel coefficients do not to vary} is assumed to be at least $2T$ symbol duration.

In Fig. 2.2, $\rho_{ij} \triangleq j - i$ denotes the length\footnote{We will drop the suffix in $\rho$ for simplicity, in places where it is understood from the context} of the link from the layer $L_i$ to $L_j$, $i < j$, and hence $\rho_{ij} \in \mathbb{N}$. For example, the length of the link $\ell_{1,1,K,N}$ from the relay $R_{11}$ to $R_{KN}$ is $\rho_{1K} = K - 1$, which is shown dashed in Fig. 2.2 for clarity.

Transmission takes place sequentially in phase-0, phase-1, $\cdots$, phase-$K$ by $L_0, L_1, \cdots, L_K$ layers respectively. Also, all the nodes that engage in transmission transmit at the same time in a particular phase. It is assumed that the signal transmitted from layer $L_i$, $i \in [0, K]$ can reach any other layer $L_j$, $j \in [i+1, K+1]$, where $L_j$ is a forward layer present between the transmitting layer and the destination layer. This is not an impractical assumption, as the amplitude of the received signal is accordingly reduced by taking into account the channel variance with a suitable path loss exponent.

Most of the literature involving multiple layers of relays use only the signal reaching at a particular layer from the preceding layer to construct the transmit signals (i.e., they use only signal from $L_{k-1}$ at $L_k$) as in [21]. On the contrary, in this thesis the transmit signals at $L_k$ are constructed using signals that have reached from $L_j$, $j \in [0, k-1]$, although signal from $L_j$, $j \in [0, k-2]$ reaches $L_k$ with lower power compared to that from $L_{k-1}$. These low power signals are the leaked signals as introduced in Chapter 1. So, any signal that travels through a link with length $\rho > 1$ is a leaked signal and the corresponding link is the weak link as introduced in Chapter 1.

Let $L$ be the set of all links and $L_{ij} \subset L$, $i < j$ be the set of links from $L_i$ to $L_j$ and we will call it as the link set. These link sets, $L_{ij}$, can be of four types, viz. (i) from the source layer to any relay layer, (ii) from any relay layer to any other relay layer, (iii) from any relay layer to the destination layer, and (iv) from the source layer to the destination layer. These are given respectively as follows:

$$L_{0,j} = \{\ell_{0,1,j,1}, \ldots, \ell_{0,M,j,N}\}, \quad j \in [1, K],$$
$$L_{i,j} = \{\ell_{i,1,j,1}, \ldots, \ell_{i,N,j,N}\}, \quad i \in [1, K-1], \quad j \in [i+1, K],$$
$$L_{i,K+1} = \{\ell_{i,1,K+1,1}, \ldots, \ell_{i,N,K+1,M}\}, \quad i \in [1, K],$$
and
$$L_{0,K+1} = \{\ell_{0,i,K+1,j}\}, \quad i, j \in [1, M].$$

Cardinality of these link sets, $|L_{0,j}|, |L_{i,j}|, |L_{i,K+1}|, \text{ and } |L_{0,K+1}|$ can be seen to be $MN$, $N^2$, $MN$, and $M^2$ respectively. All the links in any of these link sets $L_{k,l}$, have the same length $l - k$.

Now let $L_\rho$ be a class of subsets, $L_{i,j}$, of $L$ with length $\leq \rho$, which we will call as link class. Hence these classes can be written as

$$L_1 = \{L_{0,1}, L_{1,2}, L_{2,3}, \ldots, L_{K-1,K}, L_{K,K+1}\}$$
$$L_2 = \{L_{0,0}, L_{0,2}, L_{1,2}, \ldots, L_{K-1,K+1}, L_{K,K+1}\}$$
$$\vdots$$
$$L_{K+1} = \{L_{0,1}, L_{0,2}, L_{0,3}, \ldots, L_{K-1,K+1}, L_{K,K+1}\}$$

with $L_i \subset L_j$, if $i < j$.

**Claim 1.** The total number of link sets $L_{ij}$, is $\omega = (K+1)(K+2)/2$ and the cardinality $|L_\rho| \leq \omega$ with the equality being true when $\rho = K + 1$. In general, with $\rho \leq K + 1$, the
The system model discussed in this Chapter is more generalized and the existing models in the literature are special cases of it. For example, Gupta and Kumar [36] considered a model similar to this but with a single source-destination pair (i.e., $M = 1$), proposed a DF relaying strategy and obtained an achievable rate region, while Borade et al. [21] used an AF strategy with single source-destination pair (i.e., $M = 1$) and did not consider leaked signals (i.e., only signals that travel through the link with length $\rho = 1$ were considered) and analyzed the diversity-multiplexing trade-off when $K$ increases. Jing and Hassibi [11], Khajehnouri et al. [27], Krishna et al. [28], Lee et al. [29–32], and Gomadam et al. [33] used system models with one or two layers of relays between the source and the destination layers ($K = 1$ or 2) and did not consider any leaked signals (i.e., not considering signals travelling through any $\rho > 1$ link). Our work [37] considered a system model with $K = 2$ and leaked signals through the links with length $\rho = 2$.

In the next section, we will discuss the existing work by some of the above authors and highlight the drawbacks in them. (Here we have changed the symbol notation of these authors to be consistent with this thesis.) Later, we will compare their results with ours in Chapters 3 and 4, once we develop the proposed strategies.

2.2 Literature review

Here we will restrict our attention to some of the available literature which have source-destination pairs and a set of relays to convey the source transmitted signal to the destination. Also the scope includes only the following:

- Relays use AF strategy to convey information
- Relays use precoder matrices to prepare the received signal before forwarding it
- Relays decide on the precoder matrix by an intuitive way

Proof: See Appendix A.
• Relays decide on the precoder matrix by optimizing a parameter

We include the literature with the above scope, as this thesis is focused on AF strategy of combining signals at the relay layer and forwarding the composite signal to the subsequent layers.

We will consider two categories of the existing work by the nature of the precoder used by the relays: (1) Systems which use Ad hoc precoders and (2) Systems which use optimal precoders. The ad hoc precoders are in general do not require extra resources like CSI. The optimum precoders on the other hand may require CSI but would get better performance of the system than ad hoc precoder systems.

In the next section, we will see an ad hoc precoder system in the literature.

2.2.1 Ad hoc precoder systems

First, we will see the work by Jing and Hassibi [11], the primary scope of which was to show that the diversity and coding gains obtained using relays are almost same as that obtained in a MIMO system. The authors did not focus getting an optimum precoder matrix used by the relays and hence they considered a random unitary matrix as the precoder for each of the relays.

They used a system model which had one source-destination pair and one set of relays as shown in Fig. 2.3. Hence the system model used by these authors is a special case of that we consider in this thesis shown in Fig. 2.2 with only $S_1$, $L_1$, and $D_1$ taken from it.

![System model with one layer of relays and one source-destination pair.](image)

The channel variance from $S_1$ to $L_1$ and $L_1$ to $D_1$ was assumed to be constant at unity by the
The scheme consisted of two phases: in phase-0, $S_1$ transmits a vector $\alpha_0 s \in \mathbb{C}^{T \times 1}$ in $T$ time duration, which is assumed to be the coherence interval of the channel. Here $\alpha_0 = \sqrt{p_0 T}$ and is used to restrict the average power transmitted by $S_1$ to be $p_0 T$ during phase-0. In phase-1, each of the $L_1$ layer relays $R_{1n}$, $n \in [1, N]$, encode their received signals using a precoder matrix $F_{1n} \in \mathbb{C}^{T \times T}$, of their own and transmit

$$t_{1n} = \alpha_1 F_{1n} r_{1n}^{(0)}$$

to $D_1$, where $r_{1n}^{(0)}$ is the received signal vector in phase-0 by the relay $R_{1n}$ given by

$$r_{1n}^{(0)} = \alpha_0 s h_{0,1,1,n} + u_{1n}^{(0)}.$$

Here $\alpha_1 = p_1 / (N(p_0 + 1))$ is used to restrict the average power transmitted by any relay to be $p_1 T / N$ during phase-1. The authors assumed a total average power of $p_0 + p_1 = P$ Watts and allocated $p_0 = p_1 = P/2$ Watts, which they obtained by minimizing the pair-wise error probability (PEP), which is also shown to maximize receive SNR at $D_1$. The received vector at $D_1$ is shown to be

$$r_{21}^{(1)} = \alpha_0 \alpha_1 S h_{0,1,2,1} + w,$$  \hspace{1cm} (2.2)

where

$$S = [F_{11}s, \ldots, F_{1N}s], \quad h_{0,1,2,1} = \begin{bmatrix} h_{0,1,1,1} & h_{1,1,2,1} \\ \vdots & \vdots \\ h_{0,1,1,N} & h_{1,1,2,1} \end{bmatrix}, \quad \text{and} \quad w = \alpha_1 \sum_{n=1}^{N} h_{1,n,2,1} F_{1n} u_{1n}^{(0)} + u_{21}^{(1)}.$$

The matrix $S \in \mathbb{C}^{T \times N}$ is analogous to the STC obtained by a MIMO system and here it is achieved by the relays distributively and hence the authors call it DSTC. From the PEP expression, the authors showed that this system achieves the same diversity and coding gain as that of a multiple-input single-output (MISO) system at high transmit powers. Let us call this basic protocol as Jing-Hassibi Scheme (JHS).

In another work of Jing and Hassibi [12], they proved that JHS can be extended to a system which has multiple antenna radio nodes. With $A_S$ and $A_D$ being the number of antennas at the source and the destination respectively, the authors have proved that a diversity of $\min\{A_S, A_D\} \sum_{n=1}^{N} R_{1n}$ if $A_S \neq A_D$ and $A_S (1 - (1/A_S)(\log \log P / \log P)) \sum_{n=1}^{N} R_{1n}$ if $A_S = A_D$ can be achieved, with $R_{1n}$ being the number of antennas in the $n$th relay $R_{1n}$. 

\[\text{PEP} - \text{Probability that the decoder at } D_1 \text{ mistakes one transmitted symbol for another}\]

\[\text{This has multiple antennas at transmitter and single antenna at the receiver}\]
In the next section, we will see some of the existing works where relay precoders are obtained by optimizing a parameter, when the transmitted signal is a scalar, i.e., \( T = 1 \).

### 2.2.2 Optimal precoder systems

The main differences between an ad hoc system and an optimal system that we consider here are:

1. The precoders used in this section are derived by optimizing a parameter

2. A precoder matrix is used by all the relays together in a layer - depending upon whether this matrix is diagonal or not, the relays can act independently or interact amongst themselves respectively

3. The radio nodes transmit a scalar, i.e., \( T = 1 \)

As mentioned above, the focus of the literature [27–33] which we will see in this section is the design of the precoder used by the relay layer, by optimizing a parameter and eventually minimize the BER at the destination. This optimization invariably has been taken up at the destination and we will refer to those systems which minimize MSE at the destination as MMSE at destination (MMSED) schemes.

Khajehnouri and Sayed [27] used the system model shown in Fig. 2.3, minimized an objective function at the destination given by

\[
J_D = E |\eta s - h_{1,2}t_1|^2
\]  

(2.3)

and found the precoder matrix for \( L_1 \) with no power constraint. Here \( s \) is the signal that is transmitted by the source \( S_1 \) and \( \eta \) can be selected according to the need. Also \( h_{1,2} \in \mathbb{C}^{1 \times N} \) and \( t_1 \in \mathbb{C}^{N \times 1} \) are the channel coefficients from \( L_1 \) to \( D_1 \) and the \( L_1 \) transmitted vectors respectively. These vectors are given by \( h_{1,2} = [h_{1,1,2,1,1}, \ldots, h_{1,1,2,2,1}] \) and \( t_1 = F_1r_1^{(0)} \) respectively, where \( r_1^{(0)} = \alpha_0 s h_{0,1} + u_1^{(0)} \) is the received vector at \( L_1 \) in phase-0 and \( F_1 \) is the precoder that is obtained by minimizing \( J_D \) shown in (2.3). Here \( \alpha_0 = \sqrt{p_0} \) is used by \( S_1 \) to restrict its average power to \( p_0 \). Also \( u_1^{(0)} \) is the noise vector added by the relay front end receivers at \( L_1 \) and \( h_{0,1} \in \mathbb{C}^{N \times 1} \) is the vector of channel coefficients given by \( h_{0,1} = [h_{0,1,1,1,1}, \cdots, h_{0,1,1,N}]^T \). Assuming that the relays do not cooperate, the authors arrived at the  \( i \)th diagonal element of the
optimum precoder matrix $\hat{F}_1$ as

$$\hat{f}_{ii} = \frac{\eta \sigma_s^2 h_{0,1,1,i}^* h_{1,i,2,1}^*}{\sigma_s^2 \| h_{0,1,1} \|^2 + \sigma_u^2},$$

(2.4)

where $\sigma_u^2$ and $\sigma_s^2$ are the noise and signal variances respectively.

Now $\eta$ can be selected to be $\eta = \frac{SNR_t \sigma_u^2}{\sigma_s^2}$ to achieve a target SNR of $SNR_t$ at the destination and if $\eta$ is selected to be unity, then $J_D$ becomes MSE at the destination.

The authors also proved that the minimum $J_D$ thus obtained tends to a constant value as $N$ increases and that the power consumption per relay decreases. Further they took up maximizing the received power at the destination while imposing power constraint on the relays and arrived at the diagonal element of the relay precoder.

The second article which we take up for discussion here is by Krishna et al. [28], which also adopted the minimum MSE (MMSE) strategy of [27] we discussed in this section. The authors used a similar system model shown in Fig. 2.3 but with multiple source-destination pairs (i.e., $M > 1$) as shown in Fig. 2.4. Here, another difference is that the relays can exchange information amongst themselves or cooperate. Also, the relays have a constraint on the average power they transmit to $p_1$. These authors used a similar objective function $J_D$, as shown in (2.3).
while including multiple source-destination pairs as

$$J_D = \sum_{m=1}^{M} E |s_m - \mathbf{h}_{1,m,t_1}|^2.$$  \hfill (2.5)

In this scheme, the sources $\mathbf{S}_1, \ldots, \mathbf{S}_M$ transmit $\mathbf{s} = \{s_1, \ldots, s_M\}$ in phase-0. Here $\mathbf{h}_{1,m} \in \mathbb{C}^{1 \times N}$ is the vector of channel coefficients from $\mathbf{L}_1$ to $\mathbf{D}_m$ given by $\mathbf{h}_{1,m} = [h_{1,1,m}, \ldots, h_{1,N,m}]$. The authors minimized MSE shown in (2.5) and derived a non-diagonal precoder matrix for cooperative relays with average power constraint $p_1$ as

$$\hat{\mathbf{F}}_1 = \left[ \mathbf{H}_{1,2}^{H} \mathbf{H}_{1,2} + \tilde{\lambda} \mathbf{I}_N \right]^{-1} \mathbf{H}_{1,2}^{H} \mathbf{R}_{r_1(0)}^{-1} \mathbf{r}_1(0)^{H} \sigma_s^2.$$  \hfill (2.6)

where $\tilde{\lambda}$ is the Lagrange multiplier obtained from

$$\sum_{i=1}^{N} B_{ii} \left( \lambda_i + \tilde{\lambda} \right)^{-2} \sigma_s^4 = p_1.$$  \hfill (2.7)

Here, $B_{ii}$ is the $i$th diagonal element of

$$\mathbf{B} = \mathbf{Q}^H \mathbf{H}_{1,2}^{H} \mathbf{H}_{0,1}^{H} \mathbf{R}_{r_1(0)}^{-1} \mathbf{H}_{0,1} \mathbf{H}_{1,2} \mathbf{Q}$$
where $\mathbf{Q}$ is obtained from $\mathbf{H}_{1,2}^{H} \mathbf{H}_{1,2} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^H$.

Also $\lambda_i$ is the $i$th eigen value of $\mathbf{H}_{1,2}^{H} \mathbf{H}_{1,2}$ and $\mathbf{H}_{i,j}$, $i, j \in [0, 2]$, is already defined in this Chapter under the section 2.1. The autocovariance matrix $\mathbf{R}_{r_1(0)}$ is given by $\mathbf{R}_{r_1(0)} = E[\mathbf{r}_1(0) \mathbf{r}_1(0)^{H}]$. Let us call this scheme which uses (2.6) for realizing its relay precoder as MMSED-Krishna or MMSED-K in short.

To compare their results with a noncooperative relays system, these authors modified the $i$th diagonal element of the precoder matrix shown in (2.4) to maintain transmitted power to $p_1$ in Khajehnouri-Sayed scheme [27] as

$$f_{ii} = \frac{p_1 \frac{1}{2} h_{0,1,i,j}^* h_{1,i,2,1}^*}{|h_{1,i,2,1}|^2 \left[ \sum_{j=1}^{N} |h_{0,1,1,j}|^2 (p_0 |h_{0,1,1,j}|^2 + \sigma_s^2) \left( p_0 \frac{1}{2} h_{0,1,1,j}^* \right) \right]^{\frac{1}{2}}}$$  \hfill (2.8)

for noncooperative relays. Let us call this scheme which uses (2.8) for realizing its relay precoder as MMSED-Khajehnouri-Krishna or MMSED-KK in short.

The third article we will discuss here is by Lee et al. [29], who adopted a similar strategy as of those discussed so far [27, 28] to obtain relay precoders. The system model they considered is shown in Fig. 2.3. They included power constraint for the relays while optimizing when the relays cooperate as well as when they do not. They also included channel uncertainties into their calculations and hence is more general than [27,28], but considered only one source-destination
pair. For the noncooperative relays, they derived the $i$th diagonal element of the optimum relay precoder in their equation (27) as

$$f_{1i} = \frac{p_1^{\frac{1}{2}}h_{0,1,1,i}^* h_{1,1,2,i}^*}{\left( \sum_{j=1}^{N} |h_{0,1,1,j}|^2 |h_{1,1,2,j}|^2 \left( p_0 |h_{0,1,1,j}|^2 + \sigma_u^2 \right) \right)^{\frac{1}{2}}}$$

(2.9)

where we have removed the channel uncertainty parameters as they are not in the scope of this thesis. Let us call this scheme which uses (2.9) for realizing its relay precoder as MMSED-Lee or MMSED-L in short. We will see later that MMSED-Lee exploits the available global CSI effectively than MMSED-KK.

In another contribution in this area, Lee et al. [30] included receive power constraint at the destination instead of the relay transmit power constraint and obtained optimum relay precoders. Furthermore, in [31] Lee et al. considered a jamming environment along with uncertainty in channel, and found the optimum precoders by minimizing MSE with receive power constraint, when there are multiple source-destination pairs.

All the articles we discussed above in this section considered a single layer of relays (i.e., $K = 1$). Two of the examples of three-hop networks or two relay layer network (i.e., $K = 2$) are by Gomadam and Jafar [33] and Lee et al. [32]. Gomadam and Jafar obtained relay precoders by maximizing the receive signal-to-noise ratio (SNR) at the destination, while considering correlated noise. Lee et al. [32] considered relay cooperation with and without power constraints and obtained relay precoder by minimizing MSE at the destination. In both [33] and [32], the optimum precoder is implemented at the relays using an iterative algorithm as it does not have a closed-form solution.

### 2.3 Limitations in the existing systems

#### 2.3.1 Ad hoc precoder systems

The focus of Jing-Hassibi’s works [11, 12] in their scheme JHS, is to prove the efficacy of using relays and obtaining an STC distributively. Also, though there is a passing mention of using the leaked signal at the destination, it is not given rigorous treatment. Further, the schemes work only for a system when there is a single layer of relays present between the source and the destination. One of the aims of this thesis is to extend JHS to operate in a two layer system and propose intuitive combining schemes which would include leaked signals at the relays to obtain
better BER performance.

2.3.2 Optimal precoder systems

While discussing the optimal precoder systems, we showed three equations of MMSED-K, MMSED-KK, and MMSED-L in (2.6), (2.8), and (2.9) respectively. It can be seen that all these systems depend on $H_{0,1}$, $H_{1,2}$ or $h_{0,1,i,j}$, $h_{1,2,i,j}$, $j \in [1, N]$ which are termed global CSI\(^5\) at the relay layer $L_1$ or the relay $R_{1i}$, $i \in [1, N]$. This may not be feasible in many situations for the relays to obtain it. The feasibility of this depends on the application scenario. Another difficulty these systems face is that there is no closed-form solutions available if the number of relay layers is more than one (i.e., $K > 1$).

MMSED-K is derived for any number of source-destination pairs as can be seen from (2.6), unlike the other two - MMSED-KK and MMSED-L. But we can note that to get the Lagrange multiplier $\tilde{\lambda}$, one has to resort to solving (2.7). Depending upon the number of source-destination pairs, i.e., $M$, the difficulty increases. For example, if $M = 2$ is considered, then the polynomial degree is $2M = 4$ and hence a quartic equation in $\tilde{\lambda}$ needs to be solved. Hence, we can say that there is no closed-form solution for $M > 1$ in MMSED systems.

To the best of our knowledge, there is no MMSED system present in current literature which uses leaked signals. These signals which may be weak could probably increase the signal content at the receiving node. As the receiving nodes are relays and they forward the signals received to the destination, it can in turn aid the destination to decode the signal transmitted by the source effectively, leading to better performance compared to when no leaked signals are used.

This thesis addresses all the concerns discussed above. The proposed optimum strategy breaks the complex problem of finding all the precoders together at the destination into finding them at each of the corresponding relay layer stages. This breaking up of the problem is done in the proposed strategy by minimizing the MSE at the relays (We will call this strategy as MMSE at relays (MMSER) scheme) rather than at the destination. Also for the case when there are multiple source-destination pairs, the strategy uses an arbitrary combining matrix to make the MSE at relays compatible in dimension with the number of relays. The handicap for MMSER is that the relays do not have forward CSI and hence it becomes difficult to achieve performance as good as the MMSED systems in the nominal path loss (path loss exponent = 2) cases, which

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\(^5\)As explained in section 1.2, it is the knowledge of the forward and backward channel coefficients
use global CSI. But the proposed strategy makes use of the leaked signals and it is shown that the BER performance is closer to that of MMSED systems that use global CSI effectively.

So, we can cite two important reasons for not taking up MMSED though its BER performance is found to be better than MMSER in nominal path loss conditions: (1) MMSED, discussed in literature [27–32], obtains the amplifying matrix which is dependent on backward and forward channel matrices. It minimizes the MSE at the destination and the precoder matrix thus obtained has closed-form solution when there is only one layer of relays between the source and the destination. It is found to be difficult to derive a closed-form solution to this matrix when $K = 2$, as shown in Section 4.1.4. Hence, we could not take on MMSED for multi-layer cases; (2) Obtaining forward CSI is an overhead. It requires either sending the estimated channels from the receiving nodes over reliable feedback channels, or direct estimation of these channels from the backward transmission (in a time-division duplex system).

In Chapter-3, we propose ad hoc schemes which are developed heuristically and later in Chapter-4 we discuss MMSER strategy. Also we will compare the results of these systems with those of JHS, MMSED-K, MMSED-KK, and MMSED-L.
Chapter 3

Simple Ad Hoc Schemes

In this Chapter, we propose three ad hoc schemes which are arrived at heuristically that work in a three-hop system shown in Fig. 3.1. Here, we also enhance JHS, an ad hoc scheme discussed in Chapter-2 to two different forms - extended JHS (EJHS) and modified JHS (MJHS) - so that it works in a three-hop system. The BER performance of the proposed schemes, which use leaked signals, are then compared with EJHS and MJHS and shown that the proposed schemes perform better.

![System model with two layers of relays and one source-destination pair.](image)

The system model considered here shown in Fig. 3.1, is a special case of the general system model discussed in Chapter 2 shown in Fig. 2.2, with $M = 1$ and $K = 2$. All the ad hoc schemes discussed here use this system model. The proposed schemes use the signals which travel through the links in the link class $\mathcal{L}_3$ given by

$$\mathcal{L}_3 = \{L_{0,1}, L_{0,2}, L_{0,3}, L_{1,2}, L_{1,3}, L_{2,3}\}.$$  

(3.1)
The channel coefficients shown in Fig. 3.1 are assumed to be known at D_1, which are used for decoding the received signal.

Two of the proposed schemes do not use CSI and the third one uses only local backward CSI. Hence the advantage we get from these ad hoc systems is that they are simple to implement at the relays with less or no requirement of the CSI. We show that one of the schemes which does not require CSI performs better than the rest of the proposed and existing schemes.

In Chapter-2, we introduced symbols to denote various entities in the system model. We introduce more symbols that are used in the rest of this thesis. As mentioned in Chapter-2, the transmission happens in sequential phases. Let t, r, and u denote transmitted, received, and noise vectors respectively during these phases. The subscript in them represent the radio node concerned. For example, t_{ij} denotes the vector transmitted by the j-th node in the i-th layer L_i. So, t_{01} represents the transmitted vector by the first node in L_0, which is the source S_1. In the received and the noise vector symbols r and u, we also use the superscript to represent the phase in which this vector is received or added respectively. For example, r^{(k)}_{ij} and u^{(k)}_{ij} denote the vector received and the noise vector added respectively at the j-th node in layer L_i during phase-k. To be specific, r^{(1)}_{31} denotes the vector received by the first node in L_3 during phase-1, i.e., the destination D_1 receives this vector in phase-1. The superscript on t is used only when the relays transmit in more than one phase and denote the phase of transmission.

The transmit, receive, and noise vectors are given by

\[
\begin{align*}
t_{ij} &= \begin{bmatrix} t_{ij}(1) \\ \vdots \\ t_{ij}(T) \end{bmatrix}, \quad r^{(k)}_{ij} = \begin{bmatrix} r^{(k)}_{ij}(1) \\ \vdots \\ r^{(k)}_{ij}(T) \end{bmatrix}, \quad \text{and} \quad u^{(k)}_{ij} = \begin{bmatrix} u^{(k)}_{ij}(1) \\ \vdots \\ u^{(k)}_{ij}(T) \end{bmatrix}
\end{align*}
\]

respectively, where each of the components of \( u^{(k)}_{ij} \) is assumed to be an i.i.d. zero-mean complex Gaussian (ZMCG) random variable. The variance of these components \( \sigma_u^2 \), is assumed to be constant at unity. Here, T is the time duration during which the transmitting node transmits in each phase of transmission. The total average power transmitted by all transmitting nodes is assumed to be \( PT \). Let us assume that the average transmitted power per symbol duration is \( p_i \) in phase-\( i \). Hence, we have \( \sum_{i=0}^{2} p_i = P \).

In all the schemes to be discussed, various layers transmit sequentially except in MJHS, which we will see in the corresponding section. In phase-\( i \), L_i, \( i \in [0, 2] \) transmits and all other layers L_j, \( j \in [i + 1, 3] \) receive. In the proposed schemes, various receiving layers store the signals which travel through the links with length \( \rho \leq 3 \) and use them during their transmission.
Hence, the proposed schemes use all the signals that travel through the links in the link class $L$. In all the schemes, phase-0 is assumed to be the same and hence we will discuss it here. We will see phases-1 and 2 of these schemes under their corresponding sections to be discussed subsequently.

In phase-0, $S_1$ or layer $L_0$ transmits $t_{01} = \alpha_0 s = \alpha_0 [s(1), \ldots, s(T)]^T$ during a block of length $T$ duration, when the channel coefficients are assumed to remain constant. $L_1$, $L_2$, and $L_3$ or the destination $D_1$ receive. Let $s$ be normalized with $E[s^H s] = 1$.

In effect, in phase-0, $S_1$ transmits $\alpha_0 s(\tau)$ at time $\tau$, for $1 \leq \tau \leq T$. A radio node (could be relays, $R_{ij}$, $i \in [1, 2]$, $j \in [1, N]$ or $D_1$) receives $r_{ij}^{(0)}(\tau) = \alpha_0 s(\tau) h_{0,i,j} + u_{ij}^{(0)}(\tau)$ at time $\tau$ and in vector form

$$r_{ij}^{(0)} = \alpha_0 s h_{0,i,j} + u_{ij}^{(0)}.$$

(3.2)

The received vector at $D_1$ $r_{31}^{(0)}$ with $i = 3$ and $j = 1$ in (3.2), can be written as

$$r_{31}^{(0)} = \alpha_0 s h_{0,1,3,1} + u_{31}^{(0)}.$$

(3.3)

We will use the notations $x$, $z$, and $w$ for the received vectors at the destination for simplicity of notation during phase-0, phase-1, and phase-2 respectively. To be specific, following are the simpler notations of the vectors received by $D_1$ in various phases:

- Phase-0: $r_{31}^{(0)} = x = m_x + u_x$
- Phase-1: $r_{31}^{(1)} = z = m_z + u_z$
- Phase-2: $r_{31}^{(2)} = w = m_w + u_w$

Here, $m_v$ and $u_v$ denote the signal and noise components of the received vector $v$, which can be $x$ or $z$ or $w$. Therefore from (3.3),

$$m_x = \alpha_0 s h_{0,1,3,1}$$

(3.4)

and

$$u_x = u_{31}^{(0)}$$

(3.5)

are the signal and noise components of $r_{31}^{(0)}$ respectively. This is the same for all the proposed schemes and is to be used in their receivers in the maximum-likelihood (ML) decoder.
As $p_0$ is the assumed average power transmitted per symbol duration by $S_1$, we have

$$p_0 T = E \left[ t_{01}^H t_{01} \right] = E \left[ \alpha_0 s^H s \alpha_0 \right] = \alpha_0^2 \Rightarrow \alpha_0 = \left[ p_0 T \right]^{\frac{1}{2}}. \quad (3.6)$$

The average power received by $R_{ij}$ in $T$ symbol duration is given by

$$E \left[ r_{ij}^{(0)H} r_{ij}^{(0)} \right] = E \left[ (\alpha_0 s^H h_{0,1,1,j} + u_{ij}^{(0)H}) (\alpha_0 s h_{0,1,1,j} + u_{ij}^{(0)}) \right]$$

$$= \alpha_0^2 E \left[ |h_{0,1,1,j}|^2 \right] + E \left[ u_{ij}^{(0)H} u_{ij}^{(0)} \right]$$

$$= \alpha_0^2 \sigma_1^2 + T = \alpha_0^2 + T, \quad (3.7)$$

which is arrived at with the assumption that signal, noise, and channel are uncorrelated amongst each other and have zero mean. Also we have assumed that $E[|h_{0,1,1,j}|^2] = \sigma_1^2 = 1$, which we will use throughout this Chapter. Similarly the power received by $R_{2j}$ is given by

$$E \left[ r_{2j}^{(0)H} r_{2j}^{(0)} \right] = E \left[ (\alpha_0 s^H h_{0,1,2,j} + u_{2j}^{(0)H}) (\alpha_0 s h_{0,1,2,j} + u_{2j}^{(0)}) \right] = \sigma_2^2 \alpha_0^2 + T, \quad (3.8)$$

where $\sigma_2^2 = E[|h_{0,1,2,j}|^2]$ is the variance of the channel coefficient $h_{0,1,2,j}$. Also the power received by $D_1$ is given by

$$E \left[ r_{31}^{(0)H} r_{31}^{(0)} \right] = E \left[ (\alpha_0 s^H h_{0,1,3,1} + u_{31}^{(0)H}) (\alpha_0 s h_{0,1,3,1} + u_{31}^{(0)}) \right] = \sigma_3^2 \alpha_0^2 + T, \quad (3.9)$$

where $\sigma_3^2 = E[|h_{0,1,3,1}|^2]$ is the variance of the channel coefficient $h_{0,1,3,1}$. We have $\sigma_3^2 < \sigma_2^2 < \sigma_1^2 = 1$.

Each of the relays $R_{ij}$, have their own matrices $F_{ij}$, given by

$$F_{ij} = \begin{bmatrix} f_{ij}^{kl} \end{bmatrix} \quad (3.10)$$

which they use to transmit the received vector and finally produce a DSTC. Here, $k$ and $l$ denote the row and column numbers respectively. In linear dispersion space-time codes [38], these matrices are used in pairs in a MIMO system, whereas here we will use a single matrix $F_{ij}$ for a relay $R_{ij}$. Both treatments were given in [11], but while considering a single matrix, the authors considered $F_{ij}$ to be a complex unitary matrix. We will consider them to be random real orthogonal with $F_{ij} \in \mathbb{R}^{T \times T}$ and $F_{ij} F_{ij}^T = F_{ij}^T F_{ij} = I_T$ and the components $f_{ij}^{kl} \in \mathbb{R}$ are ZMG i.i.d. random variables with variance $1/T$. The performance of the system is shown to be the same using simulations in Fig. 3.14 for both real orthogonal and complex unitary matrices, in section 3.8 of this Chapter.
All the proposed schemes use leaked signals and hence they use either two or three vectors received at the destination to decode the signal transmitted by the source, depending upon whether they use $\mathcal{L}_2$ or $\mathcal{L}_3$ class of link sets respectively. We shall attach the number $\mu$ with the name of the scheme, if it uses the links from $\mathcal{L}_\mu$.

Now, we will see a detailed description of each one of the three proposed and the two derived ad hoc schemes. First three sections describe the proposed schemes and the subsequent two sections give an account of the derived schemes.

### 3.1 Relay matrix combining scheme

The relay matrix combining scheme (RMCS), one of the schemes which we propose, combines the signal vectors received at $\mathcal{L}_2$ during phase-0 and phase-1 using a matrix, before transmitting in phase-2. We will see in Chapter-4, that RMCS forms the base for our optimum schemes. Different phases of transmission and reception of this scheme are shown in Fig. 3.2 and explained below:

- **Phase-0:** $S_1$ transmits; $L_1$ and $L_2$ layer relays and $D_1$ receive
- **Phase-1:** $L_1$ layer relays transmit; $L_2$ layer relays and $D_1$ receive
- **Phase-2:** $L_2$ layer relays transmit and $D_1$ receives

![Figure 3.2: Various phases in RMCS/RSCS/RMCKCS.](image)
3.1.1 Transmit and receive vectors

In phase-1, the relays $R_{1j}, 1 \leq j \leq N$, transmit $t_{1j}(\tau)$ at time $\tau$ for $1 \leq \tau \leq T$, where $t_{1j}(\tau) = \alpha_1 \sum_{p=1}^{T} f_{1j}^{(0)}(p)$ and in vector form,

$$t_{1j} = \alpha_1 F_{1j} r_{1j}^{(0)}, \quad (3.11)$$

where $F_{1j}$ is shown in (3.10) with $i = 1$ and $\alpha_1$ is a scaling factor. The available power $p_1 T$ is equally divided amongst $N$ relays and hence we would like the power transmitted by each relay to be $p_1 T/N$, for which we use the scaling factor. Now, to get $\alpha_1$, we equate the transmitted power to $p_1 T/N$ as

$$\frac{p_1 T}{N} = E[t_{1j}^H t_{1j}] = \alpha_1^2 E[r_{1j}^{(0)H} F_{1j}^H F_{1j} r_{1j}^{(0)}] = \alpha_1^2 (\alpha_0^2 T) \text{ from (3.7).}$$

Substituting $\alpha_0^2$ from (3.6), we get

$$\frac{p_1 T}{N} = \alpha_1^2 (p_0 T + T) \Rightarrow \alpha_1 = \left[ \frac{p_1}{N(p_0 + 1)} \right]^{\frac{1}{2}}. \quad (3.12)$$

In phase-1, the relays in $L_2$ receive $r_{2j}^{(1)}, j \in [1, N]$ given by

$$r_{2j}^{(1)} = \sum_{i=1}^{N} t_{1i} h_{1,i,2,j} + u_{2j}^{(1)} = \sum_{i=1}^{N} \alpha_1 F_{1i} r_{1i}^{(0)} h_{1,i,2,j} + u_{2j}^{(1)}$$

$$= \alpha_1 \sum_{i=1}^{N} F_{1i} \left( \alpha_0 s h_{0,1,1,i} + u_{1i}^{(0)} \right) h_{1,i,2,j} + u_{2j}^{(1)} \text{ from (3.2)}$$

$$= \alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} s h_{0,1,1,i} h_{1,i,2,j} + \alpha_1 \sum_{i=1}^{N} F_{1i} u_{1i}^{(0)} h_{1,i,2,j} + u_{2j}^{(1)}. \quad (3.13)$$

This can also be written as

$$r_{2j}^{(1)} = S_1 h_{0,1,2,j} + \alpha_1 \sum_{i=1}^{N} h_{1,i,2,j} F_{1i} u_{1i}^{(0)} + u_{2j}^{(1)} \quad (3.14)$$

where

$$S_1 = \alpha_0 \alpha_1 [F_{11} s \ldots F_{1N} s] \text{ and } h_{0,1,1,2,j} = \left[ \begin{array}{c} h_{0,1,1,1} h_{1,1,2,j} \\ \vdots \\ h_{0,1,1,N} h_{1,1,2,j} \end{array} \right]. \quad (3.15)$$
In phase-1, $D_1$ receives $r_{31}^{(1)}$ given by

$$r_{31}^{(1)} = \sum_{i=1}^{N} t_{1i} h_{1,i,3,1} + u_{31}^{(1)} = \sum_{i=1}^{N} \alpha_1 F_{1i} r_{1i}^{(0)} h_{1,i,3,1} + u_{31}^{(1)}$$

$$= \alpha_1 \sum_{i=1}^{N} F_{1i} \left( \alpha_0 s h_{0,1,i,i} + u_{1i}^{(0)} \right) h_{1,i,3,1} + u_{31}^{(1)}$$

$$= \alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} s h_{0,1,i,i} h_{1,i,3,1} + \sum_{i=1}^{N} \alpha_1 F_{1i} u_{1i}^{(0)} h_{1,i,3,1} + u_{31}^{(1)}$$

$$= z = m_z + u_z,$$  \hspace{1cm} (3.16)

where

$$m_z = \alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} s h_{0,1,i,i} h_{1,i,3,1}$$  \hspace{1cm} (3.17)

and

$$u_z = \sum_{i=1}^{N} \alpha_1 F_{1i} u_{1i}^{(0)} h_{1,i,3,1} + u_{31}^{(1)}$$  \hspace{1cm} (3.18)

are the signal and noise components of the received signal respectively. This can also be written as

$$r_{31}^{(1)} = S_1 h_{0,1,3} + u_x$$  \hspace{1cm} (3.19)

where $S_1$ is given in (3.15) and

$$h_{0,1,3} = \begin{bmatrix} h_{0,1,1,1} h_{1,1,3,1} \\ \vdots \\ h_{0,1,1,N} h_{1,N,31} \end{bmatrix}.$$  \hspace{1cm} (3.20)

Like in JHS [11], it has been proved in (3.19) that the distributed space-time code in this case is $S_1$ and the equivalent channel matrix is $h_{0,1,3}$ with the equivalent noise vector $u_x$.

In phase-2, the two received vectors $r_{2j}^{(0)}$ and $r_{2j}^{(1)}$ are combined by $R_{2j}$ using a matrix before transmission. These two vectors are stacked into a single vector as

$$r_{2j} = \begin{bmatrix} r_{2j}^{(0)} \\ r_{2j}^{(1)} \end{bmatrix}.$$  \hspace{1cm} (3.21)

Then, the transmission vector is created as

$$t_{2j} = \alpha_2 F_{2j} r_{2j},$$  \hspace{1cm} (3.22)
where $\alpha_2$ is a scaling factor like $\alpha_1$ and $F_{2j} \in \mathbb{R}^{T \times 2T}$ is the relay precoder matrix of $R_{2j}$ given by

$$F_{2j} = \frac{1}{\sqrt{2}} \begin{bmatrix} f_{2j}^{11} & \ldots & f_{2j}^{1,2T} \\ \vdots & \ddots & \vdots \\ f_{2j}^{T1} & \ldots & f_{2j}^{T,2T} \end{bmatrix},$$

It can also be written in the submatrix form as

$$F_{2j} = \frac{1}{\sqrt{2}} \begin{bmatrix} F_{2j,0} & F_{2j,1} \end{bmatrix},$$

where $F_{2j,0}$ and $F_{2j,1}$ are the submatrices of $F_{2j}$ with first $T$ columns and the last $T$ columns respectively. These submatrices are chosen to be orthogonal and hence the factor $1/\sqrt{2}$ is used to make $F_{2j}^H F_{2j} = I_T$. Also we have $F_{2j}^H F_{2j} \neq I_{2T}$.

We maintain the average power transmitted by each relay in $L_2$ to be $p_2 T/N$ for $T$ channel uses.

**Claim 2.** We should select the scaling factor $\alpha_2$ in (3.22) as

$$\alpha_2 = \left[ \frac{2p_2}{N (p_0 \sigma_2^2 + 2 + p_1)} \right]^\frac{1}{2}$$

(3.24)

to keep the average power transmitted by $L_2$ channel uses to be $p_2 T/N$.

**Proof:** See Appendix B.

In phase-2, the vector received by $D_1$ is $r_{31}^{(2)}$, the components of which are given by

$$r_{31}^{(2)}(\tau) = \sum_{i=1}^{N} t_{2i}(\tau) h_{2,i,3,1} + u_{31}^{(2)}(\tau).$$

In vector form,

$$r_{31}^{(2)} = \sum_{i=1}^{N} t_{2i} h_{2,i,3,1} + u_{31}^{(2)} = \sum_{i=1}^{N} \alpha_2 F_{2i} r_{2i}^{(2)} h_{2,i,3,1} + u_{31}^{(2)}$$

$$= \sum_{i=1}^{N} \frac{\alpha_2}{\sqrt{2}} \left[ F_{2i,0} r_{2i}^{(0)} + F_{2i,1} r_{2i}^{(1)} \right] h_{2,i,3,1} + u_{31}^{(2)} = \frac{\alpha_2}{\sqrt{2}} \sum_{i=1}^{N} F_{2i,0} \left( \alpha_0 s h_{0,1,2,i} + u_{2i}^{(0)} \right) h_{2,i,3,1}$$

$$+ \frac{\alpha_2}{\sqrt{2}} \sum_{i=1}^{N} F_{2i,1} \left( \alpha_0 \alpha_1 \sum_{k=1}^{N} F_{1k} s h_{0,1,1,k} h_{1,k,2,i} + \alpha_1 \sum_{k=1}^{N} F_{1k} u_{1k}^{(0)} h_{1,k,2,i} + u_{1k}^{(1)} \right) h_{2,j,3,1} + u_{31}^{(2)}$$

$$= \frac{\alpha_0 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} F_{2i,0} s h_{0,1,2,i} h_{2,i,3,1} + \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i,1} F_{1k} s h_{0,1,1,k} h_{1,k,2,i} h_{2,i,3,1}$$

$$+ \frac{\alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \left[ F_{2i,0} u_{2i}^{(0)} + F_{2i,1} u_{2i}^{(1)} \right] h_{2,i,3,1} + \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,2,i} h_{2,i,3,1} F_{2i,1} F_{1k} u_{1k}^{(0)} + u_{31}^{(2)}$$

$$= w = m_w + u_w,$$

(3.25)
where

\[
m_w = \frac{\alpha_0 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} F_{2i,0} s h_{0,1,2,i} h_{2,i,3,1} + \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i,1} F_{1k} s h_{0,1,k} h_{1,k,2,i} h_{2,i,3,1} \tag{3.26}
\]

and

\[
u_w = \alpha_2 \sqrt{2 N \sum_{i=1}^{N} F_{2i,0} s h_{0,1,2,i} h_{2,i,3,1} + \alpha_0 \alpha_1 \alpha_2 \sqrt{2 N \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i,1} F_{1k} s h_{0,1,k} h_{1,k,2,i} h_{2,i,3,1}} \tag{3.27}
\]

are the signal and noise components of the received vector. Here, \( F_{2m,m}, m = 0,1 \) are the sub-matrices shown in (3.23). All the transmission vectors and the scaling factors are summarized in Table 3.1.

### Table 3.1: Transmitted vectors and scaling factors - RMCS

<table>
<thead>
<tr>
<th>Vector</th>
<th>Factor</th>
<th>Transmitted by</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_0 s)</td>
<td>(\alpha_0 = [p_0 T]^{\frac{1}{2}})</td>
<td>(S_1) in phase-0</td>
</tr>
<tr>
<td>(t_{1j} = \alpha_1 F_{1j} r_{1j})</td>
<td>(\alpha_1 = \left[ \frac{p_1}{N(p_0+1)} \right]^{\frac{1}{2}})</td>
<td>(L_1) relays in phase-1</td>
</tr>
<tr>
<td>(t_{2j} = \alpha_2 F_{2j} r_{2j})</td>
<td>(\alpha_2 = \left[ \frac{2p_2}{N(2+p_0p_2+p_1)} \right]^{\frac{1}{2}})</td>
<td>(L_2) relays in phase-2</td>
</tr>
</tbody>
</table>

#### 3.1.2 Maximum-likelihood decoder

The ML decoder maximizes the likelihood function of the received vector and estimates the transmitted signal. If \(y\) is the received vector, then the likelihood function \([39, 40]\) that \(s\) is transmitted is \(Pr(y|s)\) and the ML decoded vector is given by

\[
\hat{s} = \arg \max_s Pr(y|s). \tag{3.28}
\]

Now if \(y\) is jointly complex Gaussian, then we can write the likelihood function as \([41]\)

\[
Pr(y|s) = \frac{\exp \left[ -\frac{(y - m_y)^H P_y^{-1} (y - m_y)}{\pi 2^T |P_y|} \right]}{\pi 2^T |P_y|}, \tag{3.29}
\]

where \(m_y\) and \(P_y\) are the mean vector and the covariance matrix of \(y\). Now, we write the decoded vector as \([42]\)

\[
\hat{s} = \arg \max_s Pr(y|s) = \arg \min_s ||y'||^2, \tag{3.30}
\]
where

\[ y' = P_y^{-1} (y - m_y). \]

We will construct \( y \) from the vectors received by \( D_1 \) and prove that it is jointly Gaussian. We will also find \( m_y \) and \( P_y \) to be used to get \( \hat{s} \) from (3.30). \( y \) is constructed from either the three received vectors namely, \( r_{31}^{(0)} = x, r_{31}^{(1)} = z, \) and \( r_{31}^{(2)} = w \) or \( r_{31}^{(1)} = z \) and \( r_{31}^{(2)} = w, \) according to whether RMCS-3 or RMCS-2 is used. As mentioned earlier, the number RMCS-\( \mu, \mu = 2, 3 \) represents the number of leaked signals used in the system and that \( L_\mu \) is used. These vectors \( r_{31}^{(0)}, r_{31}^{(1)}, \) and \( r_{31}^{(2)} \) are shown in (3.3), (3.16), and (3.25) respectively.

For RMCS-3, \( y \) is obtained by stacking them as

\[
    y = \begin{bmatrix} x \\ z \\ w \end{bmatrix}.
\]

(3.31)

Then, the mean vector and covariance matrix of \( y \) are given by [42]

\[
    m_y = \begin{bmatrix} m_x \\ m_z \\ m_w \end{bmatrix} \quad \text{and} \quad P_y = \begin{bmatrix} P_x & P_{xz} & P_{xw} \\ P_{zx} & P_z & P_{zw} \\ P_{wx} & P_{wz} & P_w \end{bmatrix}
\]

(3.32)

respectively. Here, the covariance matrices \( P_x, P_z, \) and \( P_w \) are obtained from

\[
    P_x = E \left[ (x - m_x)(x - m_x)^H | s \right] = E \left[ u_x u_x^H \right],
\]

(3.33)

\[
    P_z = E \left[ (z - m_z)(z - m_z)^H | s \right] = E \left[ u_z u_z^H \right],
\]

(3.34)

and

\[
    P_w = E \left[ (w - m_w)(w - m_w)^H | s \right] = E \left[ u_w u_w^H \right]
\]

(3.35)

respectively. Also \( m_x = E[x|s], m_z = E[z|s], \) and \( m_w = E[w|s], \) are given in (3.4), (3.17), and (3.26) respectively and \( u_x, u_z, \) and \( u_w \) are given in (3.5), (3.18), and (3.27) respectively. Also \( P_{xz}, P_{xw}, \) and \( P_{zw} \) are the cross covariance matrices given by

\[
    P_{xz} = E \left[ (x - m_x)(z - m_z)^H | s \right] = E \left[ u_x u_z^H \right] = E \left[ u_x u_x^H \right]^H = P_{xz}^H,
\]

\[
    P_{xw} = E \left[ (x - m_x)(w - m_w)^H | s \right] = E \left[ u_x u_w^H \right] = E \left[ u_w u_x^H \right]^H = P_{xw}^H,
\]

\[
    P_{zw} = E \left[ (z - m_z)(w - m_w)^H | s \right] = E \left[ u_z u_w^H \right] = E \left[ u_w u_z^H \right]^H = P_{zw}^H,
\]

34
and

\[ P_{zw} = E[(z - m_z)(w - m_w)^H|s] = E[u_z u_w^H] = E[u_w u_z^H]^H = P_w^H \]

respectively. It can be seen from (3.3), (3.16), and (3.25) that \( x, z, \) and \( w \) are jointly Gaussian and hence \( y \) is jointly Gaussian as the channel coefficients are assumed to be known at \( D_1 \).

Similarly, for RMCS-2, \( y \) is obtained by stacking the vectors \( z \) and \( w \) as

\[ y = \begin{bmatrix} z \\ w \end{bmatrix}. \quad (3.36) \]

The mean vector and covariance matrix of \( y \) are given by [42]

\[ m_y = \begin{bmatrix} m_z \\ m_w \end{bmatrix} \quad \text{and} \quad P_y = \begin{bmatrix} P_z & P_{zw} \\ P_{wz} & P_w \end{bmatrix} \quad (3.37) \]

respectively.

Now, from (3.5) and (3.33), we get \( P_x \) as

\[ P_x = E[u_{3i_1}^{(0)} u_{3i_1}^{(0)H}] = I_T. \quad (3.38) \]

Also from (3.18) and (3.34), \( P_z \) is given as

\[ P_z = E[u_z u_z^H] = E\left(\sum_{i=1}^{N} \alpha_1 F_{1i} u_{1i}^{(0)} h_{1i,3,1} + u_{3i_1}^{(1)}\right)^H \left(\sum_{i=1}^{N} \alpha_1 F_{1i} u_{1i}^{(0)} h_{1i,3,1} + u_{3i_1}^{(1)}\right)
\]

\[ = \sum_{i=1}^{N} \sum_{j=1}^{N} E\left[\alpha_1^2 F_{1i} u_{1i}^{(0)} h_{1i,3,1} h_{1j,3,1}^* u_{1j}^{(0)H} F_{1j}^H\right] + E[u_{3i_1}^{(1)} u_{3i_1}^{(1)H}]
\]

\[ = \left(\alpha_1^2 \sum_{i=1}^{N} |h_{1i,3,1}|^2 + 1\right) I_T, \quad (3.39) \]

where the last step simplification used the properties of noise being i.i.d. ZMCG with unity variance and that the precoder matrices are orthogonal. Similarly \( P_w \) is obtained from (3.25) and (3.35) as

\[ P_w = E[u_w u_w^H]
\]

\[ = E\left[\frac{\alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \left[F_{2i,0} u_{2i}^{(0)} + F_{2i,1} u_{2i}^{(1)}\right] h_{2i,3,1} + \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,2,i} h_{2,i,3,1} F_{2i,1} F_{1k} u_{1k}^{(0)} + u_{31}^{(2)}\right]
\]

\[ + \left[\frac{\alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \left[F_{2i,0} u_{2i}^{(0)} + F_{2i,1} u_{2i}^{(1)}\right] h_{2i,3,1} + \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,2,i} h_{2,i,3,1} F_{2i,1} F_{1k} u_{1k}^{(0)} + u_{31}^{(2)}\right]^H
\]

\[ \text{respectively.} \]
\[
\begin{align*}
&= E \left[ \frac{\alpha_1^2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i,0} u_{2i}^{(0)} H H_{2i,k,3}^* \left( F_{2k,0}^* h_{2i,k,3,1}^{(0)} + \frac{\alpha_2^2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i,1} u_{2i}^{(1)} h_{2i,k,3,1}^{(1)} F_{2k,1}^* h_{2i,k,3,1}^* \right) + \frac{\alpha_1^2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,2,i} h_{2i,i,3,1} F_{2i,1}^* F_{1k}^* u_{1k}^{(0)} H H_{2j,1}^* h_{2j,3,1,1}^{*} F_{2j,1} \right] \\
&= E \left[ \frac{\alpha_1^2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \left( F_{2i,0} u_{2i}^{(0)} + F_{2i,1} u_{2i}^{(1)} \right) h_{2i,i,3,1}^{*} \left( h_{1,k,2,i} h_{2i,i,3,1}^{*} F_{2i,1} F_{1k} u_{1k}^{(0)} + u_{31}^{(2)} H H_{2j,1}^* h_{2j,3,1,1}^{*} F_{2j,1} \right) \right] \\
&= \left[ 1 + \alpha_2^2 \sum_{i=1}^{N} | h_{2i,i,3,1}^* |^2 \right] I_T + \frac{\alpha_1^2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{j=1}^{N} h_{1,k,2,i} h_{2i,i,3,1}^{*} h_{2j,3,1,1}^{*} F_{2i,1} F_{2j,1}^{H} H_{2j,1}. \tag{3.40}
\end{align*}
\]

where the last step simplification used the properties of noise being i.i.d. ZMCG with unity variance and that the precoder matrices are orthogonal. From (3.5) and (3.18) we get

\[
\mathbf{P}_{xx} = E \left[ u_x u_x^H \right] = E \left[ \left( \sum_{i=1}^{N} \alpha_1 \mathbf{F}_{1i} u_{1i}^{(0)} h_{1,i,i,3,1} + u_{31}^{(1)} \right) \right] \right] = \mathbf{0}_T. \tag{3.41}
\]

Similarly, \( \mathbf{P}_{ww} = E \left[ u_w u_w^H \right] = \mathbf{0}_T \) from the expressions of \( u_x \) and \( u_w \) shown in (3.5) and (3.27) respectively. Now from (3.18) and (3.27) we get

\[
\begin{align*}
\mathbf{P}_{zw} &= E \left[ u_z u_w^H \right] = E \left( \sum_{i=1}^{N} \alpha_1 \mathbf{F}_{1i} u_{1i}^{(0)} h_{1,i,i,3,1} + u_{31}^{(1)} \right) \\
&= \frac{\alpha_1^2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \mathbf{F}_{1i} u_{1i}^{(0)} h_{1,i,i,3,1}^{*} h_{2j,3,1,1}^{*} u_{1k}^{(0)} H H_{2j,1}^* h_{2j,3,1} \right] \right] \right]. \tag{3.42}
\end{align*}
\]

Here again we used i.i.d. ZMCG properties of the noise as \( u_{1i}^{(0)} \) in \( u_z \) and \( u_{1k}^{(0)} \) in \( u_w \) are the only components that are correlated. Also (3.42) can be simplified to

\[
\mathbf{P}_{zw} = \frac{\alpha_1^2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{j=1}^{N} h_{1,i,i,3,1}^{*} h_{2j,3,1,1}^{*} F_{2j,1} \right] \right] \right]. \tag{3.43}
\]

as \( E[u_{1i}^{(0)} u_{1k}^{(0)} H] \neq 0 \) only if \( i = k \) and \( \mathbf{F}_{1i} F_{1i}^{H} = \mathbf{I}_T \).

### 3.1.3 Receive SNR

Here, we will derive expressions for receive SNR. For RMCS-3, we can write \( \mathbf{y} \) as

\[
\begin{bmatrix}
\mathbf{x} \\
\mathbf{z} \\
\mathbf{w}
\end{bmatrix} = \begin{bmatrix}
\mathbf{m}_x \\
\mathbf{m}_z \\
\mathbf{m}_w
\end{bmatrix} + \begin{bmatrix}
\mathbf{u}_x \\
\mathbf{u}_z \\
\mathbf{u}_w
\end{bmatrix} = \mathbf{m} + \mathbf{u},
\]

\[36\]
where \( m_y \) and \( u_y \) given by

\[
m_y = \begin{bmatrix} m_x \\ m_z \\ m_w \end{bmatrix} \quad \text{and} \quad u_y = \begin{bmatrix} u_x \\ u_z \\ u_w \end{bmatrix}
\]  

(3.44)

are the signal and noise components of the concatenated received vector \( y \) respectively. Similarly, for RMCS-2, we can write \( y \) as

\[
y = \begin{bmatrix} z \\ w \end{bmatrix} = \begin{bmatrix} m_z \\ m_w \end{bmatrix} + \begin{bmatrix} u_z \\ u_w \end{bmatrix} = m_y + u_y,
\]

where \( m_y \) and \( u_y \) given by

\[
m_y = \begin{bmatrix} m_z \\ m_w \end{bmatrix} \quad \text{and} \quad u_y = \begin{bmatrix} u_z \\ u_w \end{bmatrix}
\]  

(3.45)

are the signal and noise components of the received vector \( y \) respectively.

Now, the received power can be written as

\[
E[y^H y] = P_s + P_n,
\]

where \( P_s \) is the received signal power given by

\[
P_s = E[m_y^H m_y] = E[m_x^H m_x] + E[m_z^H m_z] + E[m_w^H m_w] = P_s^{(1)} + P_s^{(2)}
\]  

(3.46)

and \( P_n \) is the noise power given by

\[
P_n = E[u_y^H u_y] = E[u_z^H u_z] + E[u_w^H u_w] = P_n^{(1)} + P_n^{(2)}
\]  

(3.47)

for RMCS-3. For RMCS-2 the received signal power is given by

\[
P_s = E[m_y^H m_y] = E[m_z^H m_z] + E[m_w^H m_w] = P_s^{(1)} + P_s^{(2)}
\]  

(3.48)

and the received noise power is given by

\[
P_n = E[u_y^H u_y] = E[u_z^H u_z] + E[u_w^H u_w] = P_n^{(1)} + P_n^{(2)}.
\]  

(3.49)

Here, the superscript \( (k) \), \( k \in [0, 2] \) in \( P_s \) and \( P_n \) in equations (3.46), (3.47), (3.48), and (3.49) denotes the phase number when the signal or the noise component is received.
Claim 3. The receive SNR of RMCS-3 and RMCS-2 are given by

\[ snr_{RMCS-3} = p_0(2\sigma_2^2p_1^2 + 4\sigma_2^2p_1 + p_2\sigma_2^2 + 2\sigma_3^2p_1 + 4\sigma_3^2 + p_2p_1) + p_0(2\sigma_3^2p_1 + 2\sigma_2^2p_1 + \sigma_2^2p_2 + 4\sigma_3^2 + 2\sigma_2^2\sigma_3^2) + 2\sigma_2^2\sigma_3^2p_0^3 \]

\[ (3.50) \]

\[ + 4p_0^2(2\sigma_3^2p_1 + 2\sigma_2^2p_1 + 4\sigma_3^2 + 6\sigma_2^2 + 4\sigma_3^2 + 2p_2 + 12) + 4\sigma_2^2p_1 + 6\sigma_2^2p_0^2 + 2\sigma_2^2p_1^2 + 12 \]

and

\[ snr_{RMCS-2} = (2p_1\sigma_4^2 + p_2\sigma_2^2)p_0^2 + (2\sigma_2^2p_1^2 + 4\sigma_2^2p_1 + p_2\sigma_2^2 + p_2p_1)p_0 \]

\[ (3.51) \]

\[ 4p_1 + 2p_2 + p_1p_2 + p_0(2p_1\sigma_4^2 + 4\sigma_2^2 + 4p_1 + 2p_2 + 8) + 4\sigma_2^2p_1 + 4\sigma_2^2p_0^2 + 2\sigma_2^2p_1^2 + 8 \]

respectively.

Proof: See Appendix C.

Now we can find \( \hat{p}_0 \), \( \hat{p}_1 \), and \( \hat{p}_2 \) by maximizing the receive SNR shown in (3.50) and (3.51). But as it is difficult, a fine computer search is carried out to find them as discussed in section 3.7.

3.2 Relay SNR combining scheme

Various phases of Relay SNR combining scheme (RSCS) are similar to that of RMCS shown in Fig. 3.2. The difference in this scheme is that the relays in L_2 layer combine the two received vectors using their respective SNRs.

3.2.1 Transmit and receive vectors

As mentioned earlier, in phase-2 the relays R_{2j} (1 \leq j \leq N) combine the two vectors \( r_{2j}^{(0)} \) and \( r_{2j}^{(1)} \) using their SNRs and create the transmission vector as

\[ t_{2j} = \alpha_2 F_{2j} \left[ \gamma_0 r_{2j}^{(0)} + \gamma_1 r_{2j}^{(1)} \right]. \]

(3.52)

Here, the precoder matrix \( F_{2j} \in \mathbb{C}^{T \times T} \), unlike in RMCS where \( F_{2j} \in \mathbb{C}^{T \times 2T} \). \( F_{2j} \) is orthogonal and hence \( F_{2j}^H F_{2j} = F_{2j}^H F_{2j} = I_T \). Also \( \gamma_0 \) and \( \gamma_1 \) are the SNRs of the received signals \( r_{2j}^{(0)} \) and \( r_{2j}^{(1)} \) respectively at \( R_{2j} \). These vectors are given in (3.2) and (3.13) respectively and repeated
where \( E \) complex conjugate of calculations. Here for convenience as

\[
\begin{align*}
\gamma_0 &= \frac{E \left( (\alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} h_{0i,2j} \right)^H (\alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} h_{0i,2j} \right)}{E \left( u_{2j}^{(0)} H u_{2j}^{(0)} \right)} = \frac{\alpha_0^2 \sigma_2^2}{T} = p_0 \sigma_2^2 \tag{3.55}
\end{align*}
\]

and

\[
\begin{align*}
\gamma_1 &= \frac{E \left( (\alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} u_{1i}^{(0)} h_{1i,2j} + u_{2j}^{(1)} \right)^H (\alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} u_{1i}^{(0)} h_{1i,2j} + u_{2j}^{(1)} \right)}{E \left( \alpha_0^2 \alpha_1^2 \sum_{i=1}^{N} |h_{0i,1i,j}|^2 |h_{1i,2j}|^2 \right)} = \frac{\alpha_0^2 \alpha_1^2 N T}{\alpha_0^2 \alpha_1^2 N T + \frac{p_0 p_1}{1 + p_0 + p_1}} \tag{3.56}
\end{align*}
\]

respectively. The last step in (3.55) and (3.56) are obtained by substituting the values of \( \alpha_0 \) and \( \alpha_1 \) from (3.6) and (3.12) respectively.

Now the scaling factor \( \alpha_2 \) is to be selected to maintain the power transmitted by each of the L_2 relays as \( p_2 T/N \) in T duration. Before that, we will derive one result concerning power calculations.

Let \( a, b \in \mathbb{C}^{n \times 1} \) be random complex vectors. Then

\[
E \left[ (a + b)^H (a + b) \right] = E \left[ a^H a \right] + E \left[ a^H b \right] + E \left[ b^H a \right] + E \left[ b^H b \right]
\]

\[
= E \left[ a^H a \right] + E \left[ a^H b \right] + (E \left[ a^H b \right])^H + E \left[ b^H b \right] = E \left[ a^H a \right] + 2 \Re \left[ E \left[ a^H b \right] \right] + E \left[ b^H b \right], \tag{3.57}
\]

where \( \Re[.] \) represents the real part of [.] and we have used the property that \( (E \left[ a^H b \right])^H \) is the complex conjugate of \( E \left[ a^H b \right] \). Using (3.52) and (3.57), we get

\[
\begin{align*}
\frac{p_2 T}{N} &= E \left[ r_{2j}^H t_{2j} \right] = E \left[ (\alpha_2 F_{2j} \left[ \gamma_0 r_{2j}^{(0)} + \gamma_1 r_{2j}^{(1)} \right])^H (\alpha_2 F_{2j} \left[ \gamma_0 r_{2j}^{(0)} + \gamma_1 r_{2j}^{(1)} \right]) \right] \\
&= \alpha_2^2 E \left[ \gamma_0^2 r_{2j}^{(0)H} + \gamma_1 r_{2j}^{(1)H} \gamma_0^2 r_{2j}^{(0)} + \gamma_1 r_{2j}^{(1)} \right] \\
&= \alpha_2^2 \left[ \gamma_0^2 E \left( r_{2j}^{(0)H} r_{2j}^{(0)} \right) + 2 \gamma_0 \gamma_1 \Re \left( E \left( r_{2j}^{(0)H} r_{2j}^{(1)} \right) \right) + \gamma_1^2 E \left( r_{2j}^{(1)H} r_{2j}^{(1)} \right) \right]. \tag{3.58}
\end{align*}
\]
Now from (3.53) and (3.54) we get

\[ E \left[ r_{2j}^{(0)} H_r^{(0)} r_{2j}^{(0)} \right] = \alpha_0^2 \sigma_2^2 + T, \quad E \left[ r_{2j}^{(0)} H_r^{(1)} r_{2j}^{(0)} \right] = 0, \quad \text{and} \quad E \left[ r_{2j}^{(1)} H_r^{(1)} r_{2j}^{(1)} \right] = \alpha_0^2 \alpha_1^2 N + \alpha_1^2 NT + T. \]

Substituting the above in (3.58), we get

\[
\frac{p_2 T}{N} = \alpha_2^2 \left[ \gamma_0^2 \left( \alpha_0^2 \sigma_2^2 + T \right) + \gamma_1^2 \left( \alpha_0^2 \alpha_1^2 N + \alpha_1^2 NT + T \right) \right] ^{\frac{1}{2}}.
\] (3.59)

In phase-0 and phase-1, D_1 receives \( r_{31}^{(0)} \) and \( r_{31}^{(1)} \) as shown in (3.3) and (3.16) respectively. In phase-2, it receives

\[
r_{31}^{(2)} = \sum_{j=1}^{N} t_{2j} h_{2,j,3,1} + u_{31}^{(2)} = \sum_{j=1}^{N} \alpha_2 F_{2j} \left[ \gamma_0 r_{2j}^{(0)} + \gamma_1 r_{2j}^{(1)} \right] h_{2,j,3,1} + u_{31}^{(2)}
\]

\[
= \sum_{j=1}^{N} \alpha_2 F_{2j} \gamma_0 \left( \alpha_0 s h_{0,1,2,j} + u_{2j}^{(0)} \right) h_{2,j,3,1}
\]

\[
+ \sum_{j=1}^{N} \alpha_2 F_{2j} \gamma_1 \left( \alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} s h_{0,1,1,i} h_{1,i,2,j} + \alpha_1 \sum_{i=1}^{N} F_{1i} u_{i1}^{(0)} h_{1,i,2,j} + u_{2j}^{(1)} \right) h_{2,j,3,1} + u_{31}^{(2)}
\]

\[
= \alpha_0 \alpha_2 \gamma_0 \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2j} s + \alpha_0 \alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{0,1,1,i} h_{1,i,2,j} h_{2,j,3,1} F_{2j} F_{1i} s
\]

\[
+ \alpha_2 \gamma_0 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(0)} + \alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{1,i,2,j} h_{2,j,3,1} F_{2j} F_{1i} u_{i1}^{(0)}
\]

\[
+ \alpha_2 \gamma_1 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(1)} + u_{31}^{(2)}
\]

\[
= w = m_w + u_w,
\] (3.60)

where \( m_w \) and \( u_w \) are the signal and noise components of \( r_{31}^{(2)} \) given by

\[
m_w = \alpha_0 \alpha_2 \gamma_0 \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2j} s + \alpha_0 \alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{0,1,1,i} h_{1,i,2,j} h_{2,j,3,1} F_{2j} F_{1i} s
\] (3.61)
and

\[
\mathbf{u}_w = \alpha_2 \gamma_0 \sum_{j=1}^{N} h_{2,j,3,1} \mathbf{F}_{2j} \mathbf{u}_{2j}^{(0)} + \alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{1,i,2,j} h_{2,j,3,1} \mathbf{F}_{2j} \mathbf{F}_{1i} \mathbf{u}_{1i}^{(0)} \\
+ \alpha_2 \gamma_1 \sum_{j=1}^{N} h_{2,j,3,1} \mathbf{F}_{2j} \mathbf{u}_{2j}^{(1)} + \mathbf{u}_{31}^{(2)}
\]

(3.62)

respectively. The transmission vectors and the corresponding scaling factors are summarized in Table 3.2.

<table>
<thead>
<tr>
<th>Vector</th>
<th>Scaling factor</th>
<th>Transmitted by</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{01} = \alpha_0 s)</td>
<td>(\alpha_0 = [p_0 T]^{1/2})</td>
<td>(S_1) in phase-0</td>
</tr>
<tr>
<td>(t_{1j} = \alpha_1 \mathbf{F}<em>{1j} r</em>{1j}^{(0)})</td>
<td>(\alpha_1 = \left[\frac{p_1}{N(p_0+1)}\right]^{1/2})</td>
<td>(L_1) relays in phase-1</td>
</tr>
<tr>
<td>(t_{2j} = \alpha_2 \mathbf{F}<em>{2j} \left[\gamma_0 r</em>{2j}^{(0)} + \gamma_1 r_{2j}^{(1)}\right])</td>
<td>(\alpha_2 = \left[\frac{\gamma_2}{\gamma_0^2(1+\rho_0\sigma_2^2)+\gamma_1^2(1+\rho_1)}\right]^{1/2})</td>
<td>(L_2) relays in phase-2</td>
</tr>
</tbody>
</table>

Table 3.2.

### 3.2.2 Maximum-likelihood decoder

The three received vectors at \(D_1\) for RSCS are \(r_{31}^{(0)}\), \(r_{31}^{(1)}\), and \(r_{31}^{(2)}\) as shown in (3.3), (3.16), and (3.60) respectively. We can concatenate these vectors into \(\mathbf{y}\) as shown in (3.31) and (3.36) respectively for RSCS-3 and RSCS-2. It can be seen as in RMCS, that \(\mathbf{y}\) is jointly Gaussian and that the mean vector, \(\mathbf{m}_y\) and covariance matrix, \(\mathbf{P}_y\) of \(\mathbf{y}\) are given in (3.32) for RSCS-3 and (3.37) for RSCS-2. Here, \(\mathbf{m}_x\), \(\mathbf{P}_x\), \(\mathbf{m}_z\), and \(\mathbf{P}_z\) are the same as that shown in (3.4), (3.38), (3.17), and (3.39) respectively. The mean vector \(\mathbf{m}_w\) is given in (3.61). Now the covariance matrix of \(\mathbf{w}\) is

\[
\mathbf{P}_w = E \left[\mathbf{u}_w \mathbf{u}_w^H\right].
\]

(3.63)

Now from (3.62), we get

\[
\mathbf{P}_w = E \left[\alpha_2 \gamma_0 \sum_{j=1}^{N} h_{2,j,3,1} \mathbf{F}_{2j} \mathbf{u}_{2j}^{(0)} \mathbf{u}_w^H\right] + E \left[\alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{1,i,2,j} h_{2,j,3,1} \mathbf{F}_{2j} \mathbf{F}_{1i} \mathbf{u}_{1i}^{(0)} \mathbf{u}_w^H\right] \\
+ E \left[\alpha_2 \gamma_1 \sum_{j=1}^{N} h_{2,j,3,1} \mathbf{F}_{2j} \mathbf{u}_{2j}^{(1)} + \mathbf{u}_{31}^{(2)} \mathbf{u}_w^H\right].
\]
\[
\begin{align*}
\text{Now, we can obtain the decoded vector given by} \\
\hat{s} &= \arg \max_s \Pr(y|s) = \arg \min_s \|y'\|^2, \quad (3.69)
\end{align*}
\]

where \(y' = P_y^{-\frac{1}{2}}(y - m_y)\).
3.2.3 Receive SNR

The receive SNR for RSCS-3 is given by

\[
\text{snr}_{\text{RSCS-3}} = \frac{P_s}{P_n} = \frac{P_s^{(0)} + P_s^{(1)} + P_s^{(2)}}{P_n^{(0)} + P_n^{(1)} + P_n^{(2)}},
\]

(3.70)

and that of RSCS-2 is given by

\[
\text{snr}_{\text{RSCS-2}} = \frac{P_s}{P_n} = \frac{P_s^{(1)} + P_s^{(2)}}{P_n^{(1)} + P_n^{(2)}},
\]

(3.71)

**Claim 4.** The receive SNR of RSCS-3 and RSCS-2 can be found to be

\[
\text{snr}_{\text{RSCS-3}} = \\
p_0^1\sigma_0^2 p_1 + \sigma_0^5 p_2 + \sigma_0^4 p_2 + 3\sigma_0^3 p_3 + 2\sigma_0^2 p_4 + 2\sigma_0^2 p_5 + 3\sigma_0^2 p_6 + p_0^2(\sigma_0^5 p_1 + 2\sigma_0^5 p_1 + 3\sigma_0^5 p_2) \\
+ 3\sigma_0^4 p_3 + 3\sigma_0^3 p_4 + 2\sigma_0^2 p_5 + 2\sigma_0^2 p_6 + 4\sigma_0^2 p_7 p_1 + 6\sigma_0^2 p_7 p_2 + 2\sigma_0^2 p_7 p_3 + 2\sigma_0^2 p_7 p_4 + 2\sigma_0^2 p_7 p_5 + 2\sigma_0^2 p_7 p_6 + 2\sigma_0^2 p_7 p_7 \\
+ p_0^2(2\sigma_0^2 p_1 + \sigma_0^2 p_2 + 3\sigma_0^2 p_3 + \sigma_0^2 p_4 + \sigma_0^2 p_5 + \sigma_0^2 p_6 + \sigma_0^2 p_7 + 2\sigma_0^2 p_8) \\
+ 2\sigma_0^4 p_8 p_1 + 4\sigma_0^4 p_8 p_2 + 2\sigma_0^4 p_8 p_3 + 6\sigma_0^4 p_8 p_4 + 2\sigma_0^4 p_8 p_5 + 2\sigma_0^4 p_8 p_6 + 2\sigma_0^4 p_8 p_7 + 4\sigma_0^4 p_8 p_8 \\
+ p_0(\sigma_0^5 p_1 + \sigma_0^5 p_2 + \sigma_0^5 p_3 + \sigma_0^5 p_4 + \sigma_0^5 p_5 + \sigma_0^5 p_6 + \sigma_0^5 p_7 + \sigma_0^5 p_8) \\
+ \sigma_0^5 p_1 + 2\sigma_0^5 p_2 + \sigma_0^5 p_3 + \sigma_0^5 p_4 + \sigma_0^5 p_5 + \sigma_0^5 p_6 + \sigma_0^5 p_7 + 2\sigma_0^5 p_8 p_1 \\
+ p_0(12\sigma_0^3 p_1 + 8\sigma_0^3 p_1 + 4\sigma_0^3 p_1 + 3\sigma_0^3 p_2 + p_0^3 p_2 + 9\sigma_0^3 p_2 + 3\sigma_0^3 p_3 + 3\sigma_0^3 p_4 + 3\sigma_0^3 p_5 + 3\sigma_0^3 p_6 + 3\sigma_0^3 p_7 + 3\sigma_0^3 p_8) \\
+ 3\sigma_0^3 p_1 + 5\sigma_0^2 p_1 + 2\sigma_0^2 p_1 + \sigma_0^2 p_1 + \sigma_0^2 p_1 + 4\sigma_0^2 p_1 p_1 + 4\sigma_0^2 p_1 p_2) \\
+ p_0^3(6\sigma_0^2 p_1 + \sigma_0^2 p_2 + \sigma_0^2 p_2 + 3\sigma_0^2 p_3 + 9\sigma_0^2 p_4 + 3\sigma_0^2 p_5 + 3\sigma_0^2 p_6 + 3\sigma_0^2 p_7 + 2\sigma_0^2 p_8 p_1) \\
+ 3\sigma_0^2 p_2 + 9\sigma_0^2 p_2 + 3\sigma_0^2 p_3 + 3\sigma_0^2 p_4 + 2\sigma_0^2 p_5 + 2\sigma_0^2 p_6 + 6\sigma_0^2 p_7 + 6\sigma_0^2 p_8 p_1 \\
+ \sigma_0^2 p_3 + \sigma_0^2 p_4 + \sigma_0^2 p_5 + \sigma_0^2 p_6 + \sigma_0^2 p_7 + \sigma_0^2 p_8 p_1 + 3\sigma_0^2 p_8 p_2 + 3\sigma_0^2 p_8 p_3 + 3\sigma_0^2 p_8 p_4 + 3\sigma_0^2 p_8 p_5 + 3\sigma_0^2 p_8 p_6 + 3\sigma_0^2 p_8 p_7 + 3\sigma_0^2 p_8 p_8 \\
+ \sigma_0^2 p_3 + \sigma_0^2 p_4 + \sigma_0^2 p_5 + \sigma_0^2 p_6 + \sigma_0^2 p_7 + \sigma_0^2 p_8 p_1 + 2\sigma_0^2 p_8 p_2 + 2\sigma_0^2 p_8 p_3 + 2\sigma_0^2 p_8 p_4 + 2\sigma_0^2 p_8 p_5 + 2\sigma_0^2 p_8 p_6 + 2\sigma_0^2 p_8 p_7 + 2\sigma_0^2 p_8 p_8
\]

(3.72)
and

\[ snr_{RSCS-2} = \frac{(p_1\sigma_1^2 + p_2\sigma_0^2)p_0^2 + (\sigma_0^2p_1 + 2\sigma_2^2)p_2 + 3\sigma_2^2p_2 + 2\sigma_3^2p_1 + 2\sigma_3^2p_2)p_0^2}{p_0(8\sigma_1^2p_1 + 6\sigma_0^2p_1 + \sigma_2^2p_2 + 2\sigma_2^2p_1 + 6\sigma_2^2 + 2\sigma_2^2) + 2\sigma_1^2p_1 + 2\sigma_1^2p_2 + 4\sigma_2^2p_1 + 2\sigma_2^2p_2 + \sigma_2^2p_1 + \sigma_2^2p_2} \]

(3.73)

respectively.

Proof: See Appendix D.

Now allocation of \( p_0, p_1, \) and \( p_2 \) can be done by maximizing the receive SNR shown in (3.72) and (3.73) for RSCS-3 and RSCS-2 respectively. But as it is difficult, a fine computer search is carried out to find \( \hat{p}_0, \hat{p}_1, \) and \( \hat{p}_2, \) which maximize SNR as discussed in section 3.7.

### 3.3 Relay matrix combining with known channel scheme

Various phases of relay matrix combining with known channel scheme (RMCKCS) are similar to that of RMCS shown in Fig. 3.2 of section 3.1. In this scheme, the relays \( R_{ij} \) are presumed to know the local backward CSI; \( R_{1j} \) has knowledge of \( h_{0,i,1,j} \), and \( R_{2j} \) has knowledge of \( h_{0,i,2,j} \) and \( h_{1,i,2,j}, \ i \in \{1, \ldots, N\} \).

#### 3.3.1 Transmit and receive vectors

In phase-1, the \( L_1 \) relays transmit \( t_{1j} \) where \( t_{1j} = \alpha_1 F_{1j} r_{1j}^{(0)} h_{0,1,j}^* \). Let us find \( \alpha_1 \), the scaling factor to maintain the power transmitted by the \( L_1 \) layer relays to \( p_1 T / N \) in phase-1. This power
is given by
\[
E \left[ t_{ij}^H t_{ij} \right] = E \left[ \left( \alpha_1 F_{1j} r_{1j}^{(0)} h_{0,1,1,j}^* \right)^H \left( \alpha_1 F_{1j} r_{1j}^{(0)} h_{0,1,1,j}^* \right) \right] = \alpha_1^2 E \left[ h_{0,1,1,j}^2 r_{1j}^{(0)} H r_{1j}^{(0)} \right]
\]
\[
= \alpha_1^2 E \left[ |h_{0,1,1,j}|^2 \left( \alpha_0 s h_{0,1,1,j} + u_{1j}^{(0)} \right)^H \left( \alpha_0 s h_{0,1,1,j} + u_{1j}^{(0)} \right) \right]
\]
\[
= \alpha_1^2 \left[ E \left( \alpha_0^2 |h_{0,1,1,j}|^4 + T \right) \right]. \tag{3.74}
\]

Now, we will derive a result on random variables, which is used in the subsequent discussions.

When \( x \) is \( x \sim N_{R}(0, \sigma^2) \), then \([43]\)

\[
E \left[ x^n \right] = \begin{cases} 
0, & \text{if } n \text{ is odd} \\
1 \cdot 3 \cdots (n-1) \sigma^n, & \text{if } n \text{ is even}
\end{cases} \tag{3.75}
\]

with \( n \in \mathbb{N} \). Now, if \( z = x + jy \), \( z \sim N_{C}(0, \sigma_z^2) \) with \( x \) and \( y \) i.i.d., \( x \sim N_{R}(0, \sigma^2) \) and\( y \sim N_{R}(0, \sigma^2) \), then \( \sigma_z^2 = E[|z|^2] = E[x^2 + y^2] = 2\sigma^2 \) and

\[
E \left[ |z|^4 \right] = E \left[ (x^2 + y^2)^2 \right] = E \left[ x^4 + y^4 + 2x^2y^2 \right] = 2E \left[ x^4 \right] + 2 \left[ E \left[ x^2 \right] \right]^2. \tag{3.76}
\]

Using (3.75), (3.76) becomes

\[
E \left[ |z|^4 \right] = 2 \left[ 1 \cdot 3 \cdot \sigma^4 \right] + 2 \left[ \sigma^2 \right]^2 = 8\sigma^4 = 2\sigma_z^4. \tag{3.77}
\]

Now, \( \alpha_1 \) is obtained from (3.74) as

\[
\frac{p_1 T}{N} = \alpha_1^2 \left[ E \left( \alpha_0^2 |h_{0,1,1,j}|^4 + T \right) \right] = \alpha_1^2 \left( 2\alpha_0^2 \sigma_1^4 + T \right) = \alpha_1^2 \left( 2\alpha_0^2 \sigma_2^4 + T \right)
\]

\[
\Rightarrow \alpha_1 = \left[ \frac{p_1}{N (1 + 2p_0)} \right]^{\frac{1}{2}} \tag{3.78}
\]

as \( E[|h_{0,1,1,j}|^2] = \sigma_1^2 = 1 \) and \( \alpha_0^2 = p_0 T \). The received vector at \( R_{2j} \) in phase-1 is

\[
r_{2j}^{(1)} = \sum_{i=1}^{N} t_{i,i} h_{1,i,2,j} + u_{2j}^{(1)} = \sum_{i=1}^{N} \alpha_1 F_{1i} r_{1i}^{(0)} h_{0,1,1,j}^* h_{1,i,2,j} + u_{2j}^{(1)}
\]
\[
= \sum_{i=1}^{N} \alpha_1 F_{1i} \left( \alpha_0 s h_{0,1,1,i} + u_{1i}^{(0)} \right) h_{0,1,1,i}^* h_{1,i,2,j} + u_{2j}^{(1)}
\]
\[
= \alpha_0 \alpha_1 \sum_{i=1}^{N} |h_{0,1,1,i}|^2 h_{1,i,2,j} F_{1i} s + \alpha_1 \sum_{i=1}^{N} h_{0,1,1,i}^* h_{1,i,2,j} F_{1i} u_{1i}^{(0)} + u_{2j}^{(1)}. \tag{3.79}
\]
The received vector $r_{31}^{(1)}$ at $D_1$ is given by
\[ r_{31}^{(1)} = \sum_{j=1}^{N} t_{1j} h_{1,j,3,1} + u_{31}^{(1)} = \sum_{j=1}^{N} \alpha_1 F_{1j} r_{1j}^{(0)} h_{0,1,1,j}^* h_{1,j,3,1} + u_{31}^{(1)} \]
\[ = \sum_{j=1}^{N} \alpha_1 F_{1j} \left( \alpha_0 s h_{0,1,1,j} + u_{1j}^{(0)} \right) h_{0,1,1,j}^* h_{1,j,3,1} + u_{31}^{(1)} \]
\[ = \alpha_0 \alpha_1 \sum_{j=1}^{N} |h_{0,1,1,j}|^2 h_{1,j,3,1} F_{1j} s + \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j}^* h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + u_{31}^{(1)} \]
\[ = z = m_z + u_z, \quad (3.80) \]

where
\[ m_z = \alpha_0 \alpha_1 \sum_{j=1}^{N} |h_{0,1,1,j}|^2 h_{1,j,3,1} F_{1j} s \quad (3.81) \]

and
\[ u_z = \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j}^* h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + u_{31}^{(1)} \quad (3.82) \]

are the signal and noise components respectively.

In phase-2, $L_2$ layer relays transmit $t_{2j}$ given by
\[ t_{2j} = \alpha_2 F_{2j} r_{2j}, \quad (3.83) \]

where $r_{2j}$ is a concatenated vector given by
\[ r_{2j} = \begin{bmatrix} r_{2j}^{(0)} h_{0,1,2,j}^* \\ r_{2j}^{(1)} \| h_{1,2,j} \| \end{bmatrix}. \]

Here, $F_{2j}$ is the same as that of RMCS shown in (3.23). The multiplying factor $h_{0,1,2,j}^*$ is the conjugate of the channel the transmitted signal would have gone through when $r_{2j}^{(0)}$ is received. Also $h_{1,2,j} = [h_{1,1,2,j}, \ldots, h_{1,N,2,j}]^T$ and the transmitted signal would have gone through this vector of channel coefficients $h_{1,2,j}$ when $r_{2j}^{(1)}$ is received, and hence intuitively $\| h_{1,2,j} \|$ is selected as the multiplying factor. This is the norm of the vector $h_{1,2,j}$ given by
\[ \| h_{1,2,j} \| = \left[ \sum_{p=1}^{N} |h_{1,p,2,j}|^2 \right]^{1/2}. \quad (3.84) \]

To achieve the total average power transmitted per symbol duration as $p_2/N$ in phase-2 by each of the relays, $\alpha_2$ is to be appropriately selected. We have the transmitted power in $T$ channel
Claim 5. We should select the scaling factor $\alpha_2$ in (3.83) as

$$\alpha_2 = \left[ \frac{2p_2}{N (2p_0\sigma_1^2 + \sigma_2^2 + p_1 (N + 1) + NT)} \right]^{\frac{1}{2}}$$

(3.86)

to keep the average power transmitted by $L_t$ uses as $p_2 T / N$.

Proof: See Appendix E. \[\blacksquare\]

The received vector $r_{31}^{(2)}$ at $D_1$ is given by

$$r_{31}^{(2)} = \sum_{j=1}^{N} t_{2j} h_{2,j,3,1} + u_{31}^{(2)} = \sum_{j=1}^{N} \alpha_2 F_{2j} r_{2j} h_{2,j,3,1} + u_{31}^{(2)}$$

$$= \sum_{j=1}^{N} \alpha_2 \frac{1}{\sqrt{2}} \left[ F_{2j,0} F_{2j,1} \begin{bmatrix} r_{2j}^{(0)} h_{0,1,2,j}^* & \end{bmatrix} h_{2,j,3,1} + u_{31}^{(2)} \right]$$

$$= \sum_{j=1}^{N} \alpha_2 \frac{1}{\sqrt{2}} \left[ F_{2j,0} r_{2j}^{(0)} h_{0,1,2,j} h_{2,j,3,1} + F_{2j,1} r_{2j}^{(1)} \parallel h_{1,2,j} \parallel h_{2,j,3,1} \right] + u_{31}^{(2)}$$

$$= \sum_{j=1}^{N} \frac{1}{\sqrt{2}} \alpha_2 \left[ \alpha_0 s h_{0,1,2,j} + u_{2j}^{(0)} \right] h_{0,1,2,j} h_{2,j,3,1}$$

$$+ \alpha_2 \frac{1}{\sqrt{2}} \sum_{j=1}^{N} F_{2j,1} \left( \alpha_0 \alpha_1 \sum_{i=1}^{N} |h_{0,1,1,i}|^2 h_{1,1,2,j} F_{1i} s + \alpha_1 \sum_{i=1}^{N} h_{0,1,1,i} h_{1,1,2,j} F_{1i} u_{1i} + u_{2j}^{(1)} \right)$$

$$= \frac{\alpha_0 \alpha_2}{\sqrt{2}} \sum_{j=1}^{N} |h_{0,1,2,j}|^2 h_{2,j,3,1} F_{2j,0} s + \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \sum_{i=1}^{N} \parallel h_{1,1,2,j} \parallel h_{2,j,3,1} h_{1,1,2,j} |h_{0,1,1,i}|^2 F_{2j,1} F_{1i} s$$

$$+ \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \sum_{i=1}^{N} \parallel h_{1,2,j} \parallel h_{2,j,3,1} h_{0,1,1,i} h_{1,1,2,j} F_{2j,1} F_{1i} u_{1i} + \frac{\alpha_2}{\sqrt{2}} \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2j,0} u_{2j}^{(0)}$$

$$+ \frac{\alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \parallel h_{1,2,j} \parallel h_{2,j,3,1} F_{2j,1} u_{2j}^{(1)} + u_{31}^{(2)}$$

$$= w = m_w + u_w$$

(3.87)
where

\[
m_w = \alpha_0 \alpha_2 \sum_{j=1}^{N} |h_{0,1,2,j}|^2 h_{2,j,3,1} F_{2,j,0} s
\]

\[
+ \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{j=1}^{N} \|h_{1,2,j}\| h_{2,j,3,1} h_{1,i,2,j} |h_{0,1,i,j}|^2 F_{2,j,1} F_{1,i} s
\]

(3.88)

and

\[
u_w = \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \sum_{i=1}^{N} \|h_{1,2,j}\| h_{2,j,3,1} h_{0,1,i,j} h_{1,i,2,j} F_{2,j,1} F_{1,i} u^{(0)}_{1_j} + \frac{\alpha_2}{\sqrt{2}} \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2,j,0} u^{(0)}_{2_j}
\]

\[
+ \frac{\alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \|h_{1,2,j}\| h_{2,j,3,1} F_{2,j,1} u^{(1)}_{2_j} + u^{(2)}_{31}
\]

(3.89)

are the signal and noise components of the received vector \(r_{31}^{(2)}\) respectively.

Expressions for \(\alpha_0, \alpha_1, \alpha_2\), and the transmission vectors are summarized in Table 3.3.

Table 3.3: Transmitted vectors and scaling factors - RMCKCS

<table>
<thead>
<tr>
<th>Vector</th>
<th>Factor</th>
<th>Transmitted by</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{01}) = (\alpha_0 s)</td>
<td>(\alpha_0 = \left[p_0 T\right]^2)</td>
<td>(S_1) in phase-0</td>
</tr>
<tr>
<td>(t_{1j} = \alpha_1 F_{1j} r^{(0)}<em>{2j} h^*</em>{0,1,1,j})</td>
<td>(\alpha_1 = \left[p_1 \frac{p_0}{N(1+2p_0)}\right]^2)</td>
<td>(L_1) relays in phase-1</td>
</tr>
<tr>
<td>(t_{2j} = \frac{\alpha_2}{\sqrt{2}} [F_{2,j,0} F_{2,j,1}] r^{(0)}<em>{2j} h^*</em>{0,1,2,j} \frac{r^{(1)}<em>{2j}}{|h</em>{1,2,j}|})</td>
<td>(\alpha_2) shown in (3.86)</td>
<td>(L_2) relays in phase-2</td>
</tr>
</tbody>
</table>

### 3.3.2 Maximum-likelihood decoder

The three received vectors at \(D_1\) for RMCKCS are \(r_{31}^{(0)}, r_{31}^{(1)},\) and \(r_{31}^{(2)}\), as shown in (3.3), (3.80), and (3.87) respectively. Now \(y\) can be constructed as a concatenation of these vectors for RMCKCS-3 and RMCKCS-2 as shown in (3.31) and (3.36) respectively. It can be seen as in RMCS, that \(y\) is jointly Gaussian and that the mean vector, \(m_y\) and covariance matrix, \(P_y\) of \(y\) are given in (3.32) for RMCKCS-3 and (3.37) for RMCKCS-2. Here, \(m_x\) and \(P_x\) are the same as that shown in (3.4) and (3.38) respectively. Now, the decoded vector is given by

\[
\hat{s} = \arg \max_s Pr(y|s) = \arg \min_s \|y'|^2,
\]

where \(y' = P_y^{-\frac{1}{2}}(y - m_y)\). Now, we need to find the mean and covariance matrices of the received signal vectors \(x, z,\) and \(w\) as \(P_y\) and \(m_y\) given in (3.32) and (3.37) depend on them.
The mean vectors $m_x$, $m_z$, and $m_w$ are given in (3.4), (3.81), and (3.88) respectively. $P_x$ is given in (3.38) under RMCS in section 3.1. From (3.82), $P_z$ is given by

$$P_z = E \left[ \left( \alpha_1 \sum_{j=1}^{N} h_{0,1,j}^* h_{1,j,3,i} F_{1j} u_{1j}^{(0)} + u_{31}^{(1)} \right) \left( \alpha_1 \sum_{j=1}^{N} h_{0,1,j}^* h_{1,j,3,i} F_{1j} u_{1j}^{(0)} + u_{31}^{(1)} \right)^H \right]$$

$$= \left[ 1 + \alpha_1^2 \sum_{j=1}^{N} |h_{0,1,j}|^2 |h_{1,j,3,i}|^2 \right] I_T. \quad (3.91)$$

Now $P_w$ is given by

$$P_w = E \left[ u_w u_w^H \right], \quad (3.92)$$

where $u_w$ is given in (3.89) and given below for convenience as

$$u_w = \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \sum_{i=1}^{N} \|h_{1,2,j}\| h_{2,3,1} h_{0,1,1,i} h_{1,2,j} F_{2j1} u_{1i}^{(0)} + \frac{\alpha_2}{\sqrt{2}} \sum_{j=1}^{N} h_{0,1,2,j} h_{2,3,1} F_{2j0} u_{2j}^{(0)}$$

$$+ \frac{\alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \|h_{1,2,j}\| h_{2,3,1} F_{2j1} u_{2j}^{(1)} + u_{31}^{(2)}. \quad (3.93)$$

So (3.92) becomes

$$P_w = E \left[ u_w u_w^H \right]$$

$$= \frac{\alpha_1 \alpha_2}{2} \sum_{j=1}^{N} \sum_{i=1}^{N} \|h_{1,2,j}\| h_{2,3,1} h_{0,1,1,i} h_{1,2,j} F_{2j1} u_{1i}^{(0)} F_{1i} F_{2j1}^H$$

$$+ E \left[ u_w u_{31}^{(2)H} \right]$$

$$= \frac{\alpha_1 \alpha_2}{2} \sum_{j=1}^{N} \sum_{i=1}^{N} \sum_{k=1}^{N} \|h_{1,2,j}\| \|h_{1,2,k}\| h_{2,3,1} h_{0,1,1,i} h_{1,2,k} F_{2j1} F_{2k1}^H$$

$$+ \left[ \frac{\alpha_2}{2} \sum_{j=1}^{N} (|h_{0,1,2,j}|^2 + \|h_{1,2,j}\|^2) |h_{2,3,1}|^2 + 1 \right] I_T. \quad (3.94)$$

Now the cross covariance matrix is given as $P_{zw}$

$$P_{zw} = E \left[ u_z u_w^H \right] = E \left( \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j} h_{1,j,3,i} F_{1j} u_{1j}^{(0)} + u_{31}^{(1)} \right) u_w^H \quad (3.95)$$

where $u_w$ is given in (3.93). Hence

$$P_{zw} = \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{j=1}^{N} \sum_{i=1}^{N} \|h_{1,2,j}\| h_{2,3,1} h_{0,1,1,i} |h_{1,2,k}|^2 h_{1,2,k} F_{2j1}^H. \quad (3.96)$$
3.3.3 Receive SNR

As for the other schemes, the receive SNR for RMCKCS-3 is defined as

\[ \text{snr}_{\text{RMCKCS-3}} \triangleq \frac{P_s(0) + P_s(1) + P_s(2)}{P_n(0) + P_n(1) + P_n(2)} \]  \hspace{1cm} (3.97)

and for RMCKCS-2, it is

\[ \text{snr}_{\text{RMCKCS-2}} \triangleq \frac{P_s(1) + P_s(2)}{P_n(1) + P_n(2)} \]  \hspace{1cm} (3.98)

**Claim 6.** The receive SNR of RMCKCS-3 and RMCKCS-2 can be found to be

\[ \text{snr}_{\text{RMCKCS-3}} = \frac{p_0(2p_1p_2 + N\sigma_3^2 + \sigma_3^2p_1 + 2\sigma_4^2p_1 + 2\sigma_2^2\sigma_3^2 + 2\sigma_2^2p_1^2 + 2N\sigma_2^2p_1^2 + 2Np_1^2p_2 + 2Np_2^2p_1 + N\sigma_2^2p_1^2 + 4\sigma_2^2p_1 + 2\sigma_2^2p_2 + 3\sigma_2^2 + 12\sigma_2^4p_0^2 + 8\sigma_2^4p_1^2 + N\sigma_2^2p_1^2 + Np_1p_2 + N\sigma_2^2p_1}{3N + 3p_1 + p_0(6N + 6p_1 + 6Np_1 + 2Np_2 + 2\sigma_6^2p_1 + 2\sigma_2^2p_2 + 6\sigma_2^2 + 6\sigma_2^2)} \]  \hspace{1cm} (3.99)

and

\[ \text{snr}_{\text{RMCKCS-2}} = \frac{(4p_1\sigma_6^2 + 4p_2\sigma_4^2)p_0^2 + (2p_1p_2 + 2\sigma_4^2p_1 + 2\sigma_2^4p_2 + 2N\sigma_2^2p_1 + 2Np_1p_2 + 2N\sigma_2^2p_1)p_0}{2N + 2p_1 + p_0(4N + 4p_1 + 4Np_1 + 2Np_2 + 2\sigma_2^2p_1 + 2\sigma_2^4p_2 + 4\sigma_2^2 + 4\sigma_2^2 + 2Np_1 + Np_2 + p_1p_2 + \sigma_4^2p_1 + \sigma_4^2p_2 + 8\sigma_2^4p_0^2 + \sigma_2^4p_1^2 + N\sigma_2^2p_1^2 + Np_1p_2 + N\sigma_2^2p_1}} \]  \hspace{1cm} (3.100)

respectively.

Proof: See Appendix F.  \[ \blacksquare \]

Maximizing the receive SNR shown in (3.99) and (3.99) is difficult and hence a fine computer search is carried out and optimum values of \( p_0, p_1, \) and \( p_2 \) are obtained in section 3.7.

3.4 Extended Jing-Hassibi scheme

A simple extension of JHS to include one more layer of relays is called EJHS, which is used as a benchmark for comparison. Here, we derive the requisite equations for the transmit and
receive signal vectors, likelihood function, and the SNR at the destination. Then these are used to simulate EJHS and the BER performance of this scheme is used to compare with that of the proposed schemes.

Different phases of transmission and reception in this scheme are shown in Fig. 3.3 and explained below:

- **Phase-0**: $S_1$ transmits; $L_1$ layer relays receive
- **Phase-1**: $L_1$ layer relays transmit and $L_2$ layer relays receive
- **Phase-2**: $L_2$ layer relays transmit and $D_1$ receives

We can see that the signals that are used here are only those that travel through the links in the link class $\mathcal{L}_1$.

![Figure 3.3: Various phases in EJHS.](image)

### 3.4.1 Transmit and receive vectors

Phase-1 of EJHS is exactly similar to RMCS except that $D_1$ ignores the received vector $r^{(1)}_{31}$. $L_1$ layer relay $R_{1j}$ transmits $t_{1j}$ given by (3.11) and $L_2$ layer relay $R_{2j}$ receives $r^{(1)}_{2j}$ given by (3.13).

In phase-2, $R_{2j}$ transmits

$$t_{2j} = \alpha_2 F_{2j} r^{(1)}_{2j}. \quad (3.101)$$

The power transmitted by each relay in $L_2$ is assumed to be $p_2 T / N$ and hence we will select $\alpha_2$ accordingly.

**Claim 7.** We should select the scaling factor $\alpha_2$ in (3.101) as

$$\alpha_2 = \left[ \frac{p_2}{N (1 + p_1)} \right]^{\frac{1}{2}}. \quad (3.102)$$

to keep the average power transmitted by $L_2$ layer relays for $T$ channel uses to be $p_2 T / N$. 

51
Proof: See Appendix G.

In phase-2, the vector received by $D_1$ is $r_{31}^{(2)}$, the components of which are given by

$$r_{31}^{(2)}(\tau) = \sum_{i=1}^{N} t_{2i}(\tau) h_{2,i,3,1} + u_{31}^{(2)}(\tau).$$

In vector form,

$$r_{31}^{(2)} = \sum_{i=1}^{N} t_{2i} h_{2,i,3,1} + u_{31}^{(2)} = \sum_{i=1}^{N} \alpha_2 F_{2i} r_{1i}^{(1)} h_{2,i,3,1} + u_{31}^{(2)}$$

$$= \alpha_2 \sum_{i=1}^{N} F_{2i} \left( \alpha_0 \alpha_1 \sum_{k=1}^{N} F_{1k} s h_{0,1,1,k} h_{1,k,2,i} + \alpha_1 \sum_{k=1}^{N} F_{1k} u_{1k}^{(0)} h_{1,k,2,i} + u_{1i}^{(1)} \right) h_{2,i,3,1} + u_{31}^{(2)}$$

$$= \alpha_0 \alpha_1 \alpha_2 \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i} F_{1k} s h_{0,1,1,k} h_{1,k,2,i} h_{2,i,3,1} + \alpha_1 \alpha_2 \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,2,i} h_{2,i,3,1} F_{2i} F_{1k} u_{1k}^{(0)}$$

$$+ \alpha_2 \sum_{i=1}^{N} F_{2i} u_{1i}^{(1)} h_{2,i,3,1} + u_{31}^{(2)} = w = m_w + u_w,$$

where

$$m_w = \alpha_0 \alpha_1 \alpha_2 \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i} F_{1k} s h_{0,1,1,k} h_{1,k,2,i} h_{2,i,3,1}$$

(3.104)

and

$$u_w = \alpha_1 \alpha_2 \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,2,i} h_{2,i,3,1} F_{2i} F_{1k} u_{1k}^{(0)} + \alpha_2 \sum_{i=1}^{N} F_{2i} u_{1i}^{(1)} h_{2,i,3,1} + u_{31}^{(2)}$$

(3.105)

are the signal and noise components of the received vector.

The transmission vectors and the corresponding scaling factors are summarized in Table 3.4.

<table>
<thead>
<tr>
<th>Vector</th>
<th>Factor</th>
<th>Transmitted by</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{01}$</td>
<td>$\alpha_0 s$</td>
<td>$\alpha_0 = [p_0 T]^\frac{1}{2}$</td>
</tr>
<tr>
<td>$t_{1j}$</td>
<td>$\alpha_1 F_{1j} r_{1j}^{(0)}$</td>
<td>$\alpha_1 = \left[ \frac{p_1}{N(1+p_0)} \right]^\frac{1}{2}$</td>
</tr>
<tr>
<td>$t_{2j}$</td>
<td>$\alpha_2 F_{2j} r_{2j}^{(1)}$</td>
<td>$\alpha_2 = \left[ \frac{p_2}{N(1+p_0)} \right]^\frac{1}{2}$</td>
</tr>
</tbody>
</table>
3.4.2 Maximum-likelihood decoder

In EJHS, D₁ has one receive vector z. It can be seen that this vector shown in (3.103), is complex Gaussian with mean mₜ, shown in (3.104). From (3.105), the covariance matrix Pₜ can be found to be

\[
Pₜ = E \left[ (w - mₜ)(w - mₜ)^H \right] = E \left[ uₜ uₜ^H \right]
\]

where

\[
\begin{align*}
\alpha₁ & = \frac{1}{n_2} \sum_{i=1}^{N} \sum_{k=1}^{N} h_{i,k,2,i} h_{i,3,1} F_{i} F_{i} u_{i,k}^{(0)} + \alpha₂ \sum_{i=1}^{N} F_{i} u_{i,k}^{(1)} h_{i,3,1} + u_{31}^{(2)} \\
\alpha₁ & = \frac{1}{n_2} \sum_{i=1}^{N} \sum_{k=1}^{N} h_{i,k,2,i} h_{i,3,1} F_{i} F_{i} u_{i,k}^{(0)} + \alpha₂ \sum_{i=1}^{N} F_{i} u_{i,k}^{(1)} h_{i,3,1} + u_{31}^{(2)} \\
\end{align*}
\]

Unlike in the proposed schemes seen so far, the destination in EJHS has only one received vector and the decoded vector is accordingly

\[
\hat{s} = \arg \max_{s} \Pr(w|s) = \arg \min_{s} \|w\|^2
\]

where

\[w' = P_{w}^{-\frac{1}{2}} (w - mₜ).\]

3.4.3 Receive SNR

On similar lines as was done in RMCS in section 3.1.3, the receive SNR is defined as

\[
\text{snr}_{\text{EJHS}} \triangleq \frac{P_s}{P_n},
\]

where

\[
P_s = E \left[ m_{w} m_{w}^H \right] \text{ and } P_n = E \left[ u_{w} u_{w}^H \right].
\]
Here, \(m_w\) and \(u_w\) are given in (3.104) and (3.105) respectively. So

\[
P_s = E \left( \alpha_0 \alpha_1 \alpha_2 \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2ki}^{H} F_{1k} s h_{0,1,i,k} h_{1,k,i,k} h_{2,i,3,1} \right)^H \\
= \alpha_0^2 \alpha_1^2 \alpha_2^2 E \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} s^H F_{1k}^{H} F_{2ki}^{H} h_{0,1,i,k} h_{1,k,i,k} h_{2,i,3,1} F_{2ji} F_{1j} s h_{0,1,i,k} h_{1,l,2,j} h_{2,j,3,1} \right] \\
= \alpha_0^2 \alpha_1^2 \alpha_2^2 \sum_{i=1}^{N} \sum_{k=1}^{N} E \left[ |h_{0,1,i,k}|^2 \right] E \left[ |h_{1,k,i,l}|^2 \right] E \left[ |h_{2,i,3,1}|^2 \right] \tag{3.110} \\
= \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2. \tag{3.111}
\]

In the above, (3.110) is obtained from the previous step by the property of uncorrelatedness of i.i.d. ZMCG channel coefficients. Similarly,

\[
P_n = E \left( \alpha_1 \alpha_2 \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,i,k} h_{2,i,3,1} F_{2ki}^{H} F_{1k} u^{(0)}_{1k} + \alpha_2 \sum_{i=1}^{N} F_{2i} u^{(1)}_{2i} h_{2,i,3,1} + u_{31}^{(2)} \right)^H \\
= \alpha_1^2 \alpha_2^2 \sum_{i=1}^{N} \sum_{k=1}^{N} E \left[ |h_{1,k,i,k}|^2 \right] E \left[ |h_{2,i,3,1}|^2 \right] T + \alpha_2^2 \sum_{i=1}^{N} E \left[ |h_{2,i,3,1}|^2 \right] T + T \\
= \alpha_1^2 \alpha_2^2 N^2 T + \alpha_2^2 N T + T. \tag{3.112}
\]

Hence from (3.108), the receive SNR is

\[
\text{snr}_{\text{EHS}} = \frac{\alpha_0^2 \alpha_1^2 \alpha_2^2 N^2}{\alpha_1^2 \alpha_2^2 N^2 T + \alpha_2^2 N T + T} \\
= \frac{p_0 p_1 p_2}{1 + p_0 + p_1 + p_2 + p_0 p_1 + p_1 p_2 + p_2 p_0}. \tag{3.113}
\]

The last step is arrived at using the values of \(\alpha_0\), \(\alpha_1\), and \(\alpha_2\) from Table 3.4. Substituting \(p_2 = P - p_0 - p_1\) in (3.113) we get

\[
\text{snr}_{\text{EHS}} = \frac{p_0 p_1 (P - p_0 - p_1)}{p_0 p_1 + (1 + P - p_0 - p_1) (1 + p_0 + p_1)}. \tag{3.114}
\]

Now, differentiating (3.114) with respect to (w.r.t.) \(p_0\) and \(p_1\) and equating to zero, we get

\[
\hat{p}_0 = \hat{p}_1 = \hat{p}_2 = P/3. \tag{3.115}
\]

54
Substituting (3.115) into (3.113), we get

\[ \text{snr}_\text{EJHS}_{max} = \frac{P^3}{9(3 + 3P + P^2)}, \]  

(3.116)

which is the maximum value of \( \text{snr}_\text{EJHS} \). As mentioned earlier, here, \( P \) is the total average power allocated to the system per symbol duration and it is given by \( P = p_0 + p_1 + p_2 \). Equation (3.115) is also verified in section 3.7 using simulations. Hence, the total power is divided equally amongst the three phases to get maximum SNR in EJHS.

### 3.5 Modified Jing-Hassibi scheme

This scheme, Modified Jing-Hassibi scheme (MJHS), is obtained by modifying JHS to include all the relays \( L_1 \) and \( L_2 \) to be transmitting at the same time in phase-1 and phase-2. Different phases of transmission and reception are shown in Fig. 3.4 and explained below:

- **Phase-0**: \( S_1 \) transmits; \( L_1 \) and \( L_2 \) layer relays receive
- **Phase-1**: \( L_1 \) layer and \( L_2 \) layer relays transmit; and \( D_1 \) receives
- **Phase-2**: \( L_1 \) layer and \( L_2 \) layer relays transmit; and \( D_1 \) receives

![Figure 3.4: Various phases in MJHS.](image)

In this scheme, we have phase-2 exactly similar to phase-1 so as to keep the total time duration to be \( 3T \), similar to the other schemes. Let \( p_1 T/2 \) be the power transmitted by \( L_1 \) and \( p_2 T/2 \) by \( L_2 \) relays in the phase-1. As the vectors to be transmitted by \( L_1 \) and \( L_2 \) relays in the second and third phases are identical and that the channel is assumed to have not varied\(^1\), we have equally divided the power between the second and third phases.

\(^1\)To cater for MJHS only, we selected the coherence interval to be at least \( 2T \) in the Chapter 2.
3.5.1 Transmit and receive vectors

Let

\[ t^{(k)}_{1j} = \alpha_1 F_{1j} r^{(0)}_{1j} \quad \text{and} \quad t^{(k)}_{2j} = \alpha_2 F_{2j} r^{(0)}_{2j} \]  

(3.117)

be the vectors transmitted by R_{1j} and R_{2j} relays respectively in kth phase, with k = 1, 2. From (3.7), the average power transmitted by R_{1j} in T channel uses during the kth phase is given by

\[ E \left[ t^{(k)H}_{1j} t^{(k)}_{1j} \right] = \alpha_1^2 E \left[ \left( r^{(0)H}_{1j} F^H_{1j} \right) \left( F_{1j} r^{(0)}_{1j} \right) \right] = \alpha_1^2 (\alpha_0^2 + T). \]  

(3.118)

As we have assumed that the total power transmitted by R_{1j} in phase-k, k \in [1, 2] is \( p_1 T/2N \), we have

\[ \frac{p_1 T}{2N} = \alpha_1^2 (\alpha_0^2 + T) \]

\[ \Rightarrow \alpha_1 = \left[ \frac{p_1}{2N(1 + p_0)} \right]^{\frac{1}{2}}. \]  

(3.119)

Here, \( \alpha_0 \) is substituted from (3.6).

Similarly from (3.8), the average power transmitted by R_{2j} in T channel uses during the kth phase is given by

\[ E \left[ t^{(k)H}_{2j} t^{(k)}_{2j} \right] = \alpha_2^2 E \left[ \left( r^{(0)H}_{2j} F^H_{2j} \right) \left( F_{2j} r^{(0)}_{2j} \right) \right] = \alpha_2^2 (\sigma_2^2 \alpha_0^2 + T). \]  

(3.120)

Also, we assumed that R_{2j} transmits an average power in T channel uses as \( p_2 T/2N \). Therefore, we have

\[ \frac{p_2 T}{2N} = \alpha_2^2 (\sigma_2^2 \alpha_0^2 + T) \]

\[ \Rightarrow \alpha_2 = \left[ \frac{p_2}{2N(1 + \sigma_2^2 p_0)} \right]^{\frac{1}{2}}. \]  

(3.121)

Now the received vector at D_1 in phase-k, k \in [1, 2], is given by

\[ r^{(k)}_{31} = \sum_{j=1}^{N} t^{(k)}_{1j} h_{1,j,3,1} + \sum_{j=1}^{N} t^{(k)}_{2j} h_{2,j,3,1} + u^{(k)}_{31}. \]  

(3.122)
Substituting \( t_{1j}^{(k)} \) and \( t_{2j}^{(k)} \) from (3.117) we get

\[
\begin{align*}
 r_{31}^{(k)} &= \sum_{j=1}^{N} \alpha_1 F_{1j} t_{1j}^{(0)} h_{1,j,3,1} + \sum_{j=1}^{N} \alpha_2 F_{2j} t_{2j}^{(0)} h_{2,j,3,1} + u_{31}^{(k)} \\
 &= \sum_{j=1}^{N} \alpha_1 F_{1j} \left( \alpha_0 s h_{0,1,1,j} + u_{1j}^{(0)} \right) h_{1,j,3,1} + \sum_{j=1}^{N} \alpha_2 F_{2j} \left( \alpha_0 s h_{0,1,2,j} + u_{2j}^{(0)} \right) h_{2,j,3,1} + u_{31}^{(k)} \\
 &= \alpha_0 \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j} F_{1j} s + \alpha_0 \alpha_2 \sum_{j=1}^{N} h_{0,1,2,j} F_{2j} s \\
 &\quad + \alpha_1 \sum_{j=1}^{N} h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + \alpha_2 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(0)} + u_{31}^{(k)} \\
 &= \begin{cases} 
  z = m_{z} + u_{z}, & \text{if } k = 1 \\
  w = m_{w} + u_{w}, & \text{if } k = 2
\end{cases}. \\
\tag{3.123}
\end{align*}
\]

Hence

\[
\begin{align*}
 m_{z} &= \alpha_0 \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j} F_{1j} s + \alpha_0 \alpha_2 \sum_{j=1}^{N} h_{0,1,2,j} F_{2j} s = m_{w}, \\
 u_{z} &= \alpha_1 \sum_{j=1}^{N} h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + \alpha_2 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(0)} + u_{31}^{(1)}, \\
 u_{w} &= \alpha_1 \sum_{j=1}^{N} h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + \alpha_2 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(0)} + u_{31}^{(2)}.
\tag{3.126}
\end{align*}
\]

The transmission vectors and the corresponding scaling factors are summarized in Table 3.5.

<table>
<thead>
<tr>
<th>Table 3.5: Transmitted vectors and scaling factors - MJHS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vector</strong></td>
</tr>
<tr>
<td>( t_0 = \alpha_0 s )</td>
</tr>
<tr>
<td>( t_{1j}^{(k)} = \alpha_1 F_{1j} r_{1j}^{(0)} )</td>
</tr>
<tr>
<td>( t_{2j}^{(k)} = \alpha_2 F_{2j} r_{2j}^{(0)} )</td>
</tr>
</tbody>
</table>

### 3.5.2 Maximum-likelihood decoder

The two received vectors at \( D_1 \) are \( z = r_{31}^{(1)} \) and \( w = r_{31}^{(2)} \) as shown in (3.123). Now \( y \) can be constructed as a concatenation of these vectors as shown in (3.36). It can be seen as in RMCS,
that $y$ is jointly Gaussian and that the mean vector, $m_y$ and covariance matrix, $P_y$ of $y$ are given in (3.37). Now the covariance matrices are given by

$$P_z = E \left[ u_z u_z^H \right] = E \left( \alpha_1 \sum_{j=1}^{N} h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + \alpha_2 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(0)} + u_{31}^{(1)} \right)$$

$$= \left[ \alpha_1^2 \sum_{j=1}^{N} |h_{1,j,3,1}|^2 + \alpha_2^2 \sum_{j=1}^{N} |h_{2,j,3,1}|^2 + 1 \right] I_T = P_w. \quad (3.127)$$

The cross-covariance matrix $P_{zw}$ of the received vectors is given by

$$P_{zw} = E \left[ u_z u_w^H \right] = E \left( \alpha_1 \sum_{j=1}^{N} h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + \alpha_2 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(0)} + u_{31}^{(1)} \right)$$

$$= \left[ \alpha_1^2 \sum_{j=1}^{N} |h_{1,j,3,1}|^2 + \alpha_2^2 \sum_{j=1}^{N} |h_{2,j,3,1}|^2 \right] I_T = P_x. \quad (3.129)$$

Now the decoded vector is given by

$$\hat{s} = \arg \max_s \Pr(y|s) = \arg \min_s \|y'\|^2 \quad (3.130)$$

where $y' = P_y^{-\frac{1}{2}} (y - m_y)$.

### 3.5.3 Receive SNR

As we have two received vectors at $D_1$ the receive SNR is given by

$$\text{snr}_{\text{MHIS}} = \frac{P_s}{P_n} = \frac{P_s^{(1)} + P_s^{(2)}}{P_n^{(1)} + P_n^{(2)}}, \quad (3.131)$$

where $P_s^{(1)}$, $P_s^{(2)}$, $P_n^{(1)}$, and $P_n^{(2)}$ are found as follows:

$$P_s^{(1)} = E \left[ m_z^H m_z \right]$$

$$= E \left( \alpha_0 \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j} h_{1,j,3,1} F_{1j} s + \alpha_0 \alpha_2 \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2j} s \right)^H$$

$$= \left( \alpha_0 \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j} h_{1,j,3,1} F_{1j} s + \alpha_0 \alpha_2 \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2j} s \right)$$

$$\left( \alpha_0 \alpha_1 \sum_{j=1}^{N} h_{0,1,1,j} h_{1,j,3,1} F_{1j} s + \alpha_0 \alpha_2 \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2j} s \right)^H$$

58
From Table 3.5 or from equations (3.6), (3.119), and (3.121), (3.136) can be simplified to

\[ P_s^{(2)}. \]  

Now,

\[
P_n^{(1)} = E \left[ u_w^H u_w \right] = E \left( \alpha_1 \sum_{j=1}^{N} h_{1,j,3,1} F_{1j} u_{1j}^{(0)} + \alpha_2 \sum_{j=1}^{N} h_{2,j,3,1} F_{2j} u_{2j}^{(0)} + u_{31}^{(1)} \right)^H \]

\[
= \alpha_1^2 \sum_{j=1}^{N} E \left[ |h_{1,j,3,1}|^2 \right] T + \alpha_2^2 \sum_{j=1}^{N} E \left[ |h_{2,j,3,1}|^2 \right] T + T \]

\[
= \alpha_1^2 N \sigma_2^2 T + \alpha_2^2 NT + T \]  

\[
= P_n^{(2)}. \]  

\[ P_n^{(1)} \] can also be obtained from (3.127) as

\[
P_n^{(1)} = E \left[ u_u^H u_u \right] = \text{Tr} \left[ E \left( u_u u_u^H \right) \right] = \text{Tr} \left[ P_z \right]
\]

\[
= \text{Tr} \left[ \left( \alpha_1^2 \sum_{j=1}^{N} |h_{1,j,3,1}|^2 + \alpha_2^2 \sum_{j=1}^{N} |h_{2,j,3,1}|^2 + 1 \right) I_T \right]
\]

\[
= T \left( \alpha_1^2 N \sigma_2^2 + \alpha_2^2 N + 1 \right). \]

Here, we have taken the \( E[.] \) operator as averaging on channel coefficients is not carried out in \( P_z \) as they are assumed to be known for the ML decoder. Hence, the receive SNR is given as

\[
\text{snr}_{\text{MHS}} = \frac{N \alpha_0^2 (\alpha_1^2 + \alpha_2^2) \sigma_2^2}{\alpha_1^2 NT \sigma_2^2 + \alpha_2^2 NT + T}. \]  

(3.136)

From Table 3.5 or from equations, (3.6), (3.119), and (1.121), (3.136) can be simplified to

\[
\text{snr}_{\text{MHS}} = \frac{(p_1 \sigma_1^2 + p_2 \sigma_2^2) \rho_0^2 + (p_2^2 \rho_1 + \sigma_2^2 p_2) \rho_0}{p_2 + \rho_0 (p_1 \sigma_1^2 + 2 \sigma_2^2 + p_2 + 2) + \sigma_2^2 \rho_1 + 2 \sigma_2^2 \rho_0^2 + 2}. \]  

(3.137)

Maximizing the receive SNR shown in (3.137) is difficult analytically and hence a fine computer search has been carried out as discussed in Section-3.7.
Before we discuss the simulations in Section-3.8, we define the path loss and optimum power to be allocated to various transmissions in simulations. In the next section, we will see the model used to incorporate the path loss in the signals and later, the optimum power allocation.

3.6 Signal-power path loss

Various channel links can have any variance which caters for the path loss. For all the simulations in this thesis, we incorporate the signal-power path loss of these links as in [36], i.e., if $d_{ij}$ is the distance from $L_i$ to $L_j$, then the channel coefficient $h_{i,n,j,k}$ of the link $\ell_{i,n,j,k}$ is given by

$$h_{i,n,j,k} = \frac{\xi_{i,n,j,k}}{d_{ij}^{\beta}}$$  \hspace{1cm} (3.138)

where $\{\xi_{i,n,j,k} \in \mathbb{C} : i, j \in [1, K], n, k \in [1, N]\}$ is an i.i.d. collection of random variables with $\xi_{i,n,j,k} \sim \mathcal{N}(0, 1)$ and $\beta$ is the path loss exponent. Hence, the variance of the channel coefficient $h_{i,n,j,k}$ is given by

$$\text{Var}[h_{i,n,j,k}] = \frac{1}{d_{ij}^{2}} \text{Var}[\xi_{i,n,j,k}] = \frac{1}{d_{ij}^{2}}.$$  \hspace{1cm} (3.139)

Equation (3.139) does not take into account the variation due to shadowing. If it is considered, then path loss\(^2\), the ratio between the power received to the power transmitted, is usually modelled as a random variable with lognormal distribution even when the distance between the transmitter and the receiver is fixed [44]. But for simplicity of exposition, we consider the channel variance to be just dependent on distance for the simulations as shown in (3.139).

Fig. 3.5 shows a simplified system model of Fig. 2.2. Let the distance $d_{0,K+1}$ from $L_0$ to $L_{K+1}$ be $d$. Let us also assume that the distance, $d_{k,k+1}$, from $L_k$ to $L_{k+1}$, is the same $\forall k \in [0, K]$ and hence $d_{k,k+1} = d/(K + 1)$. So, the distance between any two layers $L_i$ and $L_j$ is $d_{ij} = (j - i)d/(K + 1)$. Now the length $\rho_{ij}$ of the link $\ell_{im,jn}$, $m, n \in [1, N]$, from $L_i$ to $L_j$, can also be obtained from this distance, $d_{ij}$, by normalizing it with $d_{k,k+1}$ as

$$\rho_{ij} = \frac{d_{ij}}{d_{k,k+1}} = j - i.$$ 

\(^2\)Path loss in dB=Distance dependent factor $+ X$, where $X$ is a zero-mean Gaussian (ZMG) random variable due to shadowing
Channel variance of any link in terms of its length $\rho$ is $(K+1)^{\beta} \rho \sigma^2$, where $\sigma^2$ is the variance of the link from $L_0$ to $L_{K+1}$.

Figure 3.5: A simplified system model showing lengths and distances of various links with their channel variances. Here, $\beta$ is the path loss exponent and this model is used only for simulations.

It is assumed that the channel variance of the link, $\ell_{0,K+1}$ is $\sigma^2$. Hence $\sigma^2 = 1/d_{0,K+1}^\beta = 1/d^\beta$ from (3.139). Now, the channel variance of any link with length $\rho$ is given by

$$\sigma_{\rho}^2 = \frac{1}{d_{\rho}^\beta} = \frac{(K+1)^\beta}{\rho^\beta d^\beta} = \frac{(K+1)^\beta \sigma^2}{\rho^\beta},$$

where $\beta$ is the path loss exponent [36]. In this chapter, as mentioned earlier in page 28, we assume that the links with $\rho = 1$ have channel variance $\sigma_{\rho}^2 = \sigma_1^2 = 1$ for the simulations in Section-3.8.

### 3.7 Optimum power allocation

Allocation of power to various transmissions, namely, $p_0, p_1,$ and $p_2$ is to be done optimally to minimize BER, where $p_i$ represents the power transmitted by $L_i$ in phase-3. PEP is minimized for $K = 1$, i.e., when there is a single layer of relays, by Jing and Hassibi [11], and obtained the optimum values of $p_0, p_1,$ and $p_2$ for JHS. These authors proved that the optimum power allocation obtained by minimizing PEP also maximizes receive SNR. Hence in this thesis, receive SNR has been selected as the parameter to be maximized and expect that this gives near optimum power allocation to minimize PEP. Maximizing the receive SNR analytically is complex for all schemes except for EJHS, as mentioned earlier, and hence a fine computer search has been carried out as explained here.

We have three variables namely $p_0, p_1,$ and $p_2$, which are the powers allocated to the three transmissions used in the schemes discussed. These variables have two constraints, namely, $p_0 + p_1 + p_2 \leq P$ and $p_0, p_1, p_2 \geq 0$.

\[\text{Except in MJHS, where } p_1 \text{ and } p_2 \text{ represent powers transmitted by } L_1 \text{ and } L_2 \text{ respectively.}\]
A hashed triangular plane ABC is shown in Fig. 3.6, which is the plane $p_0 + p_1 + p_2 = P$ with the constraints $p_0, p_1, p_2 \geq 0$. Here, AB, BC, and AC are in $p_0 - p_1$, $p_1 - p_2$, and $p_0 - p_2$ planes respectively. Now the aim is to find the point on this plane, which when substituted in the SNR equation of a particular scheme, gets the maximum value. We will follow the procedure given below using a fine computer search.

We will select $p_0$ and keep varying $p_1$ with $p_2 = P - p_0 - p_1$. We cover the entire area of this hashed plane by selecting the points with a granularity. Consider the straight line shown on the plane in Fig. 3.6, DE, which is parallel to BC. The equation of this line is $p_0 + p_1 + p_2 = P$, $p_0 = p_0'$ where $0 \leq p_0' \leq P$. This line is obtained by intersecting the hashed triangular plane with $p_0 = p_0'$, which is a plane parallel to $p_1 - p_2$ plane. By varying $p_0'$, we will get more straight lines parallel to BC. With a certain granularity, we will vary $p_0'$, i.e., $p_0' = n\psi P$ where $0 \leq \psi \leq 1$ and $0 \leq n \leq \left\lfloor \frac{1}{\psi} \right\rfloor$, $n$ being an integer. Once $p_0'$ is selected, we select $p_1$ with a granularity as for the case of $p_0$ as $p_1' = m\epsilon P$ with $0 \leq m \leq \left\lfloor \frac{1-\psi}{\epsilon} \right\rfloor$, $m$ being an integer and $0 \leq \epsilon \leq 1$. Then $p_2$ is fixed as $p_2' = P - p_0' - p_1'$. Hence we can get the point $G(p_0', p_1', p_2')$ as shown in Fig. 3.6. The complete region of the plane ABC is covered in this manner and receive SNRs are calculated for each point. That point which has the maximum SNR is selected as the optimum point.
In the computer search, $\psi = 1/1000$ and $\epsilon = 1/1000$ have been used\(^4\). Hence the region is covered with a granularity of 0.001 in all the three axes. The optimum point ($\hat{p}_0, \hat{p}_1, \hat{p}_2$), is found for all the schemes discussed in the earlier sections, for various values of $P$ and $\beta$.

Figure 3.7: Receive SNR for RMCS-2 and RMCS-3 for $\beta = 2$.

Plots of the receive SNR at $D_1$, w.r.t. normalized $p_0$ and $p_1$ in terms of fraction of $P$ respectively for RMCS, RSCS, RMCKCS, and EJHS/MJHS are shown in Fig. 3.7, Fig. 3.8, Fig. 3.9, and Fig. 3.10, for $\beta = 2$ and $P = 16$ dB. In all the plots, we find two variants of the schemes, one with $\mu = 2$ and another with $\mu = 3$, except in Fig. 3.10, where one is that of EJHS and another MJHS. As noted earlier, here $\mu$ is the number of leaked signals used in the system. The optimum values of $p_0$, $p_1$, and $p_2$ are shown at the top of each of the plots. The difference in the plots show that the optimum values change when $\mu = 2$ or 3.

Figure 3.8: Receive SNR for RSCS-2 and RSCS-3 for $\beta = 2$.

In all the plots of the proposed protocols shown in Fig. 3.7, Fig. 3.8, and Fig. 3.9, it

\(^4\)In all the plots shown in figures from Fig. 3.7 to 3.10, $\psi = \epsilon = 1/100$ is used for clarity.
can be observed that when $\mu$ changes from 2 to 3, SNR is non-zero even when only source is transmitting or when $p_0 = P$, $p_1 = 0$, and $p_2 = 0$. This is because, when $\mu = 3$, we consider leaked signal from source also, whereas when $\mu = 2$, the signal which reaches the destination from source is ignored. We can also observe that in the plots of RSCS, there are two maxima. When $L_1$ layer relays start increasing the power transmitted, the SNR decreases to a local minimum point and then increases to the global maximum point, where we get the optimum value, $\hat{p}_0$, $\hat{p}_1$, $\hat{p}_2$, and then again decreases.

![Receive SNR for RMCKCS-2 and RMCKCS-3 for $\beta = 2$.](image1)

![Receive SNR for EJHS and MJHS for $\beta = 2$.](image2)

Also for EJHS, the fact that $\hat{p}_0 = P/3$, $\hat{p}_1 = P/3$, and $\hat{p}_2 = P/3$ derived in (3.115) in subsection 3.4.3 is verified using simulations and the plot is shown in the left side subfigure in Fig. 3.10.

In a similar fashion, we find the optimum values $(\hat{p}_0, \hat{p}_1, \hat{p}_2)$ for various values of $\beta$ and $P$, which we will use during simulations to get BER plots. Plots in Fig. 3.11, Fig. 3.12, and Fig.
3.13 show optimum powers $\hat{p}_0$, $\hat{p}_1$, and $\hat{p}_2$ of four of the schemes RMCS, RSCS, RMCKCS, and MJHS\(^5\).

The following can be observed from Fig. 3.11, which correspond to path loss exponent $\beta = 2$:

- **RMCS & RMCKCS**
  - For RMCS, as $P$ increases, the powers allocated to $S_1$, $L_1$, and $L_2$ become same and remain at $P/3$.
  - For RMCS, at values of $P < 20$ dB, the power transmitted by $L_2$ remains the lowest.
  - For RMCKCS, as $P$ increases, the powers allocated to $S_1$, $L_1$, and $L_2$ reach their constant value as shown.
  - Power transmitted by $S_1$ remains the maximum in RMCS compared to relays, in contrast to RMCKCS, where it is the lowest.

- **RSCS**
  - It does not transmit using $L_1$ relays to achieve high receive SNR for $P \leq 14$ dB, i.e., $p_1$ remains zero for these values of $P$.

\(^5\)Plot for EJHS is not shown as $\hat{p}_0/P$, $\hat{p}_1/P$, and $\hat{p}_2/P$ do not vary with $P$ or $\beta$ as shown in (3.115)
– Transition to the same value for both the powers of $S_1$ and $L_1$ occurs for a value of $P$ between 14 and 16 dB to get maximum SNR. For values of $P > 16$ dB, the source, $L_1$, and $L_2$ layer relays get equal distribution of power to achieve maximum SNR.

• MJHS

– It does not transmit using $L_1$ relays to achieve high receive SNR, i.e., $p_1$ remains zero for all values of $P$ in the range 10 to 24 dB.
– The source and $L_2$ layer relays keep their powers constant for varying $P$.

Figure 3.12: Plot of optimum power allocations for RMCS-3, RSCS-3, RMCKCS-3, and MJHS for $\beta = 3$.

The following can be observed from Fig. 3.12, which correspond to path loss exponent $\beta = 3$:

• RMCS

– The powers transmitted by the source $S_1$, and the relay layers $L_1$ and $L_2$ are to be kept constant as $P$ increases.
– In contrast to the case when $\beta = 2$, the source power is the lowest amongst all the powers when $\beta = 3$.

• RMCKCS
- In this scheme, the powers are almost constant with variation in $P$.
- It keeps the power allocated to $L_2$ layer relays the highest compared to $S_1$ and $L_1$.

- **RSCS**
  - Powers are kept constant with variation in $P$, with $L_2$ layer relays transmitting maximum power.

- **MJHS**
  - To get maximum receive SNR, this scheme keeps the $L_1$ relays mute throughout as in the case when $\beta = 2$, i.e., this scheme does not require those relays that are nearer to the source. The power transmitted by the source $p_0$ and that of $L_2$ relays $p_2$ are kept constant as the total power $P$ increases.

```
<table>
<thead>
<tr>
<th>Power $P$ in dB</th>
<th>Fraction of $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>15</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
</tr>
</tbody>
</table>
```

- **Figure 3.13**: Plot of optimum power allocations for RMCS-3, RSCS-3, RMCKCS-3, and MJHS for $\beta = 4$.

The following can be observed from Fig. 3.13, in which the plots are generated for the path loss exponent $\beta = 4$:

- **RMCS/RMCKCS**
  - Almost similar trend of the powers is followed with varying $P$ as that of the case when $\beta = 3$.
• MJHS/RSCS
  - Almost similar trend is followed here as in the case when $\beta = 3$, but the difference between the powers $p_0$ and $p_2$ has increased.

We can infer the following from all the above observations made on Fig. 3.11 to Fig. 3.13:

• As the path loss exponent decreases from 4 to 2, or the channel variances increase, RMCS gives more importance to the source than the relays, but values are brought to be the same value by making $p_0 = p_1 = p_2$ with the higher total average transmitting power $P$.

• Irrespective of the power loss condition (i.e., for any value of $\beta$ ($\beta = 2$ or 3 or 4) and $P$ in the range from 10 to 24 dB), MJHS keeps the $L_1$ layer relays muted and does not use them throughout. Also the difference between the powers divided between the source and $L_2$ layer relays narrows down when the channel variances increase or the path loss exponent $\beta$ decreases from 4 to 2.

• Like MJHS, which shuts down $L_1$ layer relays completely in any power loss condition, RSCS also mutes them but only when the signal from the source to the second layer or the $L_1$ layer relays to destination has sufficient strength, i.e., when $\beta = 2$ but with $P < 14$ dB.

In the next section, we will generate BER plots of all the schemes with the optimum powers obtained in this section, using simulations and compare.

### 3.8 Simulations and results

In the simulations, a block size of length $T = 5$ symbol duration and number of relays in each layer $N = 5$ have been used. Total number of Monte Carlo runs varied between 10,000 and 1,000,000 as required, to smoothen the resulting plots. Let us assume that the signal $s \in \Omega = \{s_1, \cdots, s_\zeta\} \subset \mathbb{C}^{T \times 1}$ is selected equally-likely from $\Omega$, whose cardinality is $\zeta$. As defined earlier $s = [s(1) \cdots s(T)]^T$, and $s(k) = s_r(k) + js_i(k), \ 1 \leq k \leq T$. Let us also assume that the real part $s_r(k)$ and the imaginary part $s_i(k)$ of $s(k)$ are equally likely selected from the $Q$-PAM signal set

$$\kappa \left\{ -\frac{Q-1}{2}, \cdots, -\frac{1}{2}, \frac{1}{2}, \cdots, \frac{Q-1}{2} \right\},$$
where $\kappa$ is the normalizing factor so that $E[s^H s] = 1$. Hence the cardinality $\zeta$ of $\Omega$ is $Q^{2T}$.

The value of $\kappa$ is found as follows:

\[
E[s^H s] = E \sum_{k=1}^{T} |s(k)|^2 = TE [ |s(k)|^2 ]
\]

\[
= TE [ s^2_r(k) + s^2_i(k) ]
\]

\[
= 2TE [ s^2_r(k) ]
\]

\[
= \frac{T\kappa^2}{Q} \sum_{j=1}^{Q/2} (2j - 1)^2 = \frac{T\kappa^2}{Q} \frac{(Q - 1)Q(Q + 1)}{6}
\]

\[
= 1 \Rightarrow \kappa = \left[ \frac{6}{T(Q^2 - 1)} \right]^\frac{1}{2}.
\]

$Q = 2$ has been used in all the simulations in this Chapter.

The relay matrices $F_{ij}$, $i \in [1, K]$, $j \in [1, N]$ have been selected to be real orthogonal, as the BER performance is found using simulations to be the same as that when complex unitary matrices are employed. Fig. 3.14 shows plots of BER for the JHS, obtained w.r.t. transmitted power $P$.

Here, there are three curves - one representing that using real orthogonal matrices, another when using complex unitary matrices at the relays, and the third one is created from the actual values from Fig. 3(a) of [11], where the authors have used complex unitary matrices. As

Figure 3.14: Comparison of JHS when unitary and orthogonal matrices are used.
the performances are the same for the unitary and real orthogonal, real orthogonal matrices are used in simulations.

BER plots shown in Fig. 3.15 to Fig. 3.17 are generated for $\beta = 4$, 3, and 2 respectively, i.e., from the worst case to the best case scenario of the channel. The transmission powers are allocated according to the generated values by computer search discussed in section 3.7. Here, the number $\mu$ attached with the names of the schemes signifies that this scheme uses all the signals that travel through the links in the link class $\mathcal{L}_\mu$. For example, ‘3’ in RMCKCS-3 denotes that it uses all the links in $\mathcal{L}_3$ shown in (3.1).

It can be observed that the performance of EJHS is the same in all the three figures, Fig. 3.15 to Fig. 3.17 as expected. The reason for this is because EJHS does not use any leaked signal and hence for any value of the path loss exponent $\beta$, it uses the signals that travel through the channel links whose channel variance is $\sigma_1^2 = 1$. The performance of MJHS becomes worse as $\beta$ increases, as $L_2$ stores and forwards the signals in phase-1 and phase-2 that was received from $S_1$ in phase-0. These signals have travelled through the links with channel variance $\sigma_2^2 = 0.125$ when $\beta = 3$ and $\sigma_2^2 = 0.0625$ when $\beta = 4$.

![Figure 3.15: Comparison of BER of the schemes when $\beta = 4$.](image)

The following can be observed from Fig. 3.15, plots of which are obtained by simulating the channel with the worst case scenario to have the path loss exponent $\beta = 4$:

- For all the values of the transmitted power $P$ from 10 to 24 dB, RSCS performs the best,
followed by RMCS.

- The performance of MJHS is the worst compared to all the other schemes.
- EJHS performs better than RMCKCS-3 for $P > 20$ dB.

![Performance of various schemes when pathloss exponent $\beta = 3$](image)

Figure 3.16: Comparison of BER of the schemes when $\beta = 3$.

The following can be observed from Fig. 3.16, plots of which are obtained when $\beta = 3$:

- The performance of RSCS continues to be the best when compared to all other schemes.
- RMCS-3 performs better than RSCS-2 for values $P > 18$ dB.
- Performance of RMCKCS-3 is better than EJHS for all the values of $P$ and that of RMCKCS-2 is better than EJHS for values of $P < 21$ dB.
- MJHS performs the worst as in the previous case when $\beta = 4$.

From Fig. 3.17, where $\beta = 2$, we can observe the following:

- All the schemes proposed by us perform better than EJHS and MJHS for all values of $P$.
- Performance of RMCS is the same as that of RSCS.
- Performance of RMCKCS-3 is better than that of RMCS-2 and RSCS-2 for $P < 20.5$ dB.
Performance of various schemes when pathloss exponent $\beta = 2$

• MJHS performs better than EJHS for all values of $P$.

We observe that as the channel becomes better with decreasing value of the path loss exponent $\beta$, the performance of the proposed schemes become better. The reason for this is that they use leaked signals that use the channel links whose variance increases when $\beta$ reduces. As $\beta$ increases, the channels become worse and hence performance of the proposed schemes reduce.

Nevertheless, even for the worst case scenario of $\beta = 4$, the performance of RSCS, which uses the SNR information of the received signals is the best with an advantage of $\approx 2$ dB at BER=$10^{-2}$. Also RMCS, which does not use any channel information performs almost same as EJHS for $P < 18$ dB and has an advantage of 0.5 dB at BER=$10^{-2}$.

Another observation we make here is that RMCKCS does not use the available channel information it has, to its best and there is a possible scope for improvement.

We also notice that MJHS has better performance than EJHS when $\beta = 2$ with an advantage of $\approx 0.5$ dB for all values of $P$. 

Figure 3.17: Comparison of BER of the schemes when $\beta = 2$. 

![Graph showing BER vs Power P in dB for various schemes when $\beta = 2$.]
3.9 Discussion and observations

From the simulation results, we see that the proposed schemes perform better than the enhanced existing systems for all the three cases when the path loss exponent $\beta = 2$, 3, and 4. RSCS performs the best for all the values of the path loss exponent.

There can be two reasons why the proposed schemes perform better, as they are different in only two ways compared to the existing schemes EJHS and MJHS: (1) The way the received signals are combined at the relays before they are forwarded, (2) They use leaked signals. Out of these two, we can conclude that the main contributor is the usage of leaked signals. This is because, RMCS which combines the received signals with no extra information, performs similar to RSCS, which uses the SNR information of the received signals. Particularly, we find that when $\beta = 2$ is considered, RMCS performs the same as RSCS.

In the next chapter, we propose optimum schemes and compare their performances with those of the existing optimum schemes discussed in Chapter-2.
Chapter 4

Optimum MMSE Schemes

In this Chapter, we propose schemes which use optimum relay precoders by minimizing MSE at the relays. We will show how the complex problem to find optimum relay precoder matrices by minimizing the MSE at the destination becomes simpler, when we minimize MSE at the relays. Firstly, we derive the precoder matrices for a multi-layer network with single source-destination pair and later extend it to a network with multiple source-destination pairs. We consider both the cases of the relays being cooperative amongst themselves and not, and derive precoders. We show that the precoders so obtained do not depend on forward CSI.

To compare the proposed MMSER with the existing MMSED schemes, first we enhance MMSED to work in a multi-layer network. We show that the performance of our scheme is comparable to and sometimes better than these enhanced schemes (though these enhanced schemes may not be optimal). This is shown to be possible by increasing the number of leaked signals that are used at the relays while forwarding.

One of the main differences, other than being optimum compared to the ad hoc schemes, is that the optimum schemes use the channel once in a particular phase, unlike ad hoc schemes, which transmit for $T$ channel uses in a phase. Hence the radio nodes transmit a scalar for every phase, which can be extended to transmit a vector, but not covered in this thesis. This is to keep it consistent with the existing literature on MMSE systems for easier performance comparison. Hence the precoder matrices obtained are for the group of relays in a particular layer. Also the optimum systems proposed, work in a multi-layer system ($K > 2$) unlike ad hoc schemes, which work in a two layer system.

First, we will propose an optimum scheme for a special case of having single source-destination pair in the general system model shown in Fig. 2.2 and later extend the scheme to
work in the general system model having multiple source-destination pairs.

\[ L_K - 1 \quad L_K \quad L_{K+1} \]

\[ S_1, \rho = K + 1, \rho = K - 1, \rho = K, \rho = 1, \rho = 1, \rho = 2, \rho = 2, \rho = 1, \rho = 1, \rho = 1, \rho = 1, \rho = 1, \rho = K - 1 \]

Figure 4.1: A multi-layer relay network. Here, \( S_1, L_k, R_{ki}, D_1 \), and \( \rho \) represent the source, \( k \)th layer, \( i \)th relay in \( L_k \), the destination, and the length of the corresponding link respectively.

4.1 Single source-destination pair

Let us consider a \( K + 1 \) hop relay network as shown in Fig. 4.1, which has one source, \( K \) relay and one destination layers. Here, there is only one source-destination pair \( S_1-D_1 \) in the source and the destination layers, unlike in the general system model which we saw in Fig. 2.2 in Chapter 2.

Transmission phases: As mentioned earlier, \( L_0 \) transmits in phase-0 and all the relays and the destination receive. Out of them, we will consider that only \( L_1, L_2, \ldots, L_\mu \) store the received signal for later use; \( L_1 \) transmits in phase-1, and \( L_2, \ldots, L_{\mu+1} \) receive and store for later use and so on till phase \( K \), when \( L_{K+1} \) receives from \( L_K \). Here, \( \mu \) can take any value from the set \( \{1, \ldots, K + 1\} \), whereas to the best of our knowledge, the existing literature on relay precoders consider \( \mu = 1 \). We will call this scheme as MMSER-\( \mu \), in which the relays and the destination store all signals received through the link sets in \( L_\mu \) for later use, where all links with length \( \rho \leq \mu \) are considered. So we use the symbol \( \mu \) for the maximum number of
leaked signals that are stored and used later in a relay, in the MMSER scheme, for constructing the signal to be forwarded by the relays. As mentioned earlier in Chapter 2, MMSE at relays or MMSER suggests that this optimum scheme would minimize MSE at the relays unlike the conventional optimization location, the destination.

Now, at a particular layer, we concatenate all the vectors received during various phases for transmission. But, we have the problem of having different sized vectors at each of the relay layers. In the ad hoc scheme chapter, we saw different ways to combine them. RSCS used the SNR information to add the two signals, while RMCKCS used conjugate and norm of the local channel coefficients for combining. RMCS, with no CSI, used a random orthogonal matrix and combined the received vectors. Here, we use the dimension of the precoder matrix to be similar to that used in RMCS.

Let us assume that in phase- \( k \), \( L_k \) relays transmit \( t_k = F_k r_k \), where the precoder matrix\(^1\) \( F_k = [F_{k,n}, \ldots, F_{k,k-1}] \in \mathbb{C}^{N \times (k-n)N} \) and \( r_k = [r_k^{(n)T}, \ldots, r_k^{(k-1)T}]^T \in \mathbb{C}^{(k-n)N \times 1} \) is a concatenated vector of all received vectors during the phases \( n \) to \( k-1 \). That is, these relays make use of the leaked signals which arrive through the weak links. Here, \( F_{k,i} \in \mathbb{C}^{N \times N} \) with \( n \leq i \leq k-1 \) and \( n = [k-\mu]^+ \), where \([x]^+\) denotes \( x \) if \( x \geq 0 \) and 0 if \( x < 0 \). We note that the superscript and subscript in received signal \( r \) denotes phase number of reception and the layer number respectively as mentioned in the previous Chapter on ad hoc schemes. The precoder submatrices \( F_{k,i} \) can be selected to be nondiagonal or diagonal depending upon whether the relays would cooperate or not respectively. The aim is to choose these matrices optimally.

Let us see an example of the MMSER-\( \mu \) scheme. Fig. 4.2 shows all the link sets in \( \mathcal{L}_3 = \{L_{0,1}, L_{0,2}, L_{0,3}, L_{1,2}, \ldots, L_{3,4}, L_{3,5}, L_{4,5}\} \) and \( \mathcal{L}_4 = \{L_{0,1}, L_{0,2}, L_{0,3}, L_{0,4}, \ldots, L_{3,4}, L_{3,5}, L_{4,5}\} \) in various phases for MMSER-3 and MMSER-4 respectively with four relay layers, i.e., \( K = 4 \). Dashed and firm lines are used for MMSER-3 and MMSER-4 respectively to differentiate them. Signals travelling by these link sets only are stored for forwarding in these examples. Fig. 4.2 also shows how the transmission and reception progress in time.

If one counts the number of link sets used in MMSER-3 and MMSER-4 he/she can find them as 12 (dashed lines) and 14 (firm lines) respectively from Fig. 4.2. These figures can be confirmed from (2.1) by substituting \( K = 4 \), \( \mu = 3 \) and \( K = \mu = 4 \) to get \( |\mathcal{L}_3| = 12 \) and

\(^1\)Here, we have taken cue from the ad hoc scheme RMCS discussed in the section 3.1, where the relay \( R_{2j} \) used a precoder matrix \( F_{2j} \in \mathbb{C}^{T \times 2T} \) to combine the two vectors received in earlier phases

\(^2\)\( F_{k,i} \) denotes the precoder matrix used by \( R_{k,i} \) relay in ad hoc schemes
Figure 4.2: Transmission schemes showing link sets in $L_3$ (dashed lines) for MMSER-3 and $L_4$ (firm lines) for MMSER-4 with four relay layers, i.e., $K = 4$. It also shows how transmission and reception progress in various phases. Here, $L_0$ and $L_5$ represent the source and the destination respectively.

$|L_4| = 14$ respectively. As MMSER-4 uses more link sets, the signal content at $D_1$ increases and hence its BER performance is better than MMSER-3.

Table 4.1 shows a summary of various transmit and receive signals of the layers. Similar to the ad hoc scheme, the letters, $t$, $r$, and $u$ refer to transmit, receive, and noise signals respectively. Subscripts and superscripts correspond to the layer and phase numbers respectively.

Synchronous reception and transmission are assumed at relay nodes and all noise signals are assumed complex zero–mean i.i.d. Gaussian random variables with variance $\sigma_u^2$. Unlike in the ad hoc schemes chapter where we assumed $\sigma_u^2 = 1$, here we vary it to modify the SNR.

We can note that $r_k$ is a concatenated vector of signal vectors in the layer $L_k$, received from the previous $k - n$ transmissions with $n = \lceil k - \mu \rceil^+$. Let $S_1$ transmit $t_0 = \sqrt{p_0}s$ with an average power of $p_0$ Watts, where $s \in \mathbb{C}$ is a unit variance constellation point.

Let us define MSE at relays at layer $L_k$ to be $J_k \triangleq E \|s - t_k\|^2$, where $s = [s, \cdots, s]^T \in \mathbb{C}^{N \times 1}$. Now, the aim is to minimize $J_k$ under the power constraint $E [t_k^H t_k] \leq p_k$ and find the precoder matrices $F_k, \forall k \in [1, K]$. 

78
Table 4.1: Transmitted and received signals - MMSER

<table>
<thead>
<tr>
<th>Layer</th>
<th>Transmitted scalar/vector signals</th>
<th>Received scalar/vector signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_0)</td>
<td>(t_0 = \sqrt{P_0} s, \ s \in \mathbb{C})</td>
<td>-</td>
</tr>
<tr>
<td>(L_k, 1 \leq k \leq K)</td>
<td>(\mathbf{t}_k = \mathbf{F}_k \mathbf{r}_k), where (\mathbf{r}_k^T = \left[\mathbf{r}_k^{(0)} \cdots \mathbf{r}_k^{(k-1)}^T\right]) with (n = [k - \mu]^+).</td>
<td>If (n = 0), then (\mathbf{r}<em>k^{(n)} = h</em>{0,k} t_0 + u_k^{(0)}).(\mathbf{r}<em>k^{(i)} = H</em>{i,k} \mathbf{t}_i + u_k^{(i)}) (\forall i \in [n, k - 1] \text{ and } n \neq 0).</td>
</tr>
<tr>
<td>(L_{K+1})</td>
<td>-</td>
<td>If (i = K - \mu + 1 = 0), then (\mathbf{r}<em>{K+1,1}^{(i)} = h</em>{0,K+1,1} t_0 + u_{K+1,1}^{(0)}). (\mathbf{r}<em>{K+1,1}^{(i)} = H</em>{i,K+1,1} \mathbf{t}<em>i + u</em>{K+1,1}^{(i)}) (\forall i \in [K - \mu + 1, K]) (K - \mu + 1 \neq 0).</td>
</tr>
</tbody>
</table>

Here, \(\mathbf{t}\), \(\mathbf{r}\), and \(\mathbf{u}\) represent transmit, receive, and noise signals, while subscripts and superscripts denote layers and phases respectively.

4.1.1 Precoder matrix at \(L_1\)

From the system model explained in Chapter 2, it is seen that for any value of \(\mu\), and the corresponding link class \(\mathcal{L}_\mu\), the precoder \(\mathbf{F}_1\) of \(L_1\) is the same. For any \(\mu\), \(1 \leq \mu \leq K + 1\), the first layer always stores the only received vector \(\mathbf{r}_1^{(0)}\) and transmits \(\mathbf{t}_1 = \mathbf{F}_1 \mathbf{r}_1 = \mathbf{F}_1 \mathbf{r}_1^{(0)}\). As defined earlier, the MSE \(J_1(\mathbf{F}_1)\) at \(L_1\) relays is the average of the norm square of the difference between the signal vector \(\mathbf{s} = [s, \cdots, s]^T \in \mathbb{C}^{N \times 1}\) and the transmit vector \(\mathbf{t}_1\), i.e.,

\[
J_1(\mathbf{F}_1) \triangleq E \left[ ||\mathbf{s} - \mathbf{t}_1||^2 \right] = E \left[ (\mathbf{s} - \mathbf{F}_1 \mathbf{r}_1)^H (\mathbf{s} - \mathbf{F}_1 \mathbf{r}_1) \right]. \tag{4.1}
\]

Now the optimum \(\mathbf{F}_1\) is given by \(\hat{\mathbf{F}}_1 = \arg \min_{\mathbf{F}_1} J_1(\mathbf{F}_1)\) subject to the constraint \(E [\mathbf{t}_1^H \mathbf{t}_1] \leq p_1\). Let us write the constraint function as \(C_1(\mathbf{F}_1) = E [\mathbf{t}_1^H \mathbf{t}_1] - p_1 \leq 0\) and let \(\mathcal{D} = \text{domain}(\mathbf{F}_1) \cap \text{domain}(C_1)\). This problem is an MMSE estimation problem with a convex
constraint on the estimate. It is a convex optimization problem with a unique solution as discussed in [45]. (Michaeli and Eldar [45] have shown that to solve this kind of a problem, take the unconstrained MMSE estimate and project it onto the constraint set. Here, we also scale up the projected estimate to make sure that the power transmitted by the relays is the same as the constrained power, if it is lower.) Now, given the optimization variable \( F_1 \in \mathbb{C}^{N \times N} \), the cost function \( J_1 : \mathbb{C}^{N \times N} \rightarrow \mathbb{R} \), and the inequality constraint function \( C_1 : \mathbb{C}^{N \times N} \rightarrow \mathbb{R} \), we define the Lagrangian \( L_1 : \mathbb{C}^{N \times N} \times \mathbb{R} \rightarrow \mathbb{R} \)

\[
L_1(F_1, \lambda_1) \equiv J_1(F_1) + \lambda_1 C_1(F_1),
\]

(4.2)

where \( \lambda_1 \geq 0 \) is the Lagrange multiplier and the domain of \( L_1 = D \times \mathbb{R} \).

Expanding (4.1) we get

\[
J_1(F_1) = E[s^H s - 2\Re \left( r_1^H F_1^H s \right) + r_1^H F_1^H F_1 r_1]
= N - 2\Re \left( \text{Tr} \left[ F_1^H R_{sr_1} \right] \right) + \text{Tr} \left[ F_1^H F_1 R_{r_1} \right],
\]

(4.3)

where \( \Re(.) \) is the real-part and \( \text{Tr}[.] \) denotes the trace operator. To arrive at (4.3), \( r_1^H F_1^H F_1 r_1 = \text{Tr} \left( r_1^H F_1^H F_1 r_1 \right) \) is used as it is a scalar. Also the cyclic properties of the \( \text{Tr}(.) \) function namely \( \text{Tr}(ABC) = \text{Tr}(BCA) \) is used. \( R_{sr_1} \) and \( R_{r_1} \) are the correlation matrices given by

\[
R_{sr_1} = E[ss^H] = E \left[ s \left( h_{0,1} t_0 + u_{1}^{(0)} \right)^H \right]
= R_{st_0} h_{0,1}^H \quad \text{and}
\]

\[
R_{r_1} = E[r_1 r_1^H] = h_{0,1} R_{t_0} h_{0,1}^H + \sigma_u^2 i_N
\]

(4.4)

(4.5)

respectively, where \( r_1 = r_1^{(0)} = h_{0,1} t_0 + u_{1}^{(0)} \) is used from Table 4.1. We have \( t_0 = \sqrt{p_0} s \) and hence

\[
R_{st_0} = E[st_0^*] = \sqrt{p_0} i_N \quad \text{and}
\]

\[
R_{t_0} = E[t_0 t_0^*] = p_0.
\]

(4.6)

(4.7)

Here, \( i_N \) is a vector with all components unity given by \( i_N = [1, \ldots, 1]^T \in \mathbb{R}^{N \times 1} \). Expanding and simplifying (4.2), we get

\[
L_1(F_1, \lambda_1) = N - 2\Re \left( \text{Tr} \left[ F_1^H R_{sr_1} \right] \right) + \text{Tr} \left( F_1^H F_1 R_{r_1} \right) + \lambda_1 \left[ \text{Tr} \left( F_1^H F_1 R_{r_1} \right) - p_1 \right].
\]

(4.8)

Now let us derive \( \hat{F}_1 \), for the cases of cooperative and noncooperative relays.
Cooperative relays

Cooperative relays exchange information amongst themselves and use this information in their precoders. Hence the precoder matrix is nondiagonal.

Claim 8. For the cooperative relays, we can obtain the precoder as

\[
\hat{F}_1 = \left[ \frac{p_1}{\text{Tr} \left[ R_{sr}^H R_{sr} R_{r1}^{-1} \right]} \right]^{\frac{1}{2}} R_{sr} R_{r1}^{-1}. \quad (4.9)
\]

Proof: See Appendix H.

Hence, the transmit vector of $L_1$ becomes

\[
t_1 = \left[ \frac{p_1}{\text{Tr} \left[ R_{sr}^H R_{sr} R_{r1}^{-1} \right]} \right]^{\frac{1}{2}} R_{sr} R_{r1}^{-1} r_1. \quad (4.10)
\]

Alternatively, the same solution could also be obtained as follows: Let us define the transmit vector as

\[
t_1 = \alpha_1 F'_1 r_1 \quad (4.11)
\]

and the MSE, $J'_1(F'_1)$, at $L_1$ relays as the average of the norm square of the difference between the signal vector $s = [s, \cdots, s]^T \in \mathbb{C}^{N \times 1}$ and the scaled transmit vector $t_1/\alpha_1$; i.e.,

\[
J'_1(F'_1) \triangleq E \left\| s - \frac{t_1}{\alpha_1} \right\|^2. \quad (4.12)
\]

Claim 9. The transmit vector shown in (4.10) can also be obtained using a short-cut by first differentiating (4.12) w.r.t. $F'_1$, equating to zero and then obtain $\alpha_1$ to satisfy the constraint $E \left[ t_1^H t_1 \right] = p_1$.

Proof: See Appendix I.

This short-cut method is used later while deriving the precoder in (4.28) for $L_k$ when the relays cooperate.

Claim 10. Using (4.4) and (4.5), we can simplify the optimum precoder in (4.9) as

\[
\hat{F}_1 = \left[ \frac{p_1}{N \left( p_0 \| h_{01} \|^2 + \sigma_0^2 \right)} \right]^{\frac{1}{2}} i_N h_{01}^H \quad (4.13)
\]

Proof: See Appendix J.

It can be seen from (4.13) that $\hat{F}_1$ does not depend on forward channel matrices, $H_{ij}$, $j \in [2, K]$, and hence, $L_1$ does not require forward CSI, when it transmits subsequently in phase-1.
Noncooperative relays

In this case, $F_1$ is taken to be diagonal. We obtain $F_1$ using two different methods: (1) zeroising non diagonal elements method and (2) Scalar optimization method. In the first method, we start with zeroising the non diagonal elements in the cooperative precoder shown in (4.13) and then use a scaling factor to ensure that the transmitted power is $p_1$. (Lee et al. [30]$^3$ also employ this method to obtain their precoder for the noncooperative relays case.) So, the precoder estimate is obtained by projecting the cooperative solution onto the convex set defined by the diagonal and the transmit power constraints. Now, let us derive the precoder for this method.

Zeroising the non diagonal elements in the cooperative solution in (4.13), we get

$$
\hat{F}^{\text{zeroised}}_1 = \left[ \frac{p_1}{N \left( p_0 \| h_{0,1} \|^2 + \sigma_u^2 \right)} \right] \frac{1}{\| h_{0,1} \|} \begin{bmatrix}
\hat{h}_{0,1,1,1}^* & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \hat{h}_{0,1,1,N}^*
\end{bmatrix}
$$

(4.14)

as

$$
i_N h_{0,1}^H = \begin{bmatrix}
1 \\
\vdots \\
1
\end{bmatrix} \begin{bmatrix}
\hat{h}_{0,1,1,1}^* & \cdots & \hat{h}_{0,1,1,N}^*
\end{bmatrix} = \begin{bmatrix}
\hat{h}_{0,1,1,1}^* & \cdots & \hat{h}_{0,1,1,N}^* \\
\vdots & \ddots & \vdots \\
\hat{h}_{0,1,1,1}^* & \cdots & \hat{h}_{0,1,1,N}^*
\end{bmatrix}.
$$

(4.15)

Let us define the transmit vector as

$$
t^{\text{zeroised}}_1 = \alpha^{\text{zeroised}}_1 \hat{F}^{\text{zeroised}}_1 r_1
$$

(4.16)

and find the scaling factor $\alpha^{\text{zeroised}}_1$ by applying the constraint $E \left[ t^{\text{zeroised}}_1 H t^{\text{zeroised}}_1 \right] = p_1$. Hence,

$$
p_1 = E \left[ t^{\text{zeroised}}_1 H t^{\text{zeroised}}_1 \right]
$$

$$
= E \left[ \left( \alpha^{\text{zeroised}}_1 \hat{F}^{\text{zeroised}}_1 r_1 \right)^H \left( \alpha^{\text{zeroised}}_1 \hat{F}^{\text{zeroised}}_1 r_1 \right) \right]
$$

$$
= \alpha^{\text{zeroised}}_1^2 \text{Tr} \left[ \hat{F}^{\text{zeroised}}_1 r_1 H \hat{F}^{\text{zeroised}}_1 r_1 \right]
$$

$$
= \alpha^{\text{zeroised}}_1^2 \frac{p_1}{N \left( p_0 \| h_{0,1} \|^2 + \sigma_u^2 \right) \| h_{0,1} \|^2}
$$

$$
\text{Tr} \left[ \begin{bmatrix}
|h_{0,1,1,1}|^2 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & |h_{0,1,1,N}|^2
\end{bmatrix}
h_{0,1}^H R_{0,1} h_{0,1} + \sigma_u^2 I_N
\right]
$$

$^3$Authors show the difficulty in ‘scalar optimization’ in MMSED scheme and hence employ ‘zeroising non diagonal’ method.

82
In other words, the $i$th diagonal element of the precoder matrix is given by

$$f_{1i}^{\text{zeroised}} = \frac{\frac{1}{2} \sigma_u^2}{\sqrt{\sum_{j=1}^N |h_{0,1,i,j}|^2 [p_0 |h_{0,1,i,j}|^2 + \sigma_u^2]}}$$

(4.20)

The second method is called ‘scalar optimization’ and is discussed in the sequel. Here, we assume a diagonal structure for the precoder matrix $F_1$ and simplify (4.8).

**Claim 11.** We can prove that the $i$th diagonal element of $F_1$ is given by

$$f_{1i} = \frac{\frac{1}{2} h_{0,1,i,i}^*}{\left(\frac{p_0}{\lambda} \sum_{j=1}^N |h_{0,1,i,j}|^2 + \sigma_u^2\right)^\frac{1}{2}}$$

(4.21)

Proof: See Appendix K.

This can be seen to be not dependent on $h_{1i,2}$ unlike those of MMSED shown in (2.8) and (2.9). Hence, the relays do not require forward CSI, when the relay layer uses the precoder matrix formed by the diagonal elements, $f_{1i}$, $i \in [1, N]$, in MMSER.
4.1.2 Precoder matrix at $L_k$, $k \in [2, K]$

The layer $L_k$ would have received $k - n$ vectors each of size $N$ from previous transmissions, starting from phase $n$ till phase $k - 1$, where $n = \lceil k - \mu \rceil$. All these vectors are stacked together as given in table 4.1 to form $r_k \in \mathbb{C}^{(k-n)N \times 1}$. Thus

$$r_k = \begin{bmatrix} r_k^{(n)} \\ \vdots \\ r_k^{(k-1)} \end{bmatrix}.$$ 

For example, if $K \geq 7$ and $\mu = 2$, then signals received through the link sets in the link class $L_2$ are taken, and $L_2$ and $L_7$ would have the stacked vectors

$$r_2 = \begin{bmatrix} r_2^{(0)} \\ r_2^{(1)} \end{bmatrix} \quad \text{and} \quad r_7 = \begin{bmatrix} r_7^{(5)} \\ r_7^{(6)} \end{bmatrix}$$

respectively. Here, $r_2, r_7 \in \mathbb{C}^{2N \times 1}$.

$L_k$ prepares its transmission vector $t_k$ by multiplying $r_k$ by a precoder $\alpha_k F_k'$, given by

$$t_k = \alpha_k F_k' r_k \quad (4.22)$$

where $\alpha_k$ is a real scalar used to restrain power. The optimum precoder at $L_k$ is given by $\alpha_k \hat{F}_k' \in \mathbb{C}^{N \times (k-n)N}$, where $\hat{F}_k' = \arg\min_{F_k'} J'_k(F_k')$ and the cost function is defined as

$$J'_k(F_k') \triangleq E \left\| s - \frac{t_k}{\alpha_k} \right\|^2 = E \left[ s - F_k' r_k \right]^H [s - F_k' r_k]$$

$$= N - 2\Re \left( \text{Tr} \left[ F_k'^H R_{sr_k} \right] \right) + \text{Tr} \left[ F_k'^H F_k' R_{rr_k} \right] \quad (4.23)$$

with the power constraint $E \left[ t_k^H t_k \right] \leq p_k$. Here, the correlation matrices $R_{sr_k}$ and $R_{rr_k}$ are given by

$$R_{sr_k} = E \left[ s r_k^H \right] = E \left[ s \left( r_k^{(n)} H \ldots r_k^{(k-1)H} \right) \right]$$

$$= \begin{bmatrix} R_{sr_k}^{(n)} \cdots R_{sr_k}^{(k-1)} \end{bmatrix} \quad (4.24)$$

and

$$R_{rr_k} = E \left[ r_k r_k^H \right]$$

$$= \begin{bmatrix} R_{r_k}^{(n)} & \cdots & R_{r_k}^{(n)} \\ \vdots & \ddots & \vdots \\ R_{r_k}^{(k-1)n} & \cdots & R_{r_k}^{(k-1)(k-1)} \end{bmatrix} \quad (4.25)$$
respectively, where \( n = \lceil k - \mu \rceil \). Again \( R_{sr_k}^{(i)} \) and \( R_{r_k}^{(i)} r_{(j)} \), \( i, j \in [n, k - 1] \), are given by

\[
R_{sr_k}^{(i)} = E \left[ s \left( H_{i,k} t_i + u_k^{(i)} \right)^H \right] = R_{sr_k} H_{i,k} \quad (4.26)
\]

and

\[
R_{r_k}^{(i)} r_{(j)} = E \left[ (H_{i,k} t_i + u_k^{(i)}) (H_{j,k} t_j + u_k^{(j)})^H \right] \\
= H_{i,k} \left( R_{t_i t_j} H_{i,k}^H + R_{t_i u_k^{(i)} t_j} + R_{u_k^{(i)} t_j} H_{j,k}^H \right) \quad (4.27)
\]

respectively, using Table 4.1. We find that these matrices are dependent on other correlation matrices. Expressions for the correlation matrices are given in Appendix N. Now let us find the precoder for two different cases: when the relays are cooperative and noncooperative.

### Cooperative relays

Here, each of the sub-matrices \( F_{ki} \in \mathbb{C}^{N \times N} \) in \( F_k = [F_{kn} \cdots F_{k,k-1}] \), \( n \leq i \leq k - 1 \), are non diagonal. The shortcut method proved in Appendix I to find the optimum precoder is used here.

**Claim 12.** We can prove that the optimum precoder \( \hat{F}_k \) is given by

\[
\hat{F}_k = \left[ \frac{p_k}{\text{Tr} \left( R_{sr_k} R_{sr_k}^{-1} R_{r_k}^{-1} \right)} \right]^{1/2} R_{sr_k} R_{r_k}^{-1} \quad (4.28)
\]

*Proof: See Appendix L.*

We can substitute (4.28) into (4.22) and obtain the transmission vector as

\[
t_k = \left[ \frac{p_k}{\text{Tr} \left( R_{sr_k} R_{sr_k}^{-1} R_{r_k}^{-1} \right)} \right]^{1/2} R_{sr_k} R_{r_k}^{-1} r_k. \quad (4.29)
\]

### Noncooperative relays

Here, each of the sub-matrices \( F_{ki} \in \mathbb{C}^{N \times N} \) in \( F_k = [F_{kn} \cdots F_{k,k-1}] \), \( n \leq i \leq k - 1 \), are diagonal. As discussed in Section 4.1.1, we can employ ‘zeroising non diagonal’ method to obtain the precoder estimate by projecting the cooperative solution onto the convex set defined by the diagonal and the transmit power constraints. Under Section 3.8, in Fig. 4.10, we compare the BER performance of this method with that of the ‘scalar optimization’ method.

In the sequel, we will discuss the ‘scalar optimization’ method. The short-cut optimization method proved in Claim 9 is not valid when relays do not cooperate. Hence, the Lagrangian \( L_k \)
is used as was done in section 4.1.1 when \( K = 1 \). Let us define the cost function as

\[
J_k(F_k) \triangleq E \| s - t_k \|^2 = E [s - F_k r_k]^H [s - F_k r_k].
\] (4.30)

Now the optimum \( F_k \) is given by \( \hat{F}_k = \arg \min_{F_k} J_k(F_k) \) subject to the constraint \( E [t_k^H t_k] \leq p_k \). The constraint function can be written as \( C_k(F_k) = E [t_k^H t_k] - p_k \leq 0 \).

This is an optimization problem [46] with the optimization variable \( F_k \in \mathbb{C}^{N \times (k-n)N} \), the cost function \( J_k : \mathbb{C}^{N \times (k-n)N} \rightarrow \mathbb{R} \), and the inequality constraint function \( C_k : \mathbb{C}^{N \times (k-n)N} \rightarrow \mathbb{R} \) with \( n = [k - \mu]^+ \). Now the Lagrangian \( L_k : \mathbb{C}^{N \times (k-n)N} \times \mathbb{R} \rightarrow \mathbb{R} \) is defined as

\[
L_k(F_k, \lambda_k) \triangleq J_k(F_k) + \lambda_k \left[ \text{Tr} (F_k^H F_k r_k) - p_k \right],
\] (4.31)

where \( \lambda_k \geq 0 \) is the Lagrange multiplier. As \( J'_k(F'_k) = J_k(F_k) \), (4.23) is substituted into (4.31) to get

\[
L_k = N - 2\Re \left[ \text{Tr} \left( F_k^H R_{sr_k} \right) \right] + \text{Tr} (F_k^H F_k r_k) + \lambda_k \left[ \text{Tr} (F_k^H F_k r_k) - p_k \right].
\] (4.32)

Here, each of the sub-matrices \( F_{kl} \in \mathbb{C}^{N \times N} \) in \( F_k = [F_{kn} \cdots F_{k,k-1}] \), \( n \leq i \leq k - 1 \), are diagonal and \( n = [k - \mu]^+ \).

**Claim 13.** We can obtain the optimum precoder from the vector \( \hat{f}_{kl} \), which is given by

\[
\hat{f}_{kl} = \left[ \frac{p_k}{\sum_{l=1}^N \text{Tr} \left( \Upsilon_{kl}^s H \Upsilon_{kl}^s \Upsilon_{kl}^t \right)} \right]^{\frac{1}{2}} \Upsilon_{kl}^s \Upsilon_{kl}^{-1}, \quad l \in [1, N].
\] (4.33)

Here,

\[
f_{kl} = [f_{kn,l} \cdots f_{k,k-1,l}], \quad \Upsilon_{kl} = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1(k-1)} \\ \vdots & \ddots & \vdots \\ \gamma_{k1} & \cdots & \gamma_{k(k-1)} \end{bmatrix},
\] (4.34)

and

\[
\Upsilon_{kl}^s = \begin{bmatrix} \gamma_{11}^s & \cdots & \gamma_{1(k-1)}^s \\ \vdots & \ddots & \vdots \\ \gamma_{k1}^s & \cdots & \gamma_{k(k-1)}^s \end{bmatrix},
\]

where \( f_{kl} \), \( f_{kj} \), and \( \gamma_{(i,j)}^{(s)} \) represent the \( l \)-th diagonal elements of \( F_{kl} \), \( F_{kj} \), and \( R_{r_k r_{s_r k}}^{(s)} \) respectively. Also \( \gamma_{kl}^{(s)} \) is the \( l \)-th diagonal element of the correlation matrix \( R_{sr_k}^{(s)} \).

**Proof:** See Appendix M.
We notice the similarity of (4.33) with the cooperative precoder in (4.28), where \( R_{sr_k} \) and \( R_{r_k} \) are analogous to \( \Upsilon_{sk} \) and \( \Upsilon_{kl} \) respectively, except for the extra summing operator in the denominator in (4.33).

The optimum precoder \( \hat{F}_k \) at \( L_k \) is made from these optimum vectors shown in (4.33), by noting that these vectors give the \( l \)th diagonal elements of all submatrices, \( F_{ki} \), \( i \in [n,k-1] \), \( n = [k-\mu]^+ \), that make up the precoder. The optimum precoder depends on these matrices, \( \Upsilon_{sk} \) and \( \Upsilon_{kl} \), \( l \in [1,N] \) which in turn depend on the correlation matrices \( R_{sr_k} \) and \( R_{r_k} \) given in (4.24) and (4.25) respectively.

### 4.1.3 MMSER precoders independent of forward CSI

As defined earlier, forward CSI is the forward channel-state-information at the transmitting relay layers, which is required by the MMSEED schemes to be used in their precoders. For MMSER, at \( L_1 \), we have seen in (4.13) and (4.21) that the precoders do not depend on forward CSI for cooperative and noncooperative relays cases respectively. Similarly, at \( L_k \), \( k \in [2,K] \), for both cooperative and noncooperative relays in (4.28) and (4.85) respectively, we saw that the precoders depend on two correlation matrices, viz. \( R_{sr_k} \) and \( R_{r_k} \). These matrices given in (4.24) and (4.25) in turn depend on \( r_k \), which is a concatenated vector of received vectors at layer \( L_k \) and hence the precoders depend only on channel coefficients of the backward channel links. Therefore, MMSER-\( \mu \) does not require forward CSI.

### 4.1.4 Enhancement of MMSED schemes

Precoders of the MMSED schemes, MMSED-K, MMSED-KK, and MMSED-L are shown in equations (2.6), (2.8) and (2.9) respectively for \( K = 1 \). For \( K = 2 \), Lee et al. [32] obtained the MMSED precoders for the two relay layers iteratively. All these precoders depend on global CSI and perform better than MMSER system proposed in this thesis, which depends on backward CSI only.

To the best of our knowledge, there is no MMSED system in literature, which has closed form solution for \( K > 1 \), i.e., single layer MMSED principle does not have its counterpart in a multiple layer system. It is also difficult to extend the MMSED scheme to work in a multi-layer case. We show this difficulty of extending it to a simple two layer system (\( K=2 \)) in the sequel.
**Difficulty in extension of MMSED schemes**

We will take the MMSED-K system of [28] which obtained precoder for the relays, when they cooperate in a single layer of relays, i.e., $K = 1$. Using the same MMSED strategy, we will attempt to obtain the optimum precoder similar to that shown in (2.6) in Chapter 2 but with $K = 2$ here. The objective function is given in (2.5) and repeated here for convenience as

$$J_D = \sum_{m=1}^{M} E \left| s_m - h_{1,2m} t_1 \right|^2. \quad (4.35)$$

Here, we will take the simplest case of $M = 1$, i.e., to have single source-destination pair, but with $K = 2$. Also we will not assume considering any leaked signal while forwarding by the relays. So the objective function is appropriately modified as

$$J_D = E \left| s - h_{2,3,1} t_2 \right|^2, \quad (4.36)$$

where $s$ is the source transmitted signal, $t_2 \in \mathbb{C}^{N \times 1}$ is the vector transmitted by $L_2$ layer, and $h_{2,3,1}$ given by

$$h_{2,3,1} = [h_{21,31}, \ldots, h_{2N,31}] \quad (4.37)$$

is the vector of channel coefficients from $L_2$ layer to the destination as shown in Fig. 4.3. Now

![System model with two layers of relays and one source-destination pair for MMSED.](image)

Figure 4.3: System model with two layers of relays and one source-destination pair for MMSED.

the transmission vector $t_i$, $i \in [1, 2]$, of $L_i$, is given by

$$t_i = F_i r_i. \quad (4.38)$$

We do not use any superscript for the phase number in $r$ as there is no requirement to consider reception in multiple phases in a particular layer. Now the aim is to find the precoder matrices of $L_1$ and $L_2$ layers, namely $F_1$ and $F_2$ respectively.
The received vector \( r_2 \in \mathbb{C}^{N \times 1} \) in L\(_2\) layer is given by

\[
r_2 = H_{12} t_1 + u_2,
\]

where \( H_{12} \in \mathbb{C}^{N \times N} \) and \( u_2 \) are the matrix of channel coefficients from L\(_1\) to L\(_2\) and noise vector respectively. Expanding (4.39), we get

\[
r_2 = H_{12} F_1 r_1 + u_2 = H_{12} F_1 (\alpha_0 s h_{01,1} + u_1) + u_2
\]

(4.40)

where we have substituted the L\(_1\) received vector \( r_1 = \alpha_0 s h_{01,1} + u_1 \). Now expanding (4.36), we get

\[
J_D = E |s - h_{2,3,1} t_2|^2 = E [(s - h_{2,3,1} t_2)^H (s - h_{2,3,1} t_2)]
\]

\[
= 1 - 2 \Re \left( \text{Tr} \left[ R_{st_2} h_{2,3,1}^H \right] \right) + \text{Tr} \left[ h_{2,3,1}^H h_{2,3,1} R_{r_2} \right],
\]

(4.41)

where \( R_{st_2} \) and \( R_{r_2} \) are given by

\[
R_{st_2} = E [s t_2^H F_2^H] = R_{sr_2} F_2^H \text{ and } R_{r_2} = E [F_2 r_2 r_2^H F_2^H] = F_2 R_{r_2} F_2^H.
\]

(4.42)

Substituting (4.42) into (4.41), we get

\[
J_D = 1 - 2 \Re \left( \text{Tr} \left[ R_{sr_2} F_2^H h_{2,3,1} \right] \right) + \text{Tr} \left[ h_{2,3,1}^H h_{2,3,1} F_2 R_{r_2} F_2^H \right].
\]

(4.43)

Including the power constraints for the relay layers L\(_1\) and L\(_2\) as \( p_1 \) and \( p_2 \) respectively, we have the Lagrangian as

\[
\mathcal{L}_D = 1 - 2 \Re \left( \text{Tr} \left[ R_{sr_2} F_2^H h_{2,3,1} \right] \right) + \text{Tr} \left[ h_{2,3,1}^H h_{2,3,1} F_2 R_{r_2} F_2^H \right] + \lambda_1 \left[ \text{Tr} \left[ F_1^H F_1 R_{r_1} \right] - p_1 \right] + \lambda_2 \left[ \text{Tr} \left[ F_2^H F_2 R_{r_2} \right] - p_2 \right].
\]

(4.44)

Now \( \mathcal{L}_D \) is to be differentiated w.r.t. \( F_1, F_2, \lambda_1, \) and \( \lambda_2 \) and solution for them obtained. This is quite involved and difficult to arrive at as the optimum precoders \( \hat{F}_1 \) and \( \hat{F}_2 \) thus obtained are functions of each other as shown in [32] by Lee et al. Hence, these authors proposed an iterative algorithm instead of going for a closed-form solution. (Unlike MMSED, we saw in earlier sections that obtaining optimum precoders was easier in MMSER and obtained closed-form solutions to them by breaking the complexity.)

In view of this, we enhance the MMSED systems to operate in a multi-layer relay network shown in Fig. 4.1 and obtain closed-form solutions to precoders in a different way. Then, we
will compare their performance with MMSER-μ system proposed in this thesis. Let us call these systems E-MMSED, for Enhanced-MMSED systems. It is shown that the precoders obtained for E-MMSED systems also are dependent on global CSI as shown in (4.49) and (4.50) in this section.

E-MMSED Strategy

For a meaningful comparison, let us consider the total number of phases as $K + 1$, and the total power transmitted as $P$ to be the same as that used by MMSER.

Various links that are used by E-MMSED are shown in Fig. 4.4. $S_1$ transmits $t'_0 = \sqrt{p'_0}s$ repeatedly for $K_0$ times, from phase zero to phase $K_0 - 1$ and the relays follow suit transmitting a signal $t$, which is obtained by averaging their received signals, from phase $K_0$ to $K$, i.e., $K_1 = K - K_0 + 1$ times. For each of these transmissions, when $S_1$ transmits or the relays transmit, the channel does not vary as a slow varying channel is assumed. Hence, the average power $p'_0$ can be equally divided in various phases when $S_1$ transmits and $p'_r$ when the relays transmit. So $p'_0 = p_0/K_0$ Watts and $p'_r = p_r/K_1$ Watts respectively, with $p_0 + p_r = P$, the total power available. So, $D_1$ would have a vector of $K_1$ received signals which it can use for decoding.

The relays in all the layers would receive in phase $k$, $k \in [0, K_0 - 1]$, a vector $r^{(k)}$ in
\( \mathbb{C}^{KN \times 1} \) given by

\[
\mathbf{r}^{(k)} = t'_0 \mathbf{h}_0 + \mathbf{u}^{(k)}, \quad k \in [0, K_0 - 1],
\]

where

\[
\mathbf{h}_0 = \begin{bmatrix}
  h_{0,1} \\
  \vdots \\
  h_{0,K}
\end{bmatrix}
\quad \text{and} \quad
\mathbf{u}^{(k)} = \begin{bmatrix}
  u^{(k)}_1 \\
  \vdots \\
  u^{(k)}_K
\end{bmatrix}
\]

with

\[
\mathbf{h}_{0,i} = \begin{bmatrix}
  h_{0,1,i,1} \\
  \vdots \\
  h_{0,1,i,N}
\end{bmatrix}
\quad \text{and} \quad
\mathbf{u}^{(k)}_i = \begin{bmatrix}
  u^{(k)}_{i,1} \\
  \vdots \\
  u^{(k)}_{i,N}
\end{bmatrix}, \quad i \in [1, K].
\]

Let us assume that all the relays transmit together in their transmission phases as though they are in a single layer\(^4\). Let us also assume that the noise is uncorrelated, i.e.,

\[
E[\mathbf{u}^{(k)}(l)\mathbf{u}^{(l)H}] = \sigma_u^2 \delta(k - l) \mathbf{I}_{KN}.
\]

Here, \( \delta(\cdot) \) is the impulse sequence [47] given by

\[
\delta(m) = \begin{cases}
  0, & \text{if } m \neq 0 \\
  1, & \text{if } m = 0
\end{cases}
\]

The relays average all the signals received and transmit in phases \( K_0 \) to \( K \) the same signal \( \mathbf{t} = \mathbf{F} \mathbf{r}_{av} \), where

\[
\mathbf{r}_{av} = \frac{1}{K_0} \sum_{i=0}^{K_0-1} \mathbf{r}^{(i)} = t'_0 \mathbf{h}_0 + \frac{1}{K_0} \sum_{i=0}^{K_0-1} \mathbf{u}^{(i)}
\]

from (4.45).

Though we have grouped all the relays together for the transmission, we do not presume that the relays can cooperate amongst themselves. This is because it may not be feasible as they are actually present in different layers. Hence, we attempt to find a diagonal precoder matrix.

As \( \mathbf{F} \in \mathbb{C}^{KN \times KN} \) is diagonal, let us define it as \( \mathbf{F} \triangleq \text{diag}[f_{11}, \ldots, f_{1N}, f_{21}, \ldots, f_{2N}, \ldots, f_{K1}, \ldots, f_{KN}] \), where \( f_{ij} \) is the multiplying factor of the relay \( R_{ij} \).

\(^4\)Like the ad hoc scheme MJHS which we saw in section 3.5
From (4.47), $t = F r_{av}$ becomes
\[
t = t'_0 F h_0 + \frac{F}{K_0} \sum_{i=0}^{K_0-1} u^{(i)},
\] (4.48)
which is transmitted $K_1$ times, so that the total number of phases would be $K + 1$, the same as that of MMSER.

Now, let us derive precoders for enhanced MMSED-KK (henceforth called E-MMSED-KK) and enhanced MMSED-L (henceforth called E-MMSED-L) schemes.

**Precoder of E-MMSED-KK**

We take (2.8) and replace the power transmitted by $S$, $p_0$ with $p_0/K_0$ and that of the relays, $p_1$ by $p_r/K_1$ as we allocate fractions of powers to them due to their multiple transmissions. Also the noise variance, $\sigma_u^2$ is replaced by $\sigma_u^2/K_0$. This is because the noise variance at each of the relays, $R_{im}$, $i \in [1, K]$, $m \in [1, N]$, after averaging is found from (4.47) as
\[
E \left[ \left( \frac{1}{K_0} \sum_{k=0}^{K_0-1} u^{(k)}_{im} \right)^* \left( \frac{1}{K_0} \sum_{l=0}^{K_0-1} u^{(l)}_{im} \right) \right] = \frac{1}{K_0^2} \sum_{k=0}^{K_0-1} \sum_{l=0}^{K_0-1} \sigma_u^2 \delta(k - l) = \frac{\sigma_u^2}{K_0}.
\]

So, we get $f_{im}$, $i \in [1, K]$ and $m \in [1, N]$, a diagonal element of the precoder of E-MMSED-KK as
\[
f_{im} = \frac{\left[ \frac{p_1 K_0}{K_1} \right]^\frac{1}{2} h_{0,1,i,m}^* h_{i,m,K+1,1}^*/|h_{i,m,K+1,1}|^2}{\sum_{k=1}^{K} \sum_{l=1}^{N} \frac{|h_{0,1,k,l}|^2}{|h_{k,l,K+1,1}|^2} \left( p_0 |h_{0,1,k,l}|^2 + \sigma_u^2 \right)^{\frac{1}{2}}}.
\] (4.49)

**Precoder of E-MMSED-L**

Similarly to get the diagonal elements of the precoder of E-MMSED-L, the signal power transmitted and noise variance are replaced as in E-MMSED-KK into (2.9) to get its diagonal element $f_{im}$ as
\[
f_{im} = \frac{\left[ \frac{p_1 K_0}{K_1} \right]^\frac{1}{2} h_{0,1,i,m}^* h_{i,m,K+1,1}^*/|h_{i,m,K+1,1}|^2}{\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,1,k,l}|^2 |h_{k,l,K+1,1}|^2 \left( p_0 |h_{0,1,k,l}|^2 + \sigma_u^2 \right)^{\frac{1}{2}}}.
\] (4.50)

Note that in both (4.49) and (4.50), a double summation in the denominator instead of a single summation is present, when $K$ is greater than 1.
Claim 14. The $i$th, $i \in [1, K]$, $m \in [1, N]$, diagonal element of the precoder of E-MMSED-L shown in (4.50) can also be obtained starting from the basic equation of the MSE given in (2.5) with $M = 1$.

Proof: See Appendix O. ■

Also, we see in both cases of E-MMSED-KK and E-MMSED-L, the relay $R_{i,m}$ needs forward CSI $h_{i,m,K+1,1}$.

Selection of $K_0$

Let us now select the best $K_0$ and use it while comparing its performance with MMSER-$\mu$ system. As it is hard to derive BER, SNR at the destination is obtained for any $K_0$ and an attempt is made to select that value of $K_0$ that maximizes it.

In phase $k$, $k \in [K_0, K]$, $D_1$ receives a scalar

$$ r^{(k)}_{K+1} = h_{K+1}t + u^{(k)}_{K+1}, \quad (4.51) $$

where

$$ h_{K+1} = \begin{bmatrix} h_{1,K+1} & \cdots & h_{K,K+1} \end{bmatrix} \in \mathbb{C}^{1 \times KN} $$

with

$$ h_{i,K+1} = [h_{i,1,K+1,1}, \cdots, h_{i,N,K+1,1}], \quad i \in [1, K]. \quad (4.52) $$

Substituting (4.48) into (4.51), we get

$$ r^{(k)}_{K+1} = h_{K+1} \left[ t_0Fh_0 + \frac{F}{K_0} \sum_{i=0}^{K_0-1} u^{(i)} \right] + u^{(k)}_{K+1} \quad (4.53) $$

where

$$ S^{(k)} = \sqrt{\frac{p_0}{K_0}} h_{K+1}Fh_0s \quad (4.54) $$

and

$$ N^{(k)} = \frac{h_{K+1}F}{K_0} \sum_{i=0}^{K_0-1} u^{(i)} \quad (4.55) $$

93
are respectively the signal and noise components of the received signal in phase-\(k\), without considering the noise that is added at \(D_1\). Let us remove the noise added at \(D_1\), as it is not considered while deriving optimum precoder in MMSED. Now these components in \(D_1\) are concatenated into vectors as

\[
\mathbf{S} = \begin{bmatrix}
S^{(K_0)} \\
\vdots \\
S^{(K)}
\end{bmatrix}
\text{ and } \mathbf{N} = \begin{bmatrix}
N^{(K_0)} \\
\vdots \\
N^{(K)}
\end{bmatrix}.
\]

The signal and noise powers from these vectors can be written as

\[
P_S = E[\mathbf{S}^H \mathbf{S}] = E \sum_{k=K_0}^{K} |S^{(k)}|^2 \quad (4.56)
\]

and

\[
P_N = E[\mathbf{N}^H \mathbf{N}] = E \sum_{k=K_0}^{K} |N^{(k)}|^2 \quad (4.57)
\]

respectively.

**Claim 15.** *Ratio of \(P_S\) and \(P_N\) shown in (4.56) and (4.57) can be found to be*

\[
\frac{P_S}{P_N} = \frac{p_0 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,1,i,j}|^4}{\sigma_u^2 \sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,1,k,l}|^2} \quad (4.58)
\]

and

\[
\frac{P_S}{P_N} = \frac{p_0 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,1,i,j}|^4 |h_{i,j,K+1,1}|^4}{\sigma_u^2 \sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,1,k,l}|^2 |h_{k,l,K+1,1}|^2} \quad (4.59)
\]

for E-MMSED-KK and E-MMSED-L respectively.

**Proof:** *See Appendix P.*

From Claim 15, i.e., from (4.58) and (4.59), we see that we can select any \(K_0\) from \([1, K]\) for both E-MMSED-KK and E-MMSED-L and these ratios do not vary. This is also shown in the BER plots in the later part of this Chapter in Simulations section in Fig. 4.7 that for different values of \(K_0\), BER does not change.

### 4.1.5 Decoder at the destination

Here, we have one destination \(D_1\), unlike in multiple source-destination pairs case to be seen in Section-4.2, where there are multiple destinations corresponding to multiple sources. The signal received at \(D_1\) or the layer \(L_{K+1}\) is \(r_{K+1,1}\). In this section, we will replace \(K+1,1\) in the
suffix with $D_1$ to make the notation simpler, as we need to use the components of this vector. Hence the received vector is given by

$$
r_{K+1,1} = r_{D_1} = \begin{bmatrix} r_{D_1}\ldots r_{D_{1m}} \end{bmatrix}, \quad (4.60)$$

where $m$ is the total number of signals $D_1$ has received in as many number of phases. For MMSER-$\mu$, $m = \mu$, $r_{D_{11}} = r^{(K+1-\mu)}_{D_1}$, and $r_{D_{1m}} = r^{(K)}_{D_1}$. For E-MMSED schemes $m = K_1 = K - K_0 + 1$ as seen in section 4.1.4, $r_{D_{11}} = r^{(K_0)}_{D_1}$ and $r_{D_{1m}} = r^{(K)}_{D_1}$. Now, we will derive an MMSE decoder for extracting the signal from $r_{D_1}$. Let us write the decoded signal to be

$$\hat{s} \triangleq f_{D_1} r_{D_1} \quad (4.61)$$

where $f_{D_1} \in \mathbb{C}^{1 \times m}$ is the decoder vector to be obtained by minimizing the MSE $J_{D_1} \triangleq E \left[ |s - \hat{s}|^2 \right]$. Simplifying $J_{D_1}$ and substituting (4.61) into it, we get

$$J_{D_1} = E \left[ |s - \hat{s}|^2 \right]$$

$$= 1 - 2 \Re \left( \hat{s}^* s \right) + E \left( \hat{s}^* \hat{s} \right)$$

$$= 1 - 2 \Re \left( f_{D_1}^H f_{D_1} s + E \left[ f_{D_1}^H f_{D_1}^H f_{D_1} f_{D_1} r_{D_1} \right] \right)$$

$$= 1 - 2 \Re \Tr \left( f_{D_1}^H R_{s' r_{D_1}} f_{D_1} r_{D_1} \right) + \Tr \left( f_{D_1}^H f_{D_1} R_{r_{D_1} r_{D_1}} \right), \quad (4.62)$$

where

$$R_{s' r_{D_1}} = E \left[ s f_{D_1}^H \right] \in \mathbb{C}^{1 \times m}$$

$$= \begin{bmatrix} R_{s' r_{D_{11}}} & \cdots & R_{s' r_{D_{1m}}} \end{bmatrix}, \quad (4.63)$$

and

$$R_{r_{D_1}} = E \left[ r_{D_1} f_{D_1}^H \right] \in \mathbb{C}^{m \times m}$$

$$= \begin{bmatrix} R_{r_{D_{11}} r_{D_{11}}} & \cdots & R_{r_{D_{11}} r_{D_{1m}}} \\
\vdots & \ddots & \vdots \\
R_{r_{D_{1m}} r_{D_{11}}} & \cdots & R_{r_{D_{1m}} r_{D_{1m}}} \end{bmatrix}. \quad (4.64)$$

Here, we use prime (’) over $s$ in the correlation matrix $R_{s' r_{D_1}}$ to identify it to be a scalar unlike in (4.24). Equation (4.62) is similar to that of the error-performance surface of the
Wiener filter \[48\] and the optimum filter or the decoder is obtained by differentiating (4.62) w.r.t. \( f^*_{D_1} \) \[49\] and equating to zero as follows:

\[
\nabla f^*_{D_1} J_{D_1} = -R_s r_{D_1} + f_{D_1} R_{r_{D_1}} = 0
\]

\[
\Rightarrow \hat{f}_{D_1} = R_s r_{D_1} R^{-1}_{r_{D_1}}. \tag{4.65}
\]

This is similar to the Wiener solution \[48\].

**MMSER-\( \mu \)**

Here, we find \( R_s r_{D_1} \) and \( R_{r_{D_1}} \) for MMSER-\( \mu \). As mentioned earlier,

\[
r_{D_1} = \begin{bmatrix} r^{(K+1-\mu)}_{D_1} \\ \vdots \\ r^{(K)}_{D_1} \end{bmatrix}. \tag{4.66}
\]

Now, the components \( R_{s' r_{D_1}}, \ i \in [1, m] \), of the row vector \( R_{s' r_{D_1}} \) in (4.63) for MMSER-\( \mu \) can be written as

\[
R_{s' r_{D_1}}, \ j \in [K + 1 - \mu, K]
\]

with \( \mu \in [1, K + 1] \). From Table 4.1, we can write

\[
R_{s' r_{D_1}} = E \left[ s r_{D_1} \right] = E \left[ s \left( h_{j,D_1} t_j + u_{D_1} \right)^H \right] = R_{s'r_j} h_{j,D_1}^H = R_{s'r_j} F_j^H h_{j,D_1}^H,
\]

where \( R_{s'r_j} \) is any one row of \( R_{sr_j} \) in (4.24) as

\[
R_{sr_j} = E \left[ sr_j^H \right] = E \left[ \begin{bmatrix} s \\ \vdots \\ s \end{bmatrix} r_j^H \right] = \begin{bmatrix} R_{s'r_j} \\ \vdots \\ R_{s'r_j} \end{bmatrix}.
\]

Now the components of \( R_{r_{D_1}} \) in (4.64) are found for MMSER-\( \mu \), which can be written from Table 4.1 as

\[
R_{r_{D_1}} = E \left[ \left( h_{i,D_1} t_i + u^{(i)}_{D_1} \right) \left( h_{j,D_1} t_j + u^{(j)}_{D_1} \right)^H \right] = h_{i,D_1} R_{t_{D_1}} h_{j,D_1}^H + \sigma_u^2 \delta(i - j). \tag{4.67}
\]
Here, we used $R_{u_D^i,u_D^j} = 0$ as $u_D^j$ is uncorrelated with $u_D^i, \forall i \neq j$ and $R_{t,u_D^j} = 0$ as $u_D^j$ is uncorrelated with $t_i, \forall i, j$ to arrive at (4.67). Now

$$R_{t,t_j} = F_t R_{r,r_j} F_j$$

(4.68)

is given in Appendix-N.

**E-MMSED**

Here, we will find $R_{s'r_D}$ and $R_{r_D}$ for E-MMSED. As mentioned earlier,

$$r_{D_1} = \begin{bmatrix} r_{D_1}^{(K_0)} \\ \vdots \\ r_{D_1}^{(K)} \end{bmatrix}.$$  

(4.69)

Now, the components $R_{s'r_{D_1}}, i \in [1, m]$, of the row vector $R_{s'r_{D_1}}$ in (4.63) for E-MMSED can be written as

$$R_{s'r_{D_1}}, k \in [K_0, K].$$

From (4.53), these components are given as

$$R_{s'r_{D_1}} = E \left[ s \left( \frac{p_0}{K_0} \right)^{\frac{1}{2}} h_{D_1} F h_0 s + \frac{1}{K_0} h_{D_1} F \sum_{k=0}^{K_0-1} u^{(k)} u^{(k+1)} \right]^H \right]$$

$$= \left[ \frac{p_0}{K_0} \right]^{\frac{1}{2}} h_0^H F^H h_{K+1}^H.$$  

(4.70)

It can be seen that $R_{s'r_{D_1}}$ does not depend on $k$ as we have assumed that the channel coefficients do not change for the block of time when the relays transmit $K_1$ times. Now, we will find the components of $R_{r_D}$. These components can be found from (4.53) as

$$R_{r_{D_1}} = E \left( \sqrt{\frac{p_0}{K_0}} h_{D_1} F h_0 s + \frac{1}{K_0} h_{D_1} F \sum_{k=0}^{K_0-1} u^{(k)} u^{(j)} \right)^H$$

$$= \frac{p_0}{K_0} h_{D_1} F h_0 h_{D_1}^H + \frac{\sigma^2}{K_0} h_{D_1} F F^H h_{D_1}^H + \sigma^2 (i - j).$$  

(4.71)

We will use the decoder found in (4.65) in our simulations for both MMSER-$\mu$ and E-MMSED systems.
4.1.6 Simulations and results

Let us select \( s \) from the Gray coded QPSK constellation [50] with unit variance and use MMSE decoders at \( D_1 \), derived in section 4.1.5. In all the plots, SNR =\( 10 \log (1/\sigma_u^2) \) is used on the \( x \)-axis, where \( \sigma_u^2 \) is the variance of the noise added at receivers.

As discussed in Section-3.6, we incorporate the signal-power path loss in the channel variance as shown in (3.140). Unlike in Chapter-3, where we assumed that the channel variance of the link with length \( \rho = 1 \) to be \( \sigma_1^2 = 1 \), here we assume that the channel variance of the link \( \ell_{0,K+1} \) to be \( \sigma_{K+1}^2 = \sigma^2 = 1 \). Hence the channel variance of any link with length \( \rho \) is \( \sigma_\rho^2 = (K + 1)^2/\rho^2 \) from (3.140), as the path loss exponent is taken to be \( \beta = 2 \) for all the simulations in this chapter.

In all the simulations, whenever a need to increase the number of layers \( K \) arises, it is done by inserting them between \( S_1 \) and \( D_1 \) keeping the distance \( d_{0,K+1} \) constant. Therefore, as \( K \) increases, one of the negative impacts is that the noise which is added in layer \( L_i \) gets amplified through all the layers \( L_j, j > i \) and hence the received noise at \( D_1 \) increases for MMSER. This is compensated by enhancing the signal content that reaches \( D_1 \), by increasing \( \mu \).

For \( K = 1 \), Jing and Hassibi [11] derived optimum power allocation by minimizing PEP. They proved that this optimum power allocation also achieves maximum SNR at \( D_1 \). The optimum power allocation they obtained was that the total power is equally divided between \( S_1 \) and relays; i.e., \( p_0 = p_1 = P/2 \). For \( K = 2 \), JHS is extended and called extended JHS (EJHS) in [51] and proved that EJHS achieves maximum SNR at \( D_1 \) when power is equally divided amongst \( S_1, L_1 \) and \( L_2 \); i.e., \( p_0 = p_1 = p_2 = P/3 \). This is also proved in this thesis in Chapter 3. Extending this for any \( K, p_0 = P/(K + 1) \) is used in all the simulations for EJHS.

Summary of results

- **Usefulness of leaked signals, i.e. \( \mu > 1 \)**: Figs. 4.5 and 4.6 show respectively how the performance of MMSER-1 worsens and that of MMSER-2 improves when \( K \) is increased

- **Selection of \( K_0 \) and \( p_0 \) for E-MMSED-KK and L**: Figs. 4.7 and 4.8 show performance of E-MMSED-KK and L as a function of \( K_0 \) and \( p_0 \). This is used to select the best \( K_0 \) and \( p_0 \) for comparison with MMSER-\( \mu \).

- **Comparison: Single layer case**: Fig. 4.9 shows a comparison of BERs of MMSER-1 and MMSER-2 with MMSED, when the relays cooperate
Comparison: Multi-layer case: Figs. 4.11 and 4.12 show that the BER performance of MMSER outperforms that of E-MMSED-KK and approaches that of E-MMSED-L when \( \mu \) is increased from 1 to \( K + 1 \) with \( K = 3 \) and 4 respectively, when the relays do not cooperate.

![Graph showing BER and SNR for varying number of layers](image)

Figure 4.5: Plots of SNR and BER of MMSER-1 for varying number of layers \( (K) \) with total power \( P = 1 \) Watt. As \( K \) is increased, due to noise getting amplified in this AF system, performance worsens unlike MMSER-\( \mu \), \( \mu \in [2, K + 1] \) where it is compensated by leaked signals.

Usefulness of leaked signals, i.e. \( \mu > 1 \)

Fig. 4.5 shows SNR at \( D_1 \) and BER plots of MMSER-1 when \( K \) is varied. It can be observed that as \( K \) increases, the performance of MMSER-1 goes down. To corroborate this, SNR at \( D_1 \) also has been plotted, which decreases as \( K \) is increased. Similarly, Fig. 4.6 shows BER plots of MMSER-2 for varying \( K \). Unlike MMSER-1, the plots of MMSER-2 show that the performance improves if the number of layers is increased. This is because MMSER-2 combines two received signals at relays with \( \mu = 2 \) thereby increasing the signal content at \( D_1 \).

Selection of \( K_0 \) and \( p_0 \) for E-MMSED-KK and L

Fig. 4.7 shows the performance of E-MMSED-KK and E-MMSED-L for various values of the number \( K_0 \) of transmissions of \( S_1 \). As was shown in (4.58) and (4.59) the BER plots also
Figure 4.6: BER plots of MMSER-2 for varying number of layers ($K$) with total power $P = 1$ Watt. Unlike MMSER-1, the BER performance improves when $K$ is increased, as MMSER-2 uses leaked signals.

corroborate the fact that the performance does not vary with $K_0$. Hence $K_0 = 1$ is used in all the simulations of E-MMSED.

Figure 4.7: E-MMSED-KK and E-MMSED-L showing same BER performance for varying $K_0$. Total power used in the simulations is $P = 1$ Watt.
Another parameter that needs to be fixed is the power allocated to \( S_1 \) \( p_0 \), and the relays \( p_r = P - p_0 \). These are found using simulations as shown in Fig. 4.8, where \( K = 4 \), \( N = 2 \), and \( P = 1 \) Watt are used. For E-MMSED-KK, it can be seen that 20% of total power \( p_0 = P/(K + 1) = P/5 \) achieves low BER for \( \text{SNR} \leq 10 \) dB and almost same BER for \( \text{SNR} > 10 \) dB than other power allocations. Similarly \( p_0 = P/2 \) or 50% of total power achieves lowest BER for E-MMSED-L. Hence these values are used for \( p_0 \) in all subsequent simulations for E-MMSED.

![Search for optimum power allocation](image)

Figure 4.8: Search for optimum power allocation for MMSED. Shows that when power is equally distributed to \( S_1 \) and layers, BER performance of E-MMSED-KK is the best and when 50% of power is allocated to \( S_1 \), E-MMSED-L attains best BER performance. Total power used in the simulations is \( P = 1 \) Watt.

**Comparison: Single layer case**

Fig. 4.9 shows plots of BER in a single layer, when the relays cooperate amongst themselves for MMSER-1, MMSER-2, and MMSED-K. For MMSED-K, the plot is generated using the equation (20) derived by Krishna, et al. in [28], which is also repeated in (2.6), in this thesis. It can be seen that both MMSER-1 and MMSED-K have similar BER values when \( N = 1 \), though MMSED-K has forward CSI. MMSER-2 outperforms MMSED-K by around 7 dB, when there is one relay in the layer (actually there is no cooperation as \( N=1 \)) and when \( N = 2 \), the improvement in the performance of MMSED-K is high compared to those of
Figure 4.9: Cooperative relays performance comparison. Performances of MMSER-1 and MMSED-K are the same when \( N = 1 \), though MMSED-K uses global CSI as it is compensated in the decoder at \( D_1 \) for MMSER-1. For \( N = 2 \), the performance of MMSED-K is better than MMSER-1 and MMSER-2 schemes. Total power used in the simulations is \( P = 2 \) Watts.

Now, we will see the theoretical reason as to how the forward CSI advantage of MMSED-K is compensated by the MMSE decoder at \( D_1 \) in MMSER-1 and the better performance of MMSER-2 over MMSED-K for \( N = 1 \) in Fig. 4.9. Let us derive the equations of the received signal at the destination and find the decoded signal from this for both the cases of MMSER-1 and MMSED-K and compare. Consider the precoder of MMSER given in (4.13) which is reproduced below for convenience:

\[
\hat{F}_1 = \left[ \frac{p_1}{N \left( p_0 \| h_{0,1} \|^2 + \sigma_a^2 \right)} \right]^{\frac{1}{2}} i_N h_{0,1}^H \| h_{0,1} \|. \tag{4.13}
\]

When \( N = 1 \), (4.13) becomes

\[
\hat{f}_{11} = \left[ \frac{p_1}{p_0 \| h_{0,11} \|^2 + \sigma_a^2} \right]^{\frac{1}{2}} \frac{h_{0,11}^*}{\| h_{0,11} \|},
\]
where \( f_{11} \) is the amplifying factor which the relay uses before transmission. From Table 4.1, the received signal at D can be written as

\[
\hat{r}_{21}^{(1)} = h_{11,21} \hat{f}_{11} r_{11}^{(0)} + u_{21}^{(1)},
\]

where \( r_{11}^{(0)} = h_{0,1,1,1} \alpha_0 s + u_{11}^{(0)} \). From (4.61), the decoded signal is given by

\[
\hat{s} = \hat{f}_{D1} \hat{r}_{21}^{(1)}
\]

where

\[
\hat{f}_{D1} = R_{s'_{21}} R_{r_{21}}^{-1}.
\]

From (4.63) and (4.64), \( R_{s'_{21}} \) and \( R_{r_{21}} \) are given by

\[
R_{s'_{21}} = E \left[ s \left( h_{11,21} \hat{f}_{11} r_{11}^{(0)} + u_{21}^{(1)} \right)^* \right] = R_{s'_{11}} \alpha_0 h_{0,1,1,1}
\]

and

\[
R_{r_{21}} = E \left[ \left( h_{11,21} \hat{f}_{11} r_{11}^{(0)} + u_{21}^{(1)} \right) \left( h_{11,21} \hat{f}_{11} r_{11}^{(0)} + u_{21}^{(1)} \right)^* \right] = p_1 |h_{11,21}|^2 + \sigma_u^2
\]

respectively. Hence \( \hat{s} \) for MMSE-R is given by

\[
\hat{s}_{MSE} = \hat{f}_{D1} \hat{r}_{21}^{(1)}
\]

\[
= \frac{\alpha_0 h_{0,1,1,1} h_{11,21} \hat{f}_{11}^*}{p_1 |h_{11,21}|^2 + \sigma_u^2} \left[ h_{11,21} \hat{f}_{11} r_{11}^{(0)} + u_{21}^{(1)} \right]
\]

\[
= \frac{p_1 \alpha_0^2 |h_{0,1,1,1}|^2 |h_{11,21}|^2}{(p_1 |h_{11,21}|^2 + \sigma_u^2) (p_0 |h_{0,1,1,1}|^2 + \sigma_u^2)} s + \frac{\alpha_0 h_{0,1,1,1} |h_{11,21}|^2 p_1}{(p_1 |h_{11,21}|^2 + \sigma_u^2) (p_0 |h_{0,1,1,1}|^2 + \sigma_u^2)} u_{11}^{(0)}
\]

\[
+ \frac{\alpha_0 |h_{0,1,1,1}| h_{11,21}^2 p_1^2}{(p_1 |h_{11,21}|^2 + \sigma_u^2) (p_0 |h_{0,1,1,1}|^2 + \sigma_u^2)} u_{21}^{(1)}.
\]

For MMSE-K, from (2.6) and (4.61), \( \hat{s} \) can be derived in a similar manner as

\[
\hat{s}_{MME} = \frac{p_1 \alpha_0^2 |h_{0,1,1,1}|^2 |h_{11,21}|^2}{(p_1 |h_{11,21}|^2 + \sigma_u^2) (p_0 |h_{0,1,1,1}|^2 + \sigma_u^2)} s + \frac{\alpha_0 h_{0,1,1,1} |h_{11,21}|^2 p_1}{(p_1 |h_{11,21}|^2 + \sigma_u^2) (p_0 |h_{0,1,1,1}|^2 + \sigma_u^2)} u_{11}^{(0)}
\]

\[
+ \frac{\alpha_0 |h_{0,1,1,1}| h_{11,21}^2 p_1^2}{(p_1 |h_{11,21}|^2 + \sigma_u^2) (p_0 |h_{0,1,1,1}|^2 + \sigma_u^2)} u_{21}^{(1)}.
\]

The above two equations (4.72) and (4.73) are the same except for the last noise term. Even these terms in both the equations can be observed to contribute same power. Hence, MMSE-1 and MMSE-K have same BER values in Fig. 4.9 for \( N = 1 \).
In other words, MMSED-K has the forward CSI $h_{11,21}$ at the relay and uses it while forwarding the received signal, whereas MMSER-1 incorporates $h_{11,21}$ into the MMSE decoder at D once the signal is received. Over and above, MMSER-2 has the advantage of getting the leaked signal from S directly at D in phase-0. It uses both this signal and that which was available for MMSER-1 to decode $s$. Hence, it outperforms MMSED-K. [We also note that this need not be the case when there are more number of relays ($N > 1$).]

So, it is always advantageous to select MMSER over MMSED, as MMSED mandates forward CSI availability at the relay, when there is only one relay between the source and the destination.

**Comparison: Multi-layer case**

Fig. 4.10 shows two plots of the BER performance of MMSER-3 when the relays do not cooperate. One plot corresponds to the ‘zeroising non diagonal’ method and the other one to ‘scalar optimization’ method described in Section 4.1.1. It can be seen that the ‘zeroising non diagonal’ method performs better than the ‘scalar optimization’ method by approximately 1 dB of SNR throughout the range of BER values shown in Fig. 4.10. The reason for this: The cooperative relays perform better than the noncooperative relays as they use the extra information exchanged amongst themselves. ‘zeroising non diagonal’ method uses the cooperative solution with some modification, whereas the ‘scalar optimization’ method uses diagonal matrix in the derivation of the precoder to get MSE estimate. As explained in Section 4.1.1, ‘zeroising non diagonal’ method projects the cooperative solution onto the convex set defined by the diagonal and the transmit power constraints to obtain its precoder.

In Figs. 4.11 and 4.12 the BER plots of EJHS, MMSER-$\mu$, $\mu = 1$ to 4, E-MMSED-KK and E-MMSED-L when $K = 3$ and 4 respectively are shown. Here, we used $N = 2$ and a total transmitted power of $P = 1$ Watt. In both these figures, if we compare amongst the proposed schemes, we observe that there is an advantage of 6 dB of SNR at BER = $10^{-1}$ for MMSER-2 over MMSER-1. MMSER-3 has an advantage of 2 dB of SNR at BER = $10^{-2}$ when compared with the performance of MMSER-2. We also observe that the proposed schemes, MMSER-$\mu$, $\mu \in [2, 5]$, are better than E-MMSED-KK though they do not use forward CSI. The lack of forward CSI is compensated by increasing the signal content at $D_1$ by using leaked signals at the relays. Though E-MMSED-L has better performance, we see that as $\mu$ increases, the performance of the scheme proposed in this thesis becomes better and goes closer to that of
Figure 4.10: Noncooperative relays performance comparison. BER plots of MMSER-3 when the precoder is obtained using ‘scalar optimization’ and ‘zeroising non diagonal’ are shown. We use $N = 2$ and $K = 3$ in the simulations. Total power used in the simulations is $P = 1$ Watt.

Figure 4.11: BER plots of EJHS, E-MMSED, and MMSER-$\mu$, $\mu \in [1-4]$. Performance of MMSER-$\mu$ is better than E-MMSED-KK when $\mu \geq 2$ and it approaches that of E-MMSED-L when $\mu$ is increased. Total power used in the simulations is $P = 1$ Watt.
Figure 4.12: BER plots of EJHS, E-MMSED, and MMSER-µ, µ ∈ [1−5]. Performance of MMSER-µ is better than E-MMSED when µ ≥ 2 and it approaches that of E-MMSED-L when µ is increased. For BER=10^{-3}, the SNR advantage of E-MMSED-L over MMSER is reduced from 7 dB (for MMSER-2) to 1 dB (for MMSER-5). Total power used in the simulations is P = 1 Watt.

E-MMSED-L. For example, at BER = 10^{-3}, the advantage of E-MMSED-L over MMSER comes down from 2 to 1 dB of SNR, when µ is increased from 4 to 5. This can be observed by comparing both Figs. 4.11 and 4.12.

Fig. 4.13 shows the performance of E-MMSED-L and MMSER-5 for high (β = 4), medium (β = 3), and low (β = 2) path loss conditions. Unlike in the other plots, where we considered the distance between S and D to be d = 1, here d = 5 is assumed, so that the channel variance decreases with the path loss exponent β. Performance of MMSER-5 is better than E-MMSED-L, which uses global CSI, for β = 4. Also, when β = 3, its performance is better than E-MMSED-L for SNR > 9 dB. For β = 2, E-MMSED-L performs better than MMSER-5. Though it has only the backward CSI, MMSER compensates this handicap with the leaked signals and performs better than E-MMSED-L for high path loss conditions. Hence, we can draw a conclusion that worsening path loss affects MMSED scheme than the proposed MMSER scheme.
Figure 4.13: BER plots of E-MMSED-L and MMSER-5 for various values of $\beta$ with $K = 4$ and $d = 5$. For high path loss conditions, MMSER-5 performs better than E-MMSED-L. Total power used in simulations is $P = 1$ Watt.

### 4.1.7 Discussion and observations

From the simulation results, we observe the following:

In an AF relay strategy, we know that the noise gets amplified at every relay layer stage and hence at the destination there is an increase in the noise power compared to a single-hop signal [21]. This is clearly reflected in the performance of MMSER-1 in Fig. 4.5 which shows that the BER increases as the number of relay layers is increased keeping the distance between the source and the destination constant. Whereas, a reverse trend is seen in Fig. 4.6 in the performance of MMSER-2, as it exploits the leaked signals. Hence we can conclude that in an AF strategy, leaked signals are quite useful and they reduce the ill-effect of the noise content at the destination.

We have seen that the performance of E-MMSED-L is better compared to that of E-MMSED-KK and that it out performs E-MMSED-KK by at least 6 dB for all values of SNR. Even MMSER-2 with partial CSI and one leaked signal performs better than E-MMSED-KK which has global CSI, when SNR $> 5$ dB. Hence we can conclude that E-MMSED-KK does not exploit the available global CSI effectively.

To get the best performance in E-MMSED-L, total power is to be divided into two halves
and one half is to be allocated to the source and the remaining to all the relay layers evenly distributed.

When there is a single layer of relays present between the source and the destination, and if there is only one relay present in it, it is good enough for the relay to have just the backward CSI instead of the global CSI. This can be seen in Fig. 4.9 wherein the BER performances of MMSER-1 and MMSED-K are the same. This is due to the availability of the forward CSI of the relay at the destination and it effectively uses it to decode the signal. When the number of relays is more than one, then the signals get mingled up when they reach the destination from the relays, and hence the destination can not decode it effectively.

As discussed in section 4.1.6, the ‘zeroising non diagonal’ method performs better than the standard ‘scalar optimization’ method of finding the MMSER precoder. So, this can be another option to implement MMSER instead of the standard ‘scalar optimization’ in future.

The key ‘take away’ from this chapter: The proposed MMSER-μ scheme, with no forward CSI at the relays, improves its performance as the number of leaked signals μ is increased and its performance approaches that of E-MMSED-L scheme, which uses global CSI.

Now, we will extend the single source-destination MMSER-μ scheme to the case when there are multiple source-destination pairs. That is, we will use $M > 1$ and consider the more general system model shown in Fig. 2.2, in the next section.

### 4.2 Multiple source-destination pairs

The system model used here is the general framework shown in Fig. 2.2. In the single source-destination pair case, we considered the MSE at the $L_k$ layer relays to be the average of the norm of the difference between the signal vector $s \in \mathbb{C}^{N \times 1}$ and the transmitted signal $t_k \in \mathbb{C}^{N \times 1}$. In essence, we could make each of the relays to transmit a signal closer to the source transmitted signal, when we minimize this MSE. When there are $M$ source-destination pairs, we want the relays to transmit a signal that is closer to a linear combination of the $M$ source transmitted signals. Hence we select the objective function to be minimized to obtain the precoders as

$$J_k(F_k) = E \|Cs - F_kr_k\|^2,$$

(4.74)

where $C \in \mathbb{C}^{N \times M}$ and $s = [s_1, \cdots, s_M]^T \in \mathbb{C}^{M \times 1}$. Here, $C$ is an arbitrary complex matrix - we call it as combining matrix - with individual components $c_{ij}$, $i \in [1, N]$, $j \in [1, M]$. Now,
Cs is given by
\[
Cs = \begin{bmatrix}
\sum_{m=1}^{M} c_{1m} s_m \\
\vdots \\
\sum_{m=1}^{M} c_{Nm} s_m
\end{bmatrix}
\]  
and by minimizing the objective function \( J_k \), the relay \( R_{kj} \), \( j \in [1, N] \) transmits a signal closer to \( \sum_{m=1}^{M} c_{jm} s_m \), which is a linear combination of the source transmitted signals \( s_m \), \( m \in [1, M] \). Matrix \( C \) can be selected to optimize a parameter at the source layer or it could optimize a general parameter of interest.

The problem now is to minimize \( J_k \) under the power constraint \( E[t_k^H t_k] \leq p_k \) and find the precoder matrices \( F_k \), \( \forall k \in [1, K] \). The optimum \( F_k \) is given by
\[
\hat{F}_k = \arg \min_{F_k} J_k(F_k)
\]  
subject to the constraint \( E[t_k^H t_k] \leq p_k \). Here, \( J_k(F_k) \) is given in (4.74). The constraint function can be written as \( C_k(F_k) = E[t_k^H t_k] - p_k \leq 0 \).

This is an optimization problem [46] with the optimization variable \( F_k \in \mathbb{C}^{N \times (k-n)N} \), the cost function \( J_k : \mathbb{C}^{N \times (k-n)N} \rightarrow \mathbb{R} \) and the inequality constraint function \( C_k : \mathbb{C}^{N \times (k-n)N} \rightarrow \mathbb{R} \) with \( n = [k - \mu]^+ \). Now the Lagrangian \( \mathcal{L}_k : \mathbb{C}^{N \times (k-n)N} \times \mathbb{R} \rightarrow \mathbb{R} \) is defined as
\[
\mathcal{L}_k(F_k, \lambda_k) \triangleq J_k(F_k) + \lambda_k \left[ \text{Tr} \left( F_k^H F_k R_{r_k} \right) - p_k \right],
\]  
where \( \lambda_k \geq 0 \) is the Lagrange multiplier and \( J_k(F_k) \) is given in (4.74). Expanding (4.74) we get
\[
J_k(F_k) = \text{Tr} \left( C^H C R_s \right) - 2\Re \left[ \text{Tr} \left( F_k^H C R_{sr_k} \right) \right] + \text{Tr} \left( F_k^H F_k R_{r_k} \right)
\]  
\[
= \text{Tr} \left( C^H C \right) - 2\Re \left[ \text{Tr} \left( F_k^H C R_{sr_k} \right) \right] + \text{Tr} \left( F_k^H F_k R_{r_k} \right)
\]  
(4.78)
as \( R_s = E[ss^H] = I_M \). Let us substitute (4.78) into (4.77) to get
\[
\mathcal{L}_k = \text{Tr} \left( C^H C \right) - 2\Re \left[ \text{Tr} \left( C R_{sr_k} F_k^H \right) \right] + \text{Tr} \left( F_k^H F_k R_{r_k} \right) + \lambda_k \left[ \text{Tr} \left( F_k^H F_k R_{r_k} \right) - p_k \right].
\]  
(4.79)

### 4.2.1 Cooperative relays

Here, each of the sub-matrices \( F_{ki} \in \mathbb{C}^{N \times N} \) in \( F_k = [F_{kn} \cdots F_{k,k-1}] \), \( n \leq i \leq k - 1 \), are not diagonal and \( n = [k - \mu]^+ \). Hence the relays can cooperate or exchange information amongst...
themselves or intra-layer communication is possible. The information from other relays can be used by a relay to create its amplification factor and effectively can forward its received signals during transmission.

Differentiating (4.79) w.r.t. $F_k^*$ we get
\[
\nabla F_k^* \mathcal{L}_k = -CR_{sr_k} + F_kR_{rk} + \lambda_k F_kR_{rk}
\]
and equating to zero we obtain
\[
F_k = \frac{CR_{sr_k}}{1 + \lambda_k} R_{rk}^{-1}.
\]  
(4.81)

Now differentiating (4.79) w.r.t. $\lambda_k$ and equating to zero, we get
\[
\frac{\partial \mathcal{L}_k}{\partial \lambda_k} = \text{Tr} \left( F_k^H F_k R_{rk} \right) - p_k
\]
and
\[
p_k = \text{Tr} \left[ R_{sr_k}^H C^H CR_{sr_k} R_{rk}^{-1} \right] \left( 1 + \lambda_k \right)^{-2}
\]
(4.82)
respectively. Substituting $F_k$ from (4.81) into (4.82), we get
\[
p_k = \text{Tr} \left[ \frac{R_{sr_k}^H C^H CR_{sr_k} R_{rk}^{-1}}{1 + \lambda_k} \right].
\]  
(4.83)

From (4.83), we get $1 + \lambda_k$ and can be substituted into (4.81) to get the optimum $F_k$ as
\[
\hat{F}_k = \frac{p_k^\frac{1}{2} CR_{sr_k} R_{rk}^{-1}}{\left[ \text{Tr} \left( R_{sr_k}^H C^H CR_{sr_k} R_{rk}^{-1} \right) \right]^\frac{1}{2}}.
\]  
(4.84)

### 4.2.2 Noncooperative relays

Here, each of the sub-matrices $F_{ki} \in \mathbb{C}^{N \times N}$ in $F_{k} = [F_{kn} \cdots F_{k,k-1}]$, $n \leq i \leq k - 1$, are diagonal and $n = [k - \mu]^+$. Hence the relays do not cooperate or exchange information amongst themselves in each layer, i.e., intra-layer communication is not possible. The relays are independent and send the received signals after multiplying by an optimum amplification factor, which does not get as good a performance as that when the relays cooperate [28].

**Claim 16.** We can obtain the optimum precoder from the vector $\hat{f}_{kl}$, which is given by
\[
\hat{f}_{kl} = \frac{1}{p_k} \sum_{l=1}^{N} \text{Tr} \left( \Upsilon_{kl}^S \Upsilon_{kl}^{-1} \right), \quad l \in [1, N],
\]
\[(4.85)\]

where
\[
c_l = [c_{l1}, \cdots, c_{lM}], \quad f_{kl} = [f_{kn,l} \cdots f_{k,k-1,l}],
\]
\[
\Upsilon_{kl} = \begin{bmatrix}
\gamma_{(n)(n)}^{kl} & \cdots & \gamma_{(n)(k-1)}^{kl} \\
\vdots & \ddots & \vdots \\
\gamma_{(k-1)(n)}^{kl} & \cdots & \gamma_{(k-1)(k-1)}^{kl}
\end{bmatrix}, \quad \text{and} \quad \Upsilon_{kl}^S = \begin{bmatrix}
\gamma_{(n)}^{s_1} & \cdots & \gamma_{(k-1)}^{s_1} \\
\vdots & \ddots & \vdots \\
\gamma_{(n)}^{s_M} & \cdots & \gamma_{(k-1)}^{s_M}
\end{bmatrix}.
\]
Here, $\Gamma_{kj}^{m(i)}$ is the $j$th element of the row vector $R_{sm',(i)} \in \mathbb{C}^{1 \times N}$ and $f_{ki,j}$ is the $j$th diagonal element of $F_{k,i}$.

Proof: See Appendix Q. ■

Notice the similarity of (4.85) with the cooperative precoder in (4.84) where $R_{srk}$ and $R_{rk}$ are analogous to $\Upsilon_{kl}$ and $\Upsilon_{kl}$ respectively, except for the extra summing operator in the denominator in (4.85).

The optimum precoder, $\hat{F}_k$, at $L_k$ is made from the optimum vectors shown in (4.85) by noting that these vectors give the $l$th diagonal elements of all submatrices, $F_{ki}$, $i \in [n, k - 1]$, $n = [k - \mu]^+$, that make up the precoder. The optimum precoder depends on these matrices, $\Upsilon_{kl}$ and $\Upsilon_{kl}$, $l \in [1, N]$ which in turn depend on the correlation matrices $R_{srk}$ and $R_{rk}$ given in (4.24) and (4.25) respectively.

### 4.2.3 Selection of $C$, the combining matrix

As observed earlier, by minimizing the objective function $J_k$, the relay $R_{kj}$, $j \in [1, N]$ transmits a signal closer to $\sum_{m=1}^{M} c_{jm} s_m$, which is a linear combination of the source transmitted signals $s_m$, $m \in [1, M]$. Now $C$ can be selected to optimize a certain parameter.

Let us take the cooperative relays case and substitute $F_k$ in (4.78) with $\hat{F}_k$ from (4.84) to get $J_{min}$ as follows:

$$J_{min}(C) = J_k(\hat{F}_k) = \text{Tr}(C^H C) - 2\text{Re}\left[\text{Tr}(\hat{F}_k^H CR_{srk})\right] + \text{Tr}(\hat{F}_k^H \hat{F}_k R_{rk})$$

$$= \text{Tr}(C^H C) - 2\text{Re}\left[\text{Tr}\left(\frac{p_{k,1}^2 R_{rk}^{-1} R_{srk}^H C C R_{srk}}{\text{Tr}[R_{srk} R_{rk}^{-1} R_{srk}^H C C]} \right)\right] + \text{Tr}\left(\frac{p_{k,1} R_{rk}^{-1} R_{srk}^H C C R_{srk}}{\text{Tr}[R_{srk} R_{rk}^{-1} R_{srk}^H C C]} \right).$$

Now, we can find $J_{min}$ for various values of $C$ and select that value of $C$ which achieves the minimum value of $J_{min}$ to be the optimum $C$, called $\hat{C}$.

A suboptimum alternative is to select $C$ as a random unitary matrix with $C^H C = I_M$. Then (4.86) becomes

$$J_{min} = M - 2\text{Re}\left[\text{Tr}\left(\frac{p_{k,1}^2 R_{rk}^{-1} R_{srk}^H R_{srk}}{[\text{Tr}[R_{srk} R_{rk}^{-1} R_{srk}^H]]^2} \right)\right] + \text{Tr}\left(\frac{p_{k,1} R_{rk}^{-1} R_{srk}^H R_{srk}}{[\text{Tr}[R_{srk} R_{rk}^{-1} R_{srk}^H]]^2} \right),$$

which does not depend on $C$. 111
4.2.4 Decoder at the destination

Similar to the decoder we derived for the single source-destination pair \((M = 1)\) case in Section-4.1.5, we will see the decoder for the case when \(M > 1\).

Here, we have multiple source-destination pairs and each destination \(D_i, \ i \in [1, M]\), receives the vector \(r_{D_i}\) given by

\[
  r_{D_i} = \begin{bmatrix}
  r_{D_i}^{(K-\mu+1)} \\
  \vdots \\
  r_{D_i}^{(K)}
  \end{bmatrix}.
\] (4.88)

This is similar to the vector received by \(D_1\) we have in (4.66), with ‘1’ getting generalized to ‘\(i\)’ and \(i \in [1, M]\). Hence, the decoder is the same as that obtained for MMSER-\(\mu\) for \(M = 1\) case in Section-4.1.5 and we can use the solution shown in (4.65).

Before we see the simulations and results in the next section, we would like to point out that MMSED systems with \(M > 1\) can be enhanced to work in a multi-layer relays case \((K > 1)\), as was done for single source-destination case \((M = 1)\). But, the focus of this thesis is to bring out the efficacy of using leaked signals in the MMSER system and that MMSER works when there are multiple leaked signals \((\mu > 1)\), multiple relay layers\((K > 1)\), and multiple source-destination pairs \((M > 1)\). The efficacy has already been shown using simulations that the performance of MMSER approaches that of the MMSED system (in Section-4.1.6), when \(M = 1\) and that MMSER works when \(K > 1\) and \(M > 1\) (in Section-4.1). Hence, enhancing the MMSED system with \(M > 1\) is not taken up for comparison and considered out of scope of this thesis.

4.2.5 Simulations and results

In the simulations, we use all the parameters to be the same as that was used in MMSER-\(\mu\) scheme in single source-destination pair case in the previous section. The additional parameters that are to be selected are the number of source-destination pairs \(M\) and the \(C\) matrix.

Here, we have selected \(c_{ij}\) to be unity \(\forall i, j\), which means that we are forcing each of the relays to transmit a sum of the transmitted signals of the \(M\) sources. The effective BER, which is plotted, is obtained by averaging all the individual BERs of the destinations, \(D_1\) to \(D_M\).

Fig. 4.14 shows BER plots of MMSER for a system with 4 layers of relays while the relays do not cooperate. A maximum value of \(\mu\) that one can go up to is \(K + 1\) and hence we
Figure 4.14: BER plots of MMSER-5 for multiple source-destination pairs. Performance of MMSER-5 becomes better when the number of source-destination pairs reduces. Total power used in the simulations is $P = 1$ Watt and the number of relays is $N = 4$.

We have selected MMSER-5 for this system. With fixed number of relays $N = 4$, we varied the number of source-destination pairs (i.e., $M$). It can be seen that the performance of MMSER-5 becomes better, when the number of source-destination pairs $M$ reduces, with fixed number of relays $N$. Similar trend can be seen in the MMSED system with one layer of relays in the work by Krishna et al. [28] as well, shown in Fig. 3, where increasing $N$ improves the performance with fixed $M$.

Fig. 4.15 shows the BER plots of MMSER-4 and MMSER-5 for the case when the relays do not cooperate. Here, we used $N = 3$, i.e., the number of relays in each of the relay layers is three, and the number of source-destination pairs used is $M = 4$. We find that, MMSER-5 has an advantage of 3 dB at BER = $10^{-1}$ over MMSER-4.

The BER plot of MMSED-K shown in Fig. 4.16 is obtained from Fig. 3 of [28] for comparison with MMSER when the relays cooperate. MMSER-2 plot is generated with the same parameters, $K = 1$ and $N = 4$, with number of source-destination pairs $M = 2$. It can be seen that the performance of MMSER-2 is better for SNR < 7.5 dB though it has only partial CSI unlike MMSED-K, which has global CSI. It shows that for low SNR values, the leaked signals used by MMSER-2 are helpful to reduce BER to values less than that of MMSED-K.
Figure 4.15: BER plots of MMSER-4 and MMSER-5 for multiple source-destination pairs. Parameters used are $K = 4$, $M = 2$, and $N = 3$ with a total power used in the simulations is $P = 1$ Watt.

Also when $K$ is increased, there is scope to increase the number of leaked signals $\mu$ and hence MMSER is expected to perform still better as was shown in single source-destination pair case.

Figure 4.16: BER plots of MMSER-2 and E-MMSED-K for multiple source-destination pairs. Here, $K = 1$, $M = 2$, and $N = 4$ with a total power used in the simulations is $P = 2$ Watts.
4.2.6 Discussion and observations

First, we showed that MMSER-5 works in a four layer system (i.e. $K = 4$) with five source-destination pairs. The BER plots show that the performance of this system degrades as the number of source-destination pairs $M$ is increased, keeping the number of relays constant. This is because, keeping the number of independent paths from sources to destinations constant, we squeeze to accommodate more and more of source-destination pairs. Hence, there is a performance degradation as $M$ is increased. This trend of degradation can also be observed in the existing work by Krishna et al. [28].

For multiple source-destination pairs case also, we have shown that as $\mu$ increases, MMSER-$\mu$ scheme improves its performance.

Finally, in the multiple source-destination pairs case, as there are no existing MMSED systems that have solution to precoders when there are multiple relay layers present ($K > 1$), we compare the performance of MMSER-2 with the existing MMSED-K system [28] for $K = 1$. Even though MMSER scheme does not possess forward CSI, it outperforms MMSED for lower values of SNR as it exploits the leaked signal from the source at the destination. The maximum number of leaked signals and the direct signal that can be accommodated in MMSER-$\mu$ scheme is $K$ and 1 respectively. Hence, when the number of relay layers $K$ is increased, there is a scope for improvement of performance, as the number of leaked signals can be increased.

Summary

To summarize this Chapter on Optimum MMSE schemes:

- We derived MMSE precoder matrices for a multi-layer relay network when single source-destination pair is present
- We enhanced the existing MMSED schemes with single source-destination pair, to make them work in a multi-layer relay network
- We showed that by increasing the number of leaked signals, we can make the performance of the MMSER scheme with partial CSI approach that of the MMSED scheme, which uses global CSI effectively
- We extended the MMSER scheme to a case with multiple source-destination pairs
We showed using simulations that MMSER performs better than MMSED for SNR < 7.5 dB, when multiple source-destination pairs and single layer of relays are present.

In the next Chapter, we will summarize the work done in this thesis and conclude highlighting the open problems that can be addressed in future.
Chapter 5

Summary and Conclusion

This thesis is focused on the design of precoders that are used by relays and this chapter summarizes the content and brings out the need for each of the chapters we saw so far. It also highlights the open problems for future attempts and draws conclusion.

In this thesis, we assume that the source transmitted signal is conveyed sequentially by the layers of relays to the concerned destination. Unlike most of the work in literature, the proposed schemes take into account the leaked signals which travel through weak links from distant layers to the relay for constructing the signal to be forwarded. This is possible in our system model, as we assume that all the forward layers present between the transmitting layer and the destination receive signals during every transmission phase. This assumption is not far fetched as the signals are attenuated according to the path loss rule in the standard literature. All the received signals at any relay layer may not be of good strength, but are put to good use to improve the performance of the schemes proposed in this thesis.

The processing complexity at a relay in the MMSER-$\mu$ scheme is not going to be very high compared to that in MMSED systems. This is because, each of the relays is going to simply add the product of the received signals, which depends on the number of leaked signals, and the corresponding optimum weight.

5.1 Summary

In Chapter-1, we discussed the challenges faced by a communication system and how they can be mitigated by employing MIMO structures. After a brief introduction of the need for cooperative communication, we discussed the multi-layer relay network architecture and emphasized
the focus of this thesis, i.e., design of relay precoders which use AF strategy. It motivates the
topic and defines the scope of the thesis.

Chapter-2 is an important part, where we develop the system model that is used in this
thesis. The later chapters use this or special cases of this system model while giving an account
of various proposed schemes. Existing works in literature also use special cases of this system
model and hence what we proposed is a more general framework.

This model accommodates the leaked signals which is a major deviation from the existing
system models. Also, it has multiple layers of relays between a source and a destination layer
with multiple source-destination pairs.

This chapter also highlights the limitations and drawbacks in the existing literature which
are given briefly as: (1) Requirement of global CSI: All the optimum schemes minimize MSE
at the destination and hence the relay precoder matrix thus obtained depends on the channel
coefficients from the relay to the destination. Therefore, all the relays require backward as well
as forward CSI, which may not be feasible in many situations. (2) No closed-form solution to
precoders in a multi-layer network: As the precoder is obtained at the destination, it jointly
optimizes all the precoders of multiple layers leading to precoders as functions of each other.
This is not solvable and hence some authors have resorted to an iterative solution for $K = 2$.
(3) Obtaining precoder solution when number of source-destination pairs $M > 2$ is difficult:
Available multiple source-destination pair solution for a single layer of relays, requires solving
$2M$th order polynomial, where $M$ is the number of source-destination pairs. This is difficult
as $M$ increases. (4) Not considering leaked signals: Though there is a passing mention of the
source transmitted signal reaching destination in the ad hoc scheme considered in the existing
literature [11], there is no rigorous treatment. As far as we know, the optimum schemes in
literature which minimize MSE, do not use leaked signals.

In Chapter-3, we propose three ad hoc schemes. These are RMCS, RSCS, and RMCKCS
and they can be employed at the relays while combining the signals, when $K = 2$, $M = 1$,
and any $N$. Of these, RMCS and RSCS do not use CSI, while RMCKCS uses local backward
CSI. In each phase, a vector of symbols is transmitted by each of the transmitting nodes, in $T$
symbol duration. Though these simple schemes are not optimum, they give an insight into the
problem, showing how the optimum power allocations and BER performance varies when the
path loss exponent is varied. To compare these schemes with JHS, an existing scheme (uses a
system model with one layer of relays), JHS is enhanced to two different forms to work with
two relay layers ($K = 2$).

Showed using simulations that we can use random real orthogonal instead of random complex unitary precoder matrices employed in JHS at the relays and get the same BER performance. For all the schemes, ML decoders are obtained and used at the destination. Though these schemes are not optimum, power allocated to the transmitters have been found by maximizing the SNR at the destination for all the schemes. It has been shown that RMCS, a scheme that does not use any information for combining its received signals, performs similar to that of RSCS, which uses SNR information to combine the received signals when the path loss exponent $\beta = 2$. The performances of the proposed schemes are compared using simulations and shown that even in the worst case scenario of the path loss exponent taking a value $\beta = 4$, the proposed schemes perform better than the enhanced existing schemes.

In Chapter-4, we develop an optimum scheme which is based on MMSE. Keeping the four limitations present in the existing literature discussed above in mind, we propose the concept of MMSE at relays and address all of them. This concept breaks the complex problem to find precoders at the destination in an MMSED system to finding them at every relay layer stage thereby able to extend MMSE concept to multi-layer relay network. We also emphasize the need of the leaked signals that reach a particular layer from distant transmitters, which could be finally exploited to get better BER performance. The important limitation of the existing systems to have global CSI is addressed by the scheme, as the precoders obtained do not depend on the forward CSI, which is unavailable at the relays in most of the situations. We also extend the proposed scheme, which is one of its kind amongst the MMSE systems that work in a multi-layer network, to include multiple source-destination pairs.

The proposed strategy called MMSER-$\mu$ scheme has solution for the case when the relays (1) do not cooperate and (2) they cooperate.

MMSER-$\mu$ scheme has the handicap of not having forward CSI. This is compensated by the relays by constructing the transmission signal using leaked signals. This is shown using simulations that the BER performance of the system approaches that of the existing scheme, which uses global CSI, with increase in the number of leaked signals.

The MMSER scheme is then extended to work when multiple source-destination pairs are present in the system. It is proved that by using an arbitrary matrix in the objective function, the relays are made to transmit a signal that is closer to a linear combination of the source signals transmitted by the sources, when we minimize MSE at the relays. Finally, at the destination, the
signals are extracted by the corresponding destinations using an MMSE decoder. It is shown using simulations that when $\mu$ or the number of leaked signals is increased, MMSER-$\mu$ performs better with four layers of relays present between two source-destination pairs. We showed that the performance of MMSER-$\mu$ scheme improves with decreasing the number of source-destination pairs for a fixed number of relays. It is also shown that for the case when single layer with four relays is present, MMSER-2 scheme performs better than the existing system by Krishna et al. [28] for SNRs $< 7.5$ dB, when there are two source-destination pairs present.

To summarize, our contributions include the following:

- For the multi-layer relay network shown in Fig. 2.2, we proposed a novel MMSER relaying strategy, which does not require forward CSI in transmitting nodes.
- We obtained closed form solutions for the optimum relay precoders in both cooperative relays and noncooperative relays cases.
- We enhanced the MMSED strategies (though these enhancements may not be optimal) proposed in [27, 28], and [30] to work in this multi-layer network for meaningful comparison with MMSER.
- We showed using simulations that combining more number of leaked signals improves the BER performance of MMSER, which approaches that of E-MMSED scheme that uses global CSI.
- We extended the MMSER scheme to operate when multiple source-destination pairs are present. Also using simulations, we show that the MMSER scheme performs better than MMSED-K scheme for SNRs $< 7.5$ dB.

5.2 Open problems

Following are some of the open problems arising out from this thesis:

The proposed ad hoc schemes work in a system model where there are only two layers of relays between a single source-destination pair. This can be extended to work with multiple leaked signals in the general system model which has multiple layers of relays and multiple source-destination pairs - each having multiple antennas.
Presently, the MMSER scheme is derived for any number of relay layers, leaked signals, and relays in each layer with multiple source-destination pairs. All the radio nodes are considered to have single antenna each. We can extend the scheme to work with multiple antenna radio nodes.

MMSED schemes can be enhanced to work in a multi-layer relay system when there are multiple source-destination pairs. This can also be used as a bench mark to compare the performance of MMSER which works in a multi-layer system, when there are multiple source-destination pairs.

One of the benefits that MMSER brings in is, no requirement of global CSI. But if it is available, is there a way to use it, is one of the questions that comes to our mind. By considering at every relay layer stage, the MSE between the signal received at that layer and the source transmitted signal vector would enable us to find a precoder of the previous layer. This precoder would depend on the forward CSI and hence global CSI can be used. This can perform better than E-MMSED-L scheme which uses global CSI.

In the ad hoc schemes, all the relays transmit vectors of size $T$, instead of scalars. They make DSTC while using multiple channel-uses within the coherence interval. In the MMSER scheme, the relays transmit scalars and hence can be extended to transmit vectors of size $T > 1$ and make DSTC.

Two-way communication between the source and the destination [52,53] is another application where the proposed scheme can be extended to. Here, all the relays receive at the same time from both transmitters - that act as the source as well as the destination. While Roemer and Haardt [52] maximize the weighted sum of the Frobenius norms of the effective channels to find the precoder matrix, Jorswieck et al. [53] give a good account of the previous work in two-way communication. Two way communication could be taken up to incorporate with MMSER strategy. Also, instead of MMSE strategy, maximizing the weighted sum of the Frobenius norms is another modification to include in the generalized framework of the system model proposed in this thesis.

In the multiple source-destination pairs case, we have designed precoders that depend on an arbitrary matrix $C$. In the simulations, we have selected it to be a rank one matrix with all the elements having identical values. Selection of this matrix can be done optimally to obtain better performance.
5.3 Conclusion

One of the applications where this research work can be applied is WSN, which consists of spatially distributed autonomous sensors which can detect an environmental parameter and transmit it to a destination [34]. Another system where this can be put to good use is the multi-hop cellular network [26].

The ad hoc schemes proposed in this thesis can be used when there is no CSI available or only local backward CSI is available.

The MMSER scheme proposed, looks promising for the future as in many situations it may not be feasible for the relays to get global CSI. Also when there is a large number of relays spread in a vast area, forming multiple layers and then forwarding the signal optimally is possible using the MMSER technique, as there are closed form solutions available for the precoders for any number of relay layers. As the number $M$ of source-destination pairs increases, it becomes cumbersome to use the existing MMSED techniques, as it needs to solve a $2M$th order polynomial and hence the MMSER technique proposed in this thesis is a better alternative with its closed-form solution of the precoders.
Appendix A

Proof of Claim 1: Cardinality of Link Sets

Total number of link sets $L_{ij}$, $\omega = (K + 1)(K + 2)/2$:

This can be obtained by noting that $i \in [0, K]$ and $j \in [1, K + 1]$ and that $i < j$. When $i$ takes value 0, $j$ can take any $K + 1$ values from the set $\{1, \ldots, K + 1\}$, when $i$ takes value 1, $j$ can take any $K$ values from $\{2, \ldots, K + 1\}$, and so on till when $i$ takes the value $K$, $j$ takes the only value $K + 1$. Hence the total number of link sets is $(K + 1) + K + (K - 1) + \cdots + 2 + 1 = (K + 1)(K + 2)/2$.

Cardinality of $L_\mu$:

We know that $\mu$ is always less than or equal to $K + 1$ as the maximum number of leaked signals which can be considered is $K + 1$.

For any general value of $\mu \leq K + 1$ the number of link sets from $S$ is $\mu$ as we need to consider all the link sets $L_{01}, L_{02}, \ldots, L_{0\mu}$. Similarly from layer $L_1$ these link sets are $L_{12}, L_{13}, \ldots, L_{1,\mu+1}$. This goes on till layer $L_{K+1-\mu}$ when there are $\mu$ link sets, viz.

$L_{K+1-\mu, K-\mu+2}, \ldots, L_{K+1-\mu, K+1}$. For layer $L_{K+2-\mu}$ there are $\mu - 1$ link sets, viz. $L_{K+2-\mu, K+3}, \ldots, L_{K+2-\mu, K+1}$ and so on it keeps reducing when we get the only link set $L_{K+1, K+1}$ when $L_K$ is reached. If we sum up all the number of link sets sequentially, it is $\mu + \mu + \cdots (K + 2 - \mu)$ times $+(\mu - 1) + (\mu - 2) + \cdots + 1$. Hence the cardinality is

$$|L_\mu| = [\mu \times (K + 2 - \mu)] + 1 + 2 + \cdots + \mu - 1$$

$$= \mu K + \frac{3\mu}{2} - \frac{\mu^2}{2}. \quad (A.1)$$

It can be seen that (A.1) is the same as (2.1) and hence the claim follows. ■
Appendix B

Proof of Claim 2: Scaling Factor of RMCS

To get $\alpha_2$, we calculate the transmitted power of $R_{2j}$ as

$$\frac{p_{2j}T}{N} = E \left[ t_{2j}^H t_{2j} \right], \quad (B.1)$$

where

$$t_{2j} = \alpha_2 F_{2j} r_{2j} = \alpha_2 \frac{1}{\sqrt{2}} \begin{bmatrix} F_{2j,0} & F_{2j,1} \end{bmatrix} \begin{bmatrix} r_{2j}^{(0)} \\ r_{2j}^{(1)} \end{bmatrix} = \frac{\alpha_2}{\sqrt{2}} \left[ F_{2j,0} t_{2j}^{(0)} + F_{2j,1} t_{2j}^{(1)} \right]$$

$$= \frac{\alpha_2}{\sqrt{2}} F_{2j,0} \left( \alpha_0 s h_{0,1,2,j} + u_{2j}^{(0)} \right)$$

$$+ \frac{\alpha_2}{\sqrt{2}} F_{2j,1} \left( \alpha_0 \alpha_1 \sum_{i=1}^{N} F_{1i} s h_{0,1,1,i} h_{1,i,2,j} + \alpha_1 \sum_{i=1}^{N} F_{1i} u_{2j}^{(0)} h_{1,i,2,j} + u_{2j}^{(1)} \right)$$

$$= \frac{\alpha_0 \alpha_2}{\sqrt{2}} h_{0,1,2,j} F_{2j,0} s + \frac{\alpha_2}{\sqrt{2}} F_{2j,0} u_{2j}^{(0)} + \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} h_{0,1,1,i} h_{1,i,2,j} F_{2j,1} F_{1i} s$$

$$+ \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} h_{1,i,2,j} F_{2j,1} F_{1i} u_{2j}^{(0)} + \frac{\alpha_2}{\sqrt{2}} F_{2j,1} u_{2j}^{(1)} \quad \text{(B.2)}$$

Hence (B.1) becomes

$$\frac{p_{2j}T}{N} = E \left[ \frac{\alpha_0 \alpha_2}{\sqrt{2}} h_{0,1,2,j} s^H F_{2j,0}^H t_{2j} + \frac{\alpha_2}{\sqrt{2}} u_{2j}^{(0)H} F_{2j,0}^H t_{2j} + \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} h_{0,1,1,i}^* h_{1,i,2,j}^* s^H F_{1i}^H F_{2j,1} t_{2j} \right]$$

$$+ E \left[ \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} h_{1,i,2,j}^* u_{2j}^{(0)H} F_{2j,1}^H t_{2j} + \frac{\alpha_2}{\sqrt{2}} u_{2j}^{(1)H} F_{2j,1}^H t_{2j} \right]$$

$$= \frac{\alpha_0 \alpha_2}{2} E \left[ \left| h_{0,1,2,j} \right|^2 \right] + \frac{\alpha_2}{2} T + \frac{\alpha_0 \alpha_1 \alpha_2}{2} \sum_{i=1}^{N} E \left[ \left| h_{0,1,1,i} \right|^2 \left| h_{1,i,2,j} \right|^2 \right]$$

$$+ \frac{\alpha_1 \alpha_2}{2} \sum_{i=1}^{N} E \left[ \left| h_{1,i,2,j} \right|^2 T \right] + \frac{\alpha_2}{2} T$$

124
\[
\begin{align*}
\frac{\alpha_0^2 \alpha_1^2 \sigma_2^2}{2} + \frac{\alpha_0^2}{2} T + \frac{\alpha_0^2 \alpha_1^2}{2} N + \frac{\alpha_1^2 \sigma_2^2}{2} N T + \frac{\alpha_0^2}{2} T \\
= \alpha_2 \left[ \frac{\alpha_0^2 \sigma_2^2}{2} + T + \frac{\alpha_0^2 \alpha_1^2}{2} N + \frac{\alpha_1^2}{2} NT \right] \\
\Rightarrow \alpha_2 = \left[ \frac{2p_2 T}{N \left[ \alpha_0^2 \sigma_2^2 + 2T + N \alpha_0^2 \alpha_1^2 + \alpha_1^2 NT \right]} \right]^{\frac{1}{2}} \\
= \left[ \frac{2p_2}{N \left( p_0 \sigma_2^2 + 2 + p_1 \right)} \right]^{\frac{1}{2}} ,
\end{align*}
\]
\text{(B.3)}

which is the same as (3.24). To arrive at (B.3), we used the following:

- Channel coefficients are ZMCG and uncorrelated and hence \( E[h_{01,ij}^* h_{01,ik}] \neq 0 \) and \( E[h_{1j,2i}^* h_{1k,2i}] \neq 0 \) only if \( j = k \)
- \( E[u_{ij}^{(k)H} u_{ij}^{(k)}] = T \) and \( F_{1j}^H F_{1j} = I_T \)
- Signal, noise, and channel coefficients are uncorrelated
Appendix C

Proof of Claim 3: Receive SNR of RMCS

From (3.4) and (3.5), we get

\[ P_s^{(0)} = E \left[ (\alpha_0 s h_{0,1,3,1})^H (\alpha_0 s h_{0,1,3,1}) \right] \]

\[ = E \left[ \alpha_0^2 s^H s h_{0,1,3,1} h_{0,1,3,1}^* \right] = E \left[ \alpha_0^2 |h_{0,1,3,1}|^2 \right] = \alpha_0^2 \sigma_3^2 \]  \hspace{1cm} (C.1)

and

\[ P_n^{(0)} = E \left[ u_{31}^{(0)H} u_{31}^{(0)} \right] = T \]  \hspace{1cm} (C.2)

respectively. Also from (3.17) and (3.18), we get

\[ P_s^{(1)} = E \left[ \left( \sum_{i=1}^{N} \alpha_1 F_{1i} s h_{0,1,1,i} h_{1,i,3,1} \right)^H \left( \sum_{i=1}^{N} \alpha_1 F_{1i} s h_{0,1,1,i} h_{1,i,3,1} \right) \right] \]

\[ = E \left[ \alpha_1^2 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{1,j,3,1}^* h_{0,1,1,j} s^H F_{1j}^H F_{1i} s h_{0,1,1,i} h_{1,i,3,1} \right] \]  \hspace{1cm} (C.3)

and

\[ P_n^{(1)} = E \left[ \left( \sum_{i=1}^{N} \alpha_1 F_{1i} u_{1i}^{(0)} h_{1,i,3,1} \right)^H \left( \sum_{i=1}^{N} \alpha_1 F_{1i} u_{1i}^{(0)} h_{1,i,3,1} \right) \right] \]

\[ = E \left[ \alpha_1^2 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{1,j,3,1}^* u_{1j}^{(0)H} F_{1j}^H F_{1i} u_{1i}^{(0)} h_{1,i,3,1} \right] + E \left[ u_{31}^{(2)H} u_{31}^{(2)} \right] \]  \hspace{1cm} (C.4)

respectively. Now the channel coefficients are i.i.d. ZMCG and hence, unless \( i = j \), the expected values will be zero. So (C.3) and (C.4) simplify to

\[ P_s^{(1)} = \alpha_0^2 \sigma_1^2 \sum_{j=1}^{N} E \left[ |h_{1,j,3,1}|^2 \right] E \left[ |h_{0,1,1,j}|^2 \right] E \left[ s^H F_{1j}^H F_{1i} s \right] = \alpha_0^2 \sigma_1^2 N \sigma_2^2 \]  \hspace{1cm} (C.5)
and
\[ P_n^{(1)} = \alpha_2^2 \sum_{j=1}^{N} \left[ |h_{1,j,3,1}|^2 \right] E \left[ u_{1j}^H F_{ij}^H u_{1j} \right] + E \left[ u_{31}^{(2)H} u_{31}^{(2)} \right] = \alpha_1^2 T N \sigma_2^2 + T \] (C.6)

respectively. Similarly, \( P_s^{(2)} \) and \( P_n^{(2)} \) from (3.26) and (3.27) are
\[
P_s^{(2)} = E \left( \frac{\alpha_0 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} F_{2i,0}^H h_{0,1,i,2,j} h_{2,i,3,1} + \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{k=1}^{N} F_{2i,1}^H h_{0,1,k,1,k} h_{1,k,2,i} h_{2,i,3,1} \right)^H
\]
\[
= \frac{1}{2} \left[ \alpha_0^2 \alpha_2^2 N \sigma_2^2 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \right] \] (C.7)

and
\[
P_n^{(2)} = E \left( \frac{\alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \left[ F_{2i,0}^H u_{2i}^{(0)} + F_{2i,1}^H u_{2i}^{(1)} \right] h_{2,i,3,1} + \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{k=1}^{N} h_{1,k,2,i} h_{2,i,3,1} \right)^H
\]
\[
= E \left( \frac{\alpha_2^2}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ u_{2i}^{(0)} h_{2i,0} + u_{2i}^{(1)} h_{2i,1} \right] \left[ F_{2j,0}^H u_{2j}^{(0)} + F_{2j,1}^H u_{2j}^{(1)} \right] h_{2,i,3,1} h_{2,j,3,1} \right)^H
\]
\[
= E \left[ u_{31}^{(2)H} u_{31}^{(2)} \right] + E \left( \frac{\alpha_1 \alpha_2}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{m=1}^{N} \sum_{l=1}^{N} h_{1,k,2,i} h_{1,k,2,j} h_{2,i,3,1} h_{2,j,3,1} \right)^H
\]
\[
= \alpha_2^2 T N + \frac{\alpha_0^2 \alpha_2^2 T N^2}{2} + T \] (C.8)

respectively. Hence, the receive SNR from (C.1), (C.2), (C.5), (C.6), (C.7), and (C.8) for RMCS-3 is
\[
\text{snr}_{\text{RMCS-3}} = \frac{P_s}{P_n} = \frac{P_s^{(0)}}{P_n^{(0)} + P_n^{(1)} + P_n^{(2)}}
\]
\[
= \frac{\alpha_0^2 \alpha_2^2 + \alpha_0^2 \alpha_2^2 N \sigma_2^2 + \frac{1}{2} \left[ \alpha_0^2 \alpha_2^2 N \sigma_2^2 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \right]}{T + \alpha_2^2 T N \sigma_2^2 + T + \alpha_2^2 T N + \frac{\alpha_0^2 \alpha_2^2 T N^2}{2} + T}
\]
127
\[
\text{snr}_{\text{RMCS-3}} = \frac{P_s}{P_n} = \frac{P_s^{(1)} + P_s^{(2)}}{P_n^{(1)} + P_n^{(2)}} \]

\[
= \frac{\alpha_0^2 \alpha_1^2 N \sigma_0^2 + \frac{1}{2} \left[ \alpha_0^2 \alpha_2^2 N \sigma_0^2 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \right]}{2 \alpha_1^2 T N \sigma_0^2 + 2 \alpha_1^2 T N + \alpha_1^2 \alpha_2^2 T N^2} \]

\[
= \frac{\alpha_0^2 \alpha_1^2 N \sigma_0^2 + \frac{1}{2} \left[ \alpha_0^2 \alpha_2^2 N \sigma_0^2 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \right]}{2 \alpha_1^2 T N \sigma_0^2 + 2 \alpha_1^2 T N + \alpha_1^2 \alpha_2^2 T N^2}. \tag{C.10}
\]

and for RMCS-2, it is given by

\[
\text{snr}_{\text{RMCS-2}} = \frac{P_s}{P_n} = \frac{P_s^{(1)} + P_s^{(2)}}{P_n^{(1)} + P_n^{(2)}} \]

\[
= \frac{\alpha_0^2 \alpha_1^2 N \sigma_0^2 + \frac{1}{2} \left[ \alpha_0^2 \alpha_2^2 N \sigma_0^2 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \right]}{2 \alpha_1^2 T N \sigma_0^2 + 2 \alpha_1^2 T N + \alpha_1^2 \alpha_2^2 T N^2} \]

\[
= \frac{\alpha_0^2 \alpha_1^2 N \sigma_0^2 + \frac{1}{2} \left[ \alpha_0^2 \alpha_2^2 N \sigma_0^2 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \right]}{4 T + 2 \alpha_1^2 T N \sigma_0^2 + 2 \alpha_1^2 T N + \alpha_1^2 \alpha_2^2 T N^2}. \tag{C.11}
\]

Simplifying (C.10) and (C.12) by substituting the values of \(\alpha_0\), \(\alpha_1\), and \(\alpha_2\) from Table 3.1, we get

\[
\text{snr}_{\text{RMCS-3}} = \frac{p_0 (2 \sigma_2^2 p_1^2 + 4 \sigma_2^2 p_1 + 2 \sigma_2^2 p_1 + 2 \sigma_2^2 p_1 + 4 \sigma_2^2 + p_2 p_1)}{6 p_1 + 2 p_2 + p_1 p_2 + p_0 (2 p_1 \sigma_2^4 + 6 \sigma_2^2 + 6 p_1 + 2 p_2 + 12)} + 4 \sigma_2^2 p_1 + 6 \sigma_2^2 p_0 + 2 \sigma_2^2 \ast p_1^2 + 12 \tag{C.13}
\]

and

\[
\text{snr}_{\text{RMCS-2}} = \frac{(2 p_1 \sigma_2^4 + p_2 \sigma_2^2) p_0^2 + (2 \sigma_2^2 p_1^2 + 4 \sigma_2^2 p_1 + p_2 \sigma_2^2 + p_2 p_1) p_0}{4 p_1 + 2 p_2 + p_1 p_2 + p_0 (2 p_1 \sigma_2^4 + 4 \sigma_2^2 + 4 p_1 + 2 p_2 + 8) + 4 \sigma_2^2 p_1 + 4 \sigma_2^2 p_0^2 + 2 \sigma_2^2 p_1^2 + 8} \tag{C.14}
\]

respectively. So we have proved that (C.13) and (C.14) are the same as (3.50) and (3.51).
Appendix D

Proof of Claim 4: Receive SNR of RSCS

The receive SNR of RSCS-3 given in (3.70) is repeated here for convenience as

\[ \text{snr}_{\text{RSCS-3}} = \frac{P_s}{P_n} = \frac{P_s^{(0)} + P_s^{(1)} + P_s^{(2)}}{P_n^{(0)} + P_n^{(1)} + P_n^{(2)}}. \] (D.1)

Here, \( P_s^{(0)}, P_n^{(0)}, P_s^{(1)}, \) and \( P_n^{(1)} \) are the same as that for RMCS. These are given in (C.1), (C.2), (C.5), and (C.6) respectively. Now, \( P_s^{(2)} \) and \( P_n^{(2)} \) are to be found. From (3.61),

\[ P_s^{(2)} = E \left[ m_u H m_u \right] \]

\[ = E \left[ \alpha_0 \alpha_2 \gamma_0 \sum_{j=1}^{N} h_{0,1,2,j} h_{2,j,3,1} F_{2j} s + \alpha_0 \alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{0,1,i,i} h_{1,i,2,j} h_{2,j,3,1} F_{2j} F_{1i} s \right]^H \]

\[ = \alpha_0^2 \alpha_2^2 \gamma_0^2 \sum_{j=1}^{N} E \left[ |h_{0,1,2,j}|^2 |h_{2,j,3,1}|^2 \right] + \alpha_0^2 \alpha_1^2 \alpha_2^2 \gamma_1^2 \sum_{j=1}^{N} \sum_{i=1}^{N} E \left[ |h_{0,1,i,i}|^2 |h_{1,i,2,j}|^2 |h_{2,j,3,1}|^2 \right] \]

\[ = \alpha_0^2 \alpha_2^2 \gamma_0^2 N \sigma_2^2 + \alpha_0^2 \alpha_1^2 \alpha_2^2 \gamma_1^2 N^2. \] (D.2)

From \( u_w \) given in (3.67), \( P_n^{(2)} \) is found as

\[ P_n^{(2)} = E \left[ u_w^H u_w \right] \]

\[ = E \left[ \alpha_2 \gamma_0 \sum_{j=1}^{N} h_{2,j,3,1} u_w^H F_{2j} u_{2j}^{(0)} \right] + E \left[ \alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} h_{1,i,2,j} h_{2,j,3,1} u_w^H F_{2j} F_{1i} u_{1i}^{(0)} \right] \]

\[ + E \left[ \alpha_2 \gamma_1 \sum_{j=1}^{N} h_{2,j,3,1} u_w^H F_{2j} u_{2j}^{(1)} + u_w^H u_{2j}^{(2)} \right] \]

\[ = \alpha_2 \gamma_0 \sum_{j=1}^{N} E \left[ |h_{2,j,3,1}|^2 \right] T + \alpha_1 \alpha_2 \gamma_1 \sum_{j=1}^{N} \sum_{i=1}^{N} E \left[ |h_{1,i,2,j}|^2 |h_{2,j,3,1}|^2 \right] T \]
which is expanded by substituting the values of \( P_s^{(0)} \), \( P_n^{(0)} \), \( P_s^{(1)} \), \( P_n^{(1)} \), \( P_s^{(2)} \), and \( P_n^{(2)} \) from (C.1), (C.2), (C.5), (C.6), (D.2), and (D.3) respectively into (D.1), we get

\[
\text{snr}_{\text{RSCS-3}} = \frac{\alpha_0^2 \sigma_3^2 + \alpha_0^2 \alpha_1^2 N \sigma_2^2 + \alpha_0^2 \alpha_2^2 \gamma_1^2 \sigma_2^2}{T + \alpha_2^2 \gamma_0^2 N \sigma_2^2 + \alpha_1^2 \alpha_2^2 \gamma_1^2 N^2} + \alpha_0^2 \alpha_1^2 \gamma_1^2 N^2 + \alpha_0^2 \alpha_2^2 \gamma_1^2 N^2.
\]

which is expanded by substituting the values of \( \alpha_0 \), \( \alpha_1 \), and \( \alpha_2 \) from Table 3.2 and obtained as

\[
\text{snr}_{\text{RSCS-3}} = \frac{p_0^4(\sigma_8^2 p_1 + \sigma_8^2 p_2 + \sigma_2^2 \sigma_3^2 + 3 \sigma_8^2 \sigma_3^2 + 2 \sigma_8^2 \sigma_3^2 p_1) + p_0^3(\sigma_6^2 p_1 + 2 \sigma_8^2 p_1 + 3 \sigma_6^2 p_2)}{T + \alpha_2^2 \gamma_0^2 N \sigma_2^2 + \alpha_1^2 \alpha_2^2 \gamma_1^2 N^2 + \alpha_1^2 \alpha_2^2 \gamma_1^2 N^2 + \alpha_0^2 \alpha_2^2 \gamma_1^2 N^2}.
\]

Similarly for RSCS-2, the receive SNR is given in (3.71), which is repeated here for convenience as

\[
\text{snr}_{\text{RSCS-2}} = \frac{P_s}{P_n} = \frac{P_s^{(1)} + P_s^{(2)}}{P_n^{(1)} + P_n^{(2)}},
\]

where \( P_s^{(1)} \), \( P_n^{(1)} \), \( P_s^{(2)} \), and \( P_n^{(2)} \) are given in (C.5), (C.6), (D.2), and (D.3) respectively. Substituting these equations into (D.5), we get

\[
\text{snr}_{\text{RSCS-2}} = \frac{\alpha_0^2 \alpha_1^2 N \sigma_2^2 + \alpha_0^2 \alpha_2^2 \gamma_0^2 N \sigma_2^2 + \alpha_0^2 \alpha_2^2 \gamma_1^2 N^2}{\alpha_1^2 T N \sigma_2^2 + \alpha_1^2 \gamma_0^2 N \sigma_2^2 + \alpha_1^2 \alpha_2^2 \gamma_1^2 N^2 T + \alpha_2^2 \gamma_1^2 N^2 T + \alpha_0^2 \alpha_2^2 \gamma_1^2 N^2 T + \alpha_0^2 \alpha_2^2 \gamma_1^2 N^2 T}.
\]
which is expanded by substituting the values of $\alpha_0$, $\alpha_1$, and $\alpha_2$ from Table 3.2 and obtained as

$$\text{snr}_{RSCS-2} = \frac{(p_1\sigma_2^6 + p_2\sigma_2^6)p_0^4 + (\sigma_2^6 p_1 + 2\sigma_2^6 p_2 + 3\sigma_2^6 p_2 + 2\sigma_2^6 p_1 p_2)p_0^3}{p_0(8\sigma_2^4 p_1 + 6\sigma_2^6 p_1 + 3\sigma_2^4 p_2 + \sigma_2^4 p_2 + 6\sigma_2^4 + 2\sigma_2^4 + 2\sigma_2^4)} + 2\sigma_2^4 p_1^2 + 2\sigma_2^6 p_1^2 + 2\sigma_2^4 p_1 + 2\sigma_2^4 p_2 + 2\sigma_2^4 p_2 + 2\sigma_2^4 p_1 p_2 + 2\sigma_2^4 + 2\sigma_2^4 + 2\sigma_2^4$$

$$+ 4\sigma_2^4 p_1 p_2) + p_0^3(4\sigma_2^6 p_1 + \sigma_2^4 p_2 + 2\sigma_2^4 + 6\sigma_2^4) + p_0^3(4\sigma_2^4 p_1 + 9\sigma_2^6 p_1 + 2\sigma_2^4 p_2 + 3\sigma_2^4 p_2 + 6\sigma_2^4 + 6\sigma_2^4) + 2\sigma_2^4 p_1^2 + 2\sigma_2^4 p_1 + 2\sigma_2^4 p_2 + 2\sigma_2^4 p_1 + 2\sigma_2^4 p_2 + 2\sigma_2^4 p_1 p_2 + 2\sigma_2^4 + 2\sigma_2^4 + 2\sigma_2^4$$

$$+ 4\sigma_2^4 p_1 p_2) + p_0^3(4\sigma_2^6 p_1 + \sigma_2^4 p_2 + 2\sigma_2^4 + 6\sigma_2^4) + p_0^3(4\sigma_2^4 p_1 + 9\sigma_2^6 p_1 + 2\sigma_2^4 p_2 + 3\sigma_2^4 p_2 + 6\sigma_2^4 + 6\sigma_2^4) + 2\sigma_2^4 p_1^2 + 2\sigma_2^4 p_1 + 2\sigma_2^4 p_2 + 2\sigma_2^4 p_1 + 2\sigma_2^4 p_2 + 2\sigma_2^4 p_1 p_2 + 2\sigma_2^4 + 2\sigma_2^4 + 2\sigma_2^4$$

Hence we proved that (D.4) and (D.6) are the same as (3.72) and (3.73) respectively.
Appendix E

Proof of Claim 5: Scaling Factor of RMCKCS

Rewriting (3.85) as

\[
\frac{p_2 T}{N} = E \left[ t_{2j}^H t_{2j} \right],
\]

where

\[
t_{2j} = \alpha_2 \frac{1}{\sqrt{2}} \left[ F_{2j,0} r_{2j}^{(0)} h_{0,1,2,j}^* + F_{2j,1} r_{2j}^{(1)} \| h_{1,2,j} \| \right]
\]

\[
= \frac{\alpha_2}{\sqrt{2}} F_{2j,0} \left( \alpha_0 s h_{0,1,2,j} + u_{2j}^{(0)} \right) h_{0,1,2,j}^*
\]

\[
+ \frac{\alpha_2}{\sqrt{2}} F_{2j,1} \left( \alpha_0 \alpha_1 \sum_{i=1}^N |h_{0,1,1,i}|^2 h_{1,i,2,j}^* F_{1i} s + \alpha_1 \sum_{i=1}^N h_{0,1,1,i}^* h_{1,i,2,j} F_{1i} u_{1i}^{(0)} + u_{2j}^{(1)} \right) \| h_{1,2,j} \|
\]

\[
= \frac{\alpha_2}{\sqrt{2}} |h_{0,1,2,j}|^2 F_{2j,0}s + \frac{\alpha_2}{\sqrt{2}} h_{0,1,2,j}^* F_{2j,0} u_{2j}^{(0)}
\]

\[
+ \frac{\alpha_0 \alpha_2}{\sqrt{2}} \sum_{i=1}^N |h_{0,1,1,i}|^2 h_{1,i,2,j} \| h_{1,2,j} \| \| F_{2j,1} F_{1i} s + \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^N h_{0,1,1,i}^* h_{1,i,2,j} \| h_{1,2,j} \| \| F_{2j,1} F_{1i} u_{1i}^{(0)}
\]

\[
+ \frac{\alpha_2}{\sqrt{2}} \| h_{1,2,j} \| \| F_{2j,1} u_{2j}^{(1)} \|.
\]

Hence (3.85) becomes

\[
\frac{p_2 T}{N} = \frac{\alpha_0 \alpha_2}{\sqrt{2}} E \left[ |h_{0,1,2,j}|^2 s^H F_{2j,0} t_{2j} \right] + \frac{\alpha_2}{\sqrt{2}} E \left[ h_{0,1,2,j}^* u_{2j}^{(0)H} F_{2j,0} t_{2j} \right]
\]

\[
+ E \left[ \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^N |h_{0,1,1,i}|^2 h_{1,i,2,j}^* \| h_{1,2,j} \| s^H F_{1i}^H F_{2j,1} t_{2j} \right]
\]

\[
+ E \left[ \frac{\alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^N h_{0,1,1,i}^* h_{1,i,2,j} \| h_{1,2,j} \| \| u_{1i}^{(0)H} F_{1i}^H F_{2j,1} t_{2j} \right] + \frac{\alpha_2}{\sqrt{2}} E \left[ \| h_{1,2,j} \| \| u_{2j}^{(1)H} F_{2j,1} t_{2j} \right]
\]
\[
\begin{align*}
&= \frac{\alpha_0^2 \alpha_2^2}{2} E \left| h_{0,1,2,j} \right|^4 + \frac{\alpha_0^2}{2} TE \left| h_{0,1,2,j} \right|^2 + \frac{\alpha_0^2 \alpha_1^2 \alpha_2^2}{2} \sum_{i=1}^{N} E \left| h_{0,1,1,i} \right|^4 \left| h_{1,i,2,j} \right|^2 \left\| h_{1,2,j} \right\|^2 \\
&\quad + \frac{\alpha_1^2 \alpha_2^2}{2} \sum_{i=1}^{N} E \left| h_{0,1,1,i} \right|^2 \left| h_{1,i,2,j} \right|^2 \left\| h_{1,2,j} \right\|^2 T \right] + \frac{\alpha_2^2}{2} E \left[ \left\| h_{1,2,j} \right\|^2 T \right] \\
&= I + II + III + IV + V, \quad (E.3)
\end{align*}
\]

where

\[
I = \frac{\alpha_0^2 \alpha_2^2}{2} E \left| h_{0,1,2,j} \right|^4 
\]

\[
II = \frac{\alpha_0^2}{2} TE \left| h_{0,1,2,j} \right|^2 = \frac{\alpha_0^2 \sigma_2^2 T}{2} \quad (E.5)
\]

\[
III = \frac{\alpha_0^2 \alpha_1^2 \alpha_2^2}{2} \sum_{i=1}^{N} E \left| h_{0,1,1,i} \right|^4 \left| h_{1,i,2,j} \right|^2 \left\| h_{1,2,j} \right\|^2 \quad (E.6)
\]

\[
IV = \frac{\alpha_1^2 \alpha_2^2}{2} \sum_{i=1}^{N} E \left| h_{0,1,1,i} \right|^2 \left| h_{1,i,2,j} \right|^2 \left\| h_{1,2,j} \right\|^2 T \quad (E.7)
\]

\[
V = \frac{\alpha_2^2}{2} E \left[ \left\| h_{1,2,j} \right\|^2 T \right] = \frac{\alpha_2^2 T}{2} E \left[ \sum_{p=1}^{N} \left| h_{1,p,2,j} \right|^2 \right] = \frac{\alpha_2^2 TN}{2}. \quad (E.8)
\]

Now using (3.77), we can write (E.5) as

\[
I = \frac{\alpha_0^2 \alpha_2^2}{2} E \left| h_{0,1,2,j} \right|^4 = \frac{\alpha_0^2 \alpha_2^2}{2} \left[ 2\sigma_2^4 \right] = \alpha_0^2 \alpha_2^2 \sigma_2^4 \quad (E.10)
\]

and (E.7) as

\[
III = \frac{\alpha_0^2 \alpha_1^2 \alpha_2^2}{2} \sum_{i=1}^{N} 2E \left| h_{1,i,2,j} \right|^2 \left\| h_{1,2,j} \right\|^2. \quad (E.11)
\]

Now,

\[
E \left[ \left| h_{1,i,2,j} \right|^2 \left\| h_{1,2,j} \right\|^2 \right] = E \left[ h_{1,i,2,j} \left( \sum_{p=1}^{N} \left| h_{1,p,2,j} \right|^2 \right) \right] = E \left[ \left| h_{1,i,2,j} \right|^4 + \sum_{p=1}^{N} \left| h_{1,i,2,j} \right|^2 \left| h_{1,p,2,j} \right|^2 \right]
\]

\[
= 2 + (N - 1) = N + 1. \quad (E.12)
\]

Hence from (E.12), (E.11) simplifies to

\[
III = \alpha_0^2 \alpha_1^2 \alpha_2^2 \left( N^2 + N \right) \quad (E.13)
\]

and (E.8) to

\[
IV = \frac{\alpha_1^2 \alpha_2^2}{2} \sum_{i=1}^{N} E \left[ \left| h_{0,1,1,i} \right|^2 \left| h_{1,i,2,j} \right|^2 \left\| h_{1,2,j} \right\|^2 T \right] = \frac{\alpha_1^2 \alpha_2^2 T}{2} \left( N^2 + N \right). \quad (E.14)
\]
Substituting (E.10), (E.6), (E.13), (E.14), and (E.9) into (E.4), we get

\[
\frac{p_2 T}{N} = \alpha_0^2 \sigma_2^4 + \frac{\alpha_0^2 \sigma_2^2 T}{2} + \alpha_0^2 \alpha_1^2 \alpha_2^2 (N^2 + N) + \frac{\alpha_1^2 \alpha_2^2 T}{2} (N^2 + N) + \frac{\alpha_2^2 T N}{2}
\]

\[
= \frac{\alpha_2^2}{2} \left[ 2\alpha_0^2 \sigma_2^4 + \sigma_2^2 T + (2\alpha_0^2 \alpha_1^2 + \alpha_1^2 T) (N^2 + N) + NT \right]
\]

\[
\Rightarrow \alpha_2^2 = \frac{2p_2 T}{N [2\alpha_0^2 \sigma_2^4 + \sigma_2^2 T + (2\alpha_0^2 \alpha_1^2 + \alpha_1^2 T) (N^2 + N) + NT]}
\]

\[
\Rightarrow \alpha_2 = \left[ \frac{2p_2}{N (2p_0 \sigma_2^4 + \sigma_2^2 + p_1 (N + 1) + NT)} \right]^{\frac{1}{2}},
\]

(E.15)

where the last step is arrived at from the expressions of \( \alpha_0^2 \) and \( \alpha_1^2 \) given in (3.6) and (3.74) respectively.
Appendix F

Proof of Claim 6: Receive SNR of RMCKCS

The receive SNR for RMCKCS given in (3.97) and (3.98) are repeated here for convenience:

\[
\text{snr}_{\text{RMCKCS-3}} = \frac{P_s^{(0)} + P_s^{(1)} + P_s^{(2)}}{P_n^{(0)} + P_n^{(1)} + P_n^{(2)}} \tag{F.1}
\]

\[
\text{snr}_{\text{RMCKCS-2}} = \frac{P_s^{(1)} + P_s^{(2)}}{P_n^{(1)} + P_n^{(2)}} \tag{F.2}
\]

Here, \(P_s^{(0)}\) and \(P_n^{(0)}\) are given in (C.1) and (C.2) respectively. From (3.81) and (3.88),

\[
P_s^{(1)} = E \left[ m_s^H m_s \right]
= E \left( \alpha_0 \alpha_1 \sum_{j=1}^{N} |h_{0,1,1,j}|^2 h_{1,j,3,1}^* F_{1j} s \right) H \left( \alpha_0 \alpha_1 \sum_{j=1}^{N} |h_{0,1,1,j}|^2 h_{1,j,3,1} F_{1j} s \right)
= \alpha_0^2 \alpha_1^2 \sum_{j=1}^{N} E \left[ |h_{0,1,1,j}|^4 |h_{1,j,3,1}|^2 \right] = 2N \alpha_0^2 \alpha_1^2 \sigma_2^2 \tag{F.3}
\]

and

\[
P_s^{(2)} = E \left[ m_w^H m_w \right]
= E \left[ \alpha_0 \alpha_2 \frac{1}{\sqrt{2}} \sum_{j=1}^{N} |h_{0,1,2,j}|^2 h_{2,j,3,1}^* s^H F_{2j,0}^H m_w \right]
+ E \left[ \frac{\alpha_0 \alpha_1 \alpha_2}{\sqrt{2}} \sum_{i=1}^{N} \sum_{j=1}^{N} ||h_{1,2,j}|| h_{2,j,3,1}^* h_{1,i,2,j}^* |h_{0,1,1,i}|^2 s^H F_{1i}^H F_{2j,1}^H m_w \right]
= \frac{\alpha_0^2 \alpha_2^2}{2} \sum_{j=1}^{N} E \left[ |h_{0,1,2,j}|^4 |h_{2,j,3,1}|^2 \right]
\]
Substituting (C.1), (C.2), (F.3), (F.4), (F.5), and (F.6) into (F.1) and (F.2), we get

\[ \text{srr} + \sum_{j=1}^{N} 2\sigma_2^2 + \sum_{j=1}^{N} 2E \left[ |h_{1,i,j,2,j}|^2 \|h_{1,2,j}\|^2 \right] \]

\[ = \alpha_0^2 \alpha_2^2 \sigma_2^4 + \alpha_0^2 \alpha_2^2 \alpha_2^2 \sum_{j=1}^{N} \sum_{i=1}^{N} (N + 1) \text{ from (E.12)} \]

\[ = N \alpha_0^2 \alpha_2^2 \sigma_2^2 + \alpha_0^2 \alpha_2^2 \alpha_2^2 N^2 (N + 1) \quad (F.4) \]

Now the noise variances can be found by taking the Tr\[.] operation of the covariance matrices and the E[.] operation w.r.t. the channel coefficients. Hence from (3.91),

\[ P_n^{(1)} = \text{Tr} \left[ E \left( P \right) \right] = \text{Tr} \left[ E \left( \left[ 1 + \alpha_0^2 \sum_{j=1}^{N} |h_{0,1,j,1}|^2 |h_{1,j,3,1}|^2 \right] I_T \right) \right] \]

\[ = T \left( 1 + N \alpha_0^2 \sigma_2^2 \right) \]

(F.5)

Similarly from (3.94),

\[ P_n^{(2)} = \text{Tr} \left[ E \left( P_w \right) \right] = \text{Tr} \left[ E \left( \left[ 1 + \frac{\alpha_0^2}{2} \sum_{j=1}^{N} \left[ |h_{0,1,2,j}|^2 + \|h_{1,2,j}\|^2 \right] \right] I_T \right) \right] \]

\[ + \text{Tr} \left[ E \left( \left[ \frac{\alpha_0^2}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \|h_{1,2,j}\| \|h_{2,k,3,1}h_{2,j,3,1}h_{0,1,1,1}|^2 h_{1,i,2,k} h_{1,2,j}^* \right] F_{2k,1} F_{2j,1}^H \right) \right] \]

\[ = T \left( 1 + \frac{\alpha_0^2}{2} \left( \sigma_2^2 + N \right) \right) + T \left( \frac{\alpha_0^2 \alpha_2^2}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} E \left[ |h_{0,1,1,1}|^2 |h_{1,j,2,j}|^2 |h_{2,j,3,1}|^2 \|h_{1,2,j}\|^2 \right] \right) \]

\[ = T \left( 1 + \frac{\alpha_0^2}{2} \left( \sigma_2^2 + N \right) \right) + T \left( \frac{\alpha_0^2 \alpha_2^2}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (N + 1) \right) \text{ from (E.12)} \]

\[ = \frac{T}{2} \left[ 2 + \alpha_0^2 \sigma_2^2 + \alpha_0^2 \sigma_2^2 + \alpha_0^2 \sigma_2^2 + \alpha_0^2 \sigma_2^2 \right] \]

(F.6)

Substituting (C.1), (C.2), (F.3), (F.4), (F.5), and (F.6) into (F.1) and (F.2), we get

\[ \text{srr}_{\text{RMKCS-3}} = \frac{\alpha_0^2 \sigma_3^2 + 2N \alpha_0^2 \alpha_1^2 \sigma_2^2 + N \alpha_0^2 \alpha_2^2 \sigma_4 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \left( N + 1 \right)}{T + T \left( 1 + \alpha_0^2 \sigma_2^2 \right) + \frac{T}{2} \left[ 2 + \alpha_0^2 \sigma_2^2 N + \alpha_0^2 \sigma_2^2 N^2 + \alpha_0^2 \sigma_2^2 N^2 + \alpha_0^2 \alpha_2^2 N^3 \right]} \]

\[ = \frac{4 \alpha_0^2 \sigma_3^2 + 4 N \alpha_0^2 \alpha_1^2 \sigma_2^2 + 2 N \alpha_0^2 \alpha_2^2 \sigma_4 + 2 N^2 \left( N + 1 \right) \alpha_0^2 \alpha_1^2 \alpha_2^2}{4 T + 2T N \alpha_0^2 \sigma_2^2 + T \left[ 2 + \alpha_0^2 \sigma_2^2 N + \alpha_0^2 \sigma_2^2 N + \alpha_0^2 \sigma_2^2 N^2 + \alpha_0^2 \sigma_2^2 N^3 \right]} \]  

(F.7)

and

\[ \text{srr}_{\text{RMKCS-2}} = \frac{2N \alpha_0^2 \alpha_1^2 \sigma_2^2 + N \alpha_0^2 \alpha_2^2 \sigma_4 + \alpha_0^2 \alpha_1^2 \alpha_2^2 N^2 \left( N + 1 \right)}{T \left( 1 + \alpha_0^2 \sigma_2^2 \right) + \frac{T}{2} \left[ 2 + \alpha_0^2 \sigma_2^2 N + \alpha_0^2 \sigma_2^2 N^2 + \alpha_0^2 \sigma_2^2 N^2 + \alpha_0^2 \alpha_2^2 N^3 \right]} \]

\[ = \frac{4N \alpha_0^2 \alpha_1^2 \sigma_2^2 + 2N \alpha_0^2 \alpha_2^2 \sigma_4 + 2N^2 \left( N + 1 \right) \alpha_0^2 \alpha_1^2 \alpha_2^2}{2 T + 2T N \alpha_0^2 \sigma_2^2 + T \left[ 2 + \alpha_0^2 \sigma_2^2 N + \alpha_0^2 \sigma_2^2 N^2 + \alpha_0^2 \alpha_2^2 N^2 + \alpha_0^2 \alpha_2^2 N^3 \right]} \]  

(F.8)
respectively. Now, we can expand (F.7) and (F.8) by substituting the values of \( \alpha_0 \), \( \alpha_1 \), and \( \alpha_2 \) from Table 3.3 and obtain

\[
\text{snr}_{\text{RMCKCS-3}} = \frac{p_0(2p_1p_2 + N\sigma_3^2 + \sigma_3^2p_1 + 2\sigma_2^2p_1 + 2\sigma_2^2p_2 + \sigma_2^2\sigma_3^2 + 2\sigma_2^2p_1^2 + 2N\sigma_2^2p_1^2 + 2Np_1p_2 + 2N\sigma_2^2p_1 + N\sigma_2^2p_1 + p_0^2(2N\sigma_3^2 + 2\sigma_2^2p_1 + 4\sigma_3^2p_1)}{3N + p_0(6N + 6p_1 + 6Np_1 + 2Np_2 + 2\sigma_2^2p_1 + 2\sigma_2^2p_2 + 6\sigma_2^2 + 6\sigma_1^2) + 3Np_1 + Np_2 + p_1p_2 + \sigma_1^2p_1 + \sigma_2^2p_2 + 3\sigma_2^2 + 12\sigma_2^2p_1^0 + \sigma_2^2p_1^2 + N\sigma_2^2p_1^2 + Np_1p_2 + N\sigma_2^2p_1^2} \tag{F.9}
\]

and

\[
\text{snr}_{\text{RMCKCS-2}} = \frac{(4p_1\sigma_2^6 + 4p_2\sigma_2^4)p_0^2 + (2p_1p_2 + 2\sigma_1^2p_1 + 2\sigma_2^2p_2 + 2Np_1p_2 + 2N\sigma_2^2p_1p_0)}{2N + 2p_1 + p_0(4N + 4p_1 + 4Np_1 + 2Np_2 + 2\sigma_2^2p_1 + 2\sigma_2^2p_2 + 4\sigma_2^2 + 4\sigma_2^4) + 2Np_1 + Np_2 + p_1p_2 + \sigma_1^2p_1 + \sigma_2^2p_1 + 2\sigma_2^2p_2 + 8\sigma_2^2p_1^0 + \sigma_2^2p_1^2 + N\sigma_2^2p_1^2 + Np_1p_2 + N\sigma_2^2p_1^2} \tag{F.10}
\]

respectively. Hence we proved that (F.9) and (F.10) are the same as (3.99) and (3.100) respectively.
Appendix G

Proof of Claim 7: Scaling Factor of EJHS

To find $\alpha_2$, we will first calculate the transmitted power. From (3.101), it is given as

$$E \left[ t_{2i}^H t_{2i} \right] = \alpha_2^2 E \left[ r_{2i}^{(1)H} F_{2i} H F_{2i} r_{2i}^{(1)} \right] = \alpha_2^2 E \left[ r_{2i}^{(1)H} r_{2i}^{(1)} \right]$$

$$= \alpha_2^2 E \left( \alpha_0 \alpha_1 \sum_{k=1}^{N} F_{1k} s h_{0,1,k} h_{1,k,2,i} + \alpha_1 \sum_{k=1}^{N} F_{1k} u_{1k}^{(0)} h_{1,k,2,i} + u_{2i}^{(1)} \right)$$

$$= \alpha_2^2 \alpha_0^2 \alpha_1^2 E \left( \sum_{k=1}^{N} F_{1k} s h_{0,1,k} h_{1,k,2,i} + \sum_{m=1}^{N} F_{1m} u_{1m}^{(0)} h_{1,m,2,i} + u_{2i}^{(1)} \right)$$

$$+ \alpha_2^2 \alpha_1^2 E \left( \sum_{k=1}^{N} F_{1k} u_{1k}^{(0)} h_{1,k,2,i} \right)^H \left( \sum_{m=1}^{N} F_{1m} u_{1m}^{(0)} h_{1,m,2,i} \right) + \alpha_2^2 E \left[ u_{2i}^{(1)H} u_{2i}^{(1)} \right]$$

$$= \alpha_2^2 \alpha_0^2 \alpha_1^2 \sum_{k=1}^{N} \sum_{m=1}^{N} \mathbf{s}^H \mathbf{F}_{1k}^H h_{0,1,k}^* h_{1,k,2,i}^* \mathbf{F}_{1m} s h_{0,1,m} h_{1,m,2,i} + \alpha_2^2 T$$

$$= \alpha_2^2 \alpha_0^2 \alpha_1^2 N + \alpha_2^2 \alpha_1^2 N T + \alpha_2^2 T$$

$$= \alpha_2^2 \left( p_0 T + \frac{p_1}{N(1+p_0)} N + \frac{p_1}{N(1+p_0)} N T + T \right)$$

$$= \alpha_2^2 T \left[ 1 + p_1 \right]. \tag{G.1}$$

To arrive at (G.1) from the previous step, we used the properties of the channel coefficients being i.i.d. ZMCG and also signal, noise, and channel coefficients being uncorrelated amongst

$$= \alpha_2^2 T \left[ 1 + p_1 \right]. \tag{G.2}$$
each other. From (G.2), we get \( \alpha_2^2 T [1 + p_1] = p_2 T / N \), which leads to

\[
\alpha_2 = \left[ \frac{p_2}{N (1 + p_1)} \right]^{\frac{1}{2}}.
\]  

(G.3)
Appendix H

Proof of Claim 8: Optimum Precoder at $L_1$ for MMSER Scheme When the Relays Cooperate

Differentiating (4.8) w.r.t. $F_1^*$ and $\lambda_1$ [49] yield

\[
\nabla_{F_1^*} L_1 = -R_{sr_1} + F_1 R_{r_1} + \lambda_1 F_1 R_{r_1} \quad \text{and} \quad (H.1)
\]

\[
\frac{\partial L_1}{\partial \lambda_1} = \text{Tr} \left( F_1^H F_1 R_{r_1} \right) - p_1 \quad \text{(H.2)}
\]

respectively. Equating both (H.1) and (H.2) to zero, we get

\[
F_1 = \frac{1}{1 + \lambda_1} R_{sr_1} R_{r_1}^{-1} \quad \text{and} \quad \text{Tr} \left( F_1^H F_1 R_{r_1} \right) = p_1 \quad \text{(H.3)}
\]

respectively. Equation (H.3) is arrived at with the assumption that $R_{r_1}$ is nonsingular, which is true from (4.5) as $\sigma_u^2 \neq 0$.

Substituting (H.3) into (H.4) and rearranging, we get

\[
(1 + \lambda_1)^2 = \frac{1}{p_1} \text{Tr} \left[ R_{sr_1}^H R_{sr_1} R_{r_1}^{-1} \right] \quad \text{(H.5)}
\]

as $(R_{r_1}^{-1})^H = R_{r_1}$ and $\text{Tr}(ABC) = \text{Tr}(BCA)$. Finding the value of $1 + \lambda_1$ from (H.5) and substituting in (H.3), we get the optimum precoder as

\[
\hat{F}_1 = \left[ \frac{p_1}{\text{Tr} \left[ R_{sr_1}^H R_{sr_1} R_{r_1}^{-1} \right]} \right]^{\frac{1}{2}} R_{sr_1} R_{r_1}^{-1}, \quad \text{(H.6)}
\]

which is the desired result shown in (4.9) and hence the claim follows. ■
Appendix I

Proof of Claim 9: Shortcut Method to Derive Optimum Precoder

Simplifying (4.12) we get

\[ J'_1(F'_1) = E [s - F'_1 r_1]^H [s - F'_1 r_1] \]
\[ = N - 2\Re \text{Tr} \left[ F'_1^H R_{sr_1} \right] + \text{Tr} \left[ F'_1^H F'_1 R_{r_1} \right], \tag{I.1} \]

where \( \Re \) is the real-part and \( \text{Tr}[.] \) denotes the trace operator. The covariance matrices \( R_{sr_1} \) and \( R_{r_1} \) are given in (4.4) and (4.5) respectively.

Equation (I.1) is similar to that of the error-performance surface of the Wiener filter [48] and the optimum weight vector of the filter is obtained by differentiating (I.1) wrt \( F'_1 \) and equating it to zero. Operating in a similar manner, we get

\[ \nabla_{F'_1} J_1 = F_1 R_{r_1} - R_{sr_1} = 0 \Rightarrow \hat{F}_1 = R_{sr_1} R_{r_1}^{-1}. \tag{I.2} \]

This is similar to the Wiener solution [48]. We enforce the power constraint \( E \left[ t_1^H t_1 \right] \leq p_1 \) as

\[ E [\alpha_1 F_1 r_1]^H [\alpha_1 F_1 r_1] = \alpha_1^2 \text{Tr} \left[ F_1^H F_1 R_{r_1} \right] = p_1 \]

and fix the real scalar \( \alpha_1 \) to

\[ \alpha_1 = \sqrt{\frac{p_1}{\text{Tr} \left[ F_1^H F_1 R_{r_1} \right]}}. \tag{I.3} \]

Substituting (I.2) into (I.3), multiplying both equations and substituting into (4.11), we get the transmit vector as

\[ t_1 = \sqrt{\frac{p_1}{\text{Tr} \left[ R_{sr_1}^H R_{sr_1} R_{r_1}^{-1} \right]}} R_{sr_1} R_{r_1}^{-1} r_1. \tag{I.4} \]

It can be seen that (I.4) is the same as (4.10) and hence the claim follows.
Appendix J

Proof of Claim 10: Simplified Optimum Precoder at $L_1$ for MMSER Scheme When the Relays Cooperate

The product $R_{sr}^{-1} R_{r_1}$ using (4.4) and (4.5) is

$$R_{sr} R_{r_1}^{-1} = \sqrt{p_0} i_N h_{01}^H \left[ \sigma_u^2 I_N + h_{01} p_0 h_{01}^H \right]^{-1}. \quad (J.1)$$

Applying the matrix identity, $[A + BCD]^{-1} = A^{-1} - A^{-1} B [C^{-1} + DA^{-1} B]^{-1} DA^{-1}$, to the square-bracketed term in (J.1) with $A = \sigma_u^2 I_N$, $B = h_{01}$, $C = p_0$, and $D = h_{01}^H$ we get

$$R_{sr} R_{r_1}^{-1} = \sqrt{p_0} i_N h_{01}^H \left[ I_N \sigma_u^2 - \frac{h_{01}}{\sigma_u^2} \left( \frac{1}{p_0} + \frac{h_{01} h_{01}}{\sigma_u^2} \right)^{-1} \frac{h_{01}^H}{\sigma_u^2} \right]$$

$$= \sqrt{p_0} i_N \left[ \frac{1}{\sigma_u^2} - \frac{h_{01}^H}{\sigma_u^2} \left( \frac{1}{p_0} + \frac{h_{01}^H h_{01}}{\sigma_u^2} \right)^{-1} \frac{h_{01}^H}{\sigma_u^2} \right]$$

$$= \frac{\sqrt{p_0} i_N}{\sigma_u^2 + p_0 \| h_{01} \|^2} h_{01}^H. \quad (J.2)$$

Using (J.2) and (4.4) we get

$$\text{Tr} \left[ R_{sr}^H R_{sr} R_{r_1}^{-1} \right]$$

$$= \text{Tr} \left[ \left( \sqrt{p_0} i_N h_{01}^H \right)^H \frac{\sqrt{p_0} i_N}{\sigma_u^2 + p_0 \| h_{01} \|^2} i_N h_{01}^H \right]$$

$$= \frac{p_0 N \| h_{01} \|^2}{\sigma_u^2 + p_0 \| h_{01} \|^2}. \quad (J.3)$$
To get the above result, we used the cyclic properties of \( \text{Tr}[.] \) operator and the fact that \( i_N^H i_N = N \). Substituting (J.2) and (J.3) into (H.6) we get the desired result shown in (4.13) and hence the claim follows.
Appendix K

Proof of Claim 11: Diagonal Element of the Optimum Precoder at $L_1$ for MMSER Scheme When the Relays Do Not Cooperate

Using the expressions of the correlation matrices, $R_{sr_1}$ and $R_{r_1}$ from (4.4) and (4.5) respectively, we can simplify $L_1$ in (4.8) to

$$L_1 = N - 2\sqrt{p_0}R \left( \sum_{i=1}^{N} f_{1i}^* h_{0,1,1,i}^* \right) + \sum_{i=1}^{N} |f_{1i}|^2 (p_0 |h_{0,1,1,i}|^2 + \sigma_u^2) + \lambda_1 \left[ \sum_{i=1}^{N} |f_{1i}|^2 (p_0 |h_{0,1,1,i}|^2 + \sigma_u^2) - p_1 \right].$$  \hspace{1cm} (K.1)

Differentiating (K.1) w.r.t. the conjugate of $f_{1i}$ and $\lambda_1$ yield

$$\frac{\partial L_1}{\partial f_{1i}^*} = -\sqrt{p_0} h_{0,1,1,i}^* + f_{1i} (p_0 |h_{0,1,1,i}|^2 + \sigma_u^2) + \lambda_1 \left[ f_{1i} (p_0 |h_{0,1,1,i}|^2 + \sigma_u^2) \right]$$  \hspace{1cm} (K.2)

and

$$\frac{\partial L_1}{\partial \lambda_1} = \sum_{i=1}^{N} |f_{1i}|^2 (p_0 |h_{0,1,1,i}|^2 + \sigma_u^2) - p_1$$  \hspace{1cm} (K.3)

respectively. Equating both (K.2) and (K.3) to zero, we get

$$f_{1i} = \frac{p_0^{\frac{1}{2}} h_{0,1,1,i}^*}{(1 + \lambda_1) (p_0 |h_{0,1,1,i}|^2 + \sigma_u^2)}$$  \hspace{1cm} (K.4)
and

\[ p_1 = \sum_{i=1}^{N} |f_{ii}|^2 \left( p_0 |h_{0,1,1,i}|^2 + \sigma_u^2 \right) \]  \hspace{1cm} (K.5)

respectively. From (K.4) and (K.5) we get

\[ f_{1i} = \left[ \frac{p_1}{\sum_{j=1}^{N} \left| h_{0,1,1,j} \right|^2 \left( p_0 \left| h_{0,1,1,i} \right|^2 + \sigma_u^2 \right)} \right]^{\frac{1}{2}} \frac{h_{0,1,1,i}}{p_0 \left| h_{0,1,1,i} \right|^2 + \sigma_u^2}, \]  \hspace{1cm} (K.6)

which is the desired result shown in (4.21) and hence the claim follows.
Appendix L

Proof of Claim 12: Optimum Precoder at \( L_k \) for MMSER Scheme When the Relays Cooperate

Differentiating (4.23) w.r.t. \( F'_k \) and equating to zero we get

\[
\nabla_{F'_k} J'_k = -R_{sr_k} + F'_k R_{r_k} \quad \text{and} \quad F'_k = R_{sr_k} R^{-1}_{r_k}
\]

respectively. Now the power constraint to find \( \alpha_k \) is used as

\[
E[\mathbf{t}_k^H \mathbf{t}_k] = p_k \\
\Rightarrow \alpha_k^2 E[\mathbf{t}_k^H F'_k F'_k \mathbf{t}_k] = p_k \\
\Rightarrow \alpha_k^2 \operatorname{Tr}[F'_k F'_k R_{r_k}] = p_k \\
\Rightarrow \alpha_k = \left[ \frac{p_k}{\operatorname{Tr}[F'_k F'_k R_{r_k}]} \right]^{\frac{1}{2}}.
\]

(L.2)

Now substituting (L.1) into (L.2) we get

\[
\alpha_k = \left[ \frac{p_k}{\operatorname{Tr}(R_{sr_k}^H R_{sr_k} R_{r_k}^{-1})} \right]^{\frac{1}{2}}. \tag{L.3}
\]

Now multiplying (L.3) and (L.1) we obtain the precoder as

\[
\alpha_k F'_k = \left[ \frac{p_k}{\operatorname{Tr}(R_{sr_k}^H R_{sr_k} R_{r_k}^{-1})} \right]^{\frac{1}{2}} R_{sr_k} R_{r_k}^{-1} = \hat{F}_k, \tag{L.4}
\]

which is the desired result shown in (4.28) and hence the claim follows. ■
Appendix M

Proof of Claim 13: Optimum Precoder at $L_k$ for MMSER Scheme When the Relays Do Not Cooperate

To simplify (4.32), let us consider

$$
F_k^H F_k R_{r_k} = \begin{bmatrix}
F_{kn}^H \\
\vdots \\
F_{k,k-1}^H
\end{bmatrix}
\begin{bmatrix}
F_{kn} \cdots F_{k,k-1} R_{r_k}
\end{bmatrix}
= 
\begin{bmatrix}
F_{kn} F_{kn} \cdots F_{kn} F_{k,k-1} \\
\vdots \\
F_{k,k-1} F_{kn} \cdots F_{k,k-1} F_{k,k-1}
\end{bmatrix}
R_{r_k}.
$$

(M.1)

Substituting $R_{r_k}$ from (4.25) into (M.1), we get

$$
F_k^H F_k R_{r_k} = \begin{bmatrix}
\Gamma_{k,n,n} \cdots \Gamma_{k,n,k-1} \\
\vdots \\
\Gamma_{k,k-1,n} \cdots \Gamma_{k,k-1,k-1}
\end{bmatrix}
$$

(M.2)

where

$$
\Gamma_{k,\eta,\nu} = \sum_{i=n}^{k-1} F_{k\eta}^H F_{ki} R_{r_k(i)} R_{r_k(\nu)}.
$$

Taking $\text{Tr}(\cdot)$ operator on (M.2), we get

$$
\text{Tr} \left( F_k^H F_k R_{r_k} \right) = \sum_{j=n}^{k-1} \text{Tr} (\Gamma_{k,j,j})
$$
\[
\begin{align*}
&= \sum_{j=n}^{k-1} \sum_{i=n}^{k-1} \text{Tr} \left( F_{kj}^H F_{ki} R_{r_k}^{(i)(j)} \right) \\
&= \sum_{j=n}^{k-1} \sum_{i=n}^{k-1} N \sum_{l=1}^{N} f_{k,j,l}^* f_{ki,l} \gamma_{kl}^{(i)(j)},
\end{align*}
\] (M.3)

where \( f_{ki,l} \), \( f_{kj,l} \) and \( \gamma_{kl}^{(i)(j)} \) represent the \( l \)th diagonal elements of \( F_{ki} \), \( F_{kj} \) and \( R_{r_k}^{(i)(j)} \) respectively. To arrive at (M.3), the fact that the matrices \( F_{ki} \), \( i \in [n, k-1] \) are diagonal is used.

Finally, to simplify (4.32) \( F_k^H R_{sr_k} \) is found as

\[
\begin{bmatrix}
F_{kn}^H \\
\vdots \\
F_{k,k-1}^H
\end{bmatrix}
= \begin{bmatrix}
R_{sr_k}^{(n)} & \cdots & R_{sr_k}^{(k-1)} \\
\vdots & \ddots & \vdots \\
R_{sr_k}^{(n)} & \cdots & F_{k,k-1}^H R_{sr_k}^{(k-1)}
\end{bmatrix}. \tag{M.4}
\]

Taking \( \text{Tr}(.) \) operator on (M.4) we get

\[
\text{Tr} \left( F_k^H R_{sr_k} \right) = \sum_{j=n}^{k-1} \sum_{i=n}^{k-1} N \sum_{l=1}^{N} f_{kj,l}^* \gamma_{kl}^{(i)(j)}, \tag{M.5}
\]

where \( \gamma_{kl}^{(i)(j)} \) is the \( l \)th diagonal element of the correlation matrix \( R_{sr_k}^{(i)(j)} \). From (M.3) and (M.5), (4.32) becomes

\[
\mathcal{L}_k = N - 2R \left[ \sum_{j=n}^{k-1} \sum_{l=1}^{N} f_{kj,l}^* \gamma_{kl}^{(i)(j)} \right] + (1 + \lambda_k) \sum_{j=n}^{k-1} \sum_{i=n}^{k-1} \sum_{l=1}^{N} f_{kj,l}^* f_{ki,l} \gamma_{kl}^{(i)(j)} - \lambda_k p_k. \tag{M.6}
\]

Differentiating (M.6) w.r.t. the conjugate of the precoder matrix diagonal element, \( f_{kj,l}^* \), \( j \in [n, k-1] \), \( l \in [1, N] \) and the Lagrangian multiplier \( \lambda_k \), we get

\[
\frac{\partial \mathcal{L}_k}{\partial f_{kj,l}^*} = -\gamma_{kl}^{(i)(j)} + (1 + \lambda_k) \sum_{i=n}^{k-1} f_{ki,l} \gamma_{kl}^{(i)(j)}, \quad j \in [1, k-1], \quad l \in [1, N] \tag{M.7}
\]

and

\[
\frac{\partial \mathcal{L}_k}{\partial \lambda_k} = \sum_{j=n}^{k-1} \sum_{i=n}^{k-1} \sum_{l=1}^{N} f_{kj,l}^* f_{ki,l} \gamma_{kl}^{(i)(j)} - p_k. \tag{M.8}
\]
respectively. Equating (M.7) and (M.8) to zero, we get
\[
\sum_{i=n}^{k-1} f_{ki,l} \gamma_{kl}^{(i)(j)} = \frac{\gamma_{kl}}{1 + \lambda_k}, \quad j \in [n, k - 1], \ l \in [1, N] \tag{M.9}
\]
and
\[
p_k = \sum_{j=n}^{k-1} \sum_{i=n}^{k-1} f_{kj,l} f_{ki,l} \gamma_{kl}^{(i)(j)} \tag{M.10}
\]
respectively. Equation (M.9) can be written in matrix form as
\[
f_{kl} \Upsilon_{kl} = \frac{1}{1 + \lambda_k} \Upsilon_{kl}^s, \ l \in [1, N], \tag{M.11}
\]
where
\[
f_k = [f_{kn,l} \cdots f_{k,k-1,l}], \ \Upsilon_{kl} = \begin{bmatrix} \gamma_{kl}^{(n)(n)} & \cdots & \gamma_{kl}^{(n)(k-1)} \\ \vdots & \ddots & \vdots \\ \gamma_{kl}^{(k-1)(n)} & \cdots & \gamma_{kl}^{(k-1)(k-1)} \end{bmatrix}, \tag{M.12}
\]
and
\[
\Upsilon_{kl}^s = [\gamma_{kl}^{s(n)} \cdots \gamma_{kl}^{s(k-1)}].
\]
Similarly, (M.10) can be written in matrix form as
\[
p_k = \sum_{l=1}^{N} Tr \left( f_{kl}^H f_{kl} \Upsilon_{kl} \right). \tag{M.13}
\]
From (M.11), we get
\[
f_{kl} = \frac{1}{1 + \lambda_k} \Upsilon_{kl}^s \Upsilon_{kl}^{-1} \tag{M.14}
\]
as \Upsilon_{kl} is a nonsingular matrix which depends on \( R_{sr_k} \) and \( R_{r_k} \). Substituting (M.14) into (M.13), we get
\[
p_k = \frac{1}{(1 + \lambda_k)^2} \sum_{l=1}^{N} Tr \left( \Upsilon_{kl}^s H \Upsilon_{kl}^s \Upsilon_{kl}^{-1} \right). \tag{M.15}
\]
From (M.14) and (M.15), the optimum \( \hat{f}_{kl} \) is given by
\[
\hat{f}_{kl} = \left[ \frac{p_k}{\sum_{l=1}^{N} Tr \left( \Upsilon_{kl}^s H \Upsilon_{kl}^s \Upsilon_{kl}^{-1} \right)} \right]^2 \Upsilon_{kl}^s \Upsilon_{kl}^{-1}, \ l \in [1, N], \tag{M.16}
\]
which is the desired result shown in (4.33) and hence the claim follows.
Appendix N

Expressions of the Correlation Matrices

\( R_{sr_k} \) and \( R_{r_k} \)

We have seen in the derivations of precoders for both cooperative and noncooperative relays cases in (4.28) and (4.33), that they depend on two correlation matrices, viz. \( R_{sr_k} \) and \( R_{r_k} \). In this Appendix we get the expressions for them. We use Table 4.1 for transmit and receive vectors while finding expressions and assume that a general MMSER-\( \mu \) system with a class of link sets \( \mathcal{L}_\mu \) is considered.

N.1 Correlation matrix \( R_{sr_k} \in \mathbb{C}^{N \times (k-n)} \), \( k \in [1, K] \)

\( R_{sr_k} \) gives the correlation between the signal vector \( s = [s, \cdots , s]^T \in \mathbb{C}^{N \times 1} \) and the vector \( r_k \) created by stacking the vectors received by the \( L_k \) layer in various phases till phase \( k - 1 \). It is given by

\[
R_{sr_k} = E \left[ sr_k^H \right] = E \left[ s (r_k^{(n)H} \cdots r_k^{(k-1)H}) \right] = \begin{bmatrix} R_{sr_k^{(n)}} & \cdots & R_{sr_k^{(k-1)}} \end{bmatrix} \tag{N.1}
\]

where \( n = [k - \mu]^+ \). Now \( R_{sr_k^{(i)}} \), \( i \in [n, k - 1] \), is given by

\[
R_{sr_k^{(i)}} = E \left[ sr_k^{(i)H} \right] \tag{N.2}
\]

and for \( i = 0 \), it is given by

\[
R_{sr_k^{(0)}} = E \left[ sr_k^{(0)H} \right] = E \left[ s (h_{0,1,k} t_0 + u_k^{(0)})^H \right] = R_{st0} h_{0,1,k}^H, \text{ where } R_{st0} \text{ is given in (4.6)}. \tag{N.3}
\]
For $i \in [n, k - 1]$, $n \neq 0$, it is given by
\[
R_{sr}^{(i)} = E \left[ s_k^{(i)} H \right] = E \left[ s \left( H_{i,k} t_i + u_k^{(i)} \right) H \right] = R_{st_i} H_{i,k}^H
\]  
\[\text{(N.4)}\]
where
\[
R_{st_i} = E \left[ s_k H \right] = R_{sr_i} F_i^H
\]  
\[\text{(N.5)}\]
and $R_{su_k}^{(i)} = 0$ as the signal $s$ is uncorrelated with the noise $u_k^{(i)}$. Here, as in (N.1)
\[
R_{sr_i} = \left[ R_{sr_i}^{(m)} \cdots R_{sr_i}^{(1-1)} \right]
\]  
\[\text{(N.6)}\]
where $m = [i - \mu]^+$. Therefore $R_{sr_k}$, $k \in [1, K]$ depends on $R_{st_0}$ if $k \leq \mu$ and $R_{sr_1}$, $i \in [n, k - 1]$. Hence $L_2$ needs $R_{st_0}$ if $\mu > 1$ and $R_{sr_1}$, $L_3$ needs $R_{st_0}$ if $\mu > 2$ and $R_{sr_1}$, $R_{sr_2}$, etc. But any particular layer $L_k$ requires only $R_{sr_i}$, $i \in [n, k - 1]$, $n = [k - \mu]^+$, to make its precoder and hence this can be forwarded by the previous layers.

### N.2 Auto correlation matrix, $R_{rk} \in \mathbb{C}^{(k-n)N \times (k-n)N}$, $k \in [1, K]$

The auto-correlation matrix of the vector $r_k$ is given by
\[
R_{rk} = E \left[ r_k r_k^H \right] = \begin{bmatrix}
R_{r_k}^{(n)} & \cdots & R_{r_k}^{(n)}_{r_k-1} \\
\vdots & \ddots & \vdots \\
R_{r_k-1}^{(n)} & \cdots & R_{r_k-1}^{(n)}_{r_k-1}
\end{bmatrix},
\]  
\[\text{(N.7)}\]
where $n = [k - \mu]^+$. Here, $R_{r_k}^{(i)}_{r_k(j)}$, $i, j \in [n, k - 1]$, is given by
\[
R_{r_k}^{(i)}_{r_k(j)} = E \left[ r_k^{(i)} r_k^{(j)} H \right].
\]  
\[\text{(N.8)}\]
As $R_{rk}$ is Hermitian, it is sufficient to find only the component matrices when $i \leq j$, where we call $R_{r_k}^{(i)}_{r_k(j)}$ as a component matrix of $R_{rk}$.

**Case when $i = j$**

Let $i = j = 0$. Then the component matrix is given by
\[
R_{r_k}^{(0)}_{r_k} = E \left[ \left( h_{0,1,k} t_0 + u_0^0 \right) \left( h_{0,1,k} t_0 + u_0^0 \right)^H \right] = h_{0,1,k} R_{00} h_{0,1,k}^H + \sigma_n^2 I_N
\]  
\[\text{(N.9)}\]
where $R_{t0}$ is given in (4.7).

For $i = j \neq 0$, i.e., $i \in [n, k - 1]$, $n \neq 0$, the component matrix is given by

$$R_{(i), (i)} = E \left( H_i t_i + u_k^{(i)} \right) \left( H_i t_i + u_k^{(i)} \right)^H$$

$$= H_i R_t H_i^H + \sigma_n^2 I_N$$  \hspace{1cm} (N.10)

where

$$R_t = E \left( t_i t_i^H \right) = F_i R_{r_i} F_i^H.$$  \hspace{1cm} (N.11)

Here,

$$R_{r_i} = E \left[ r_i r_i^H \right] =
\begin{bmatrix}
R_{r_i (m), (m)} & \cdots & R_{r_i (m), (i-1)} \\
\vdots & \ddots & \vdots \\
R_{r_i (i-1), (m)} & \cdots & R_{r_i (i-1), (i-1)}
\end{bmatrix}$$  \hspace{1cm} (N.12)

where $m = [i - \mu]^+$. Therefore $R_{r_k}, k \in [1, K]$ depends on $R_{r_i}, i \in [n, k-1]$ with $n = [k - \mu]^+$.

**Case when $i < j$**

$R_{(i), (j)}, i, j \in [n, k-1], i < j$ from Table 4.1 is given by

$$R_{(i), (j)} = E \left[ r_k^{(i)} r_k^{(j)H} \right] =
\begin{bmatrix}
R_{r_k (m), (m)} & \cdots & R_{r_k (m), (j)} \\
\vdots & \ddots & \vdots \\
R_{r_k (j), (m)} & \cdots & R_{r_k (j), (j)}
\end{bmatrix}$$  \hspace{1cm} (N.13)

To arrive at (N.13) we used the facts that $R_{u_k^{(i)}, u_k^{(j)}} = 0$ as $u_k^{(i)}$ is uncorrelated with $u_k^{(j)}$, $\forall i \neq j$ and $R_{t_i u_k^{(j)}} = 0$ as $u_k^{(j)}$ is uncorrelated with $t_i$, $\forall i < k$, which is true as $i < j$ and $j$ spans till $k - 1$ and so $j < k$. Now

$$R_{t_i t_j} = F_i R_{r_i r_j} F_j^H$$  \hspace{1cm} (N.14)

where $R_{r_i r_j} \in \mathbb{C}^{(i-l)N \times (j-p)N}$, $l = [i - \mu]^+, p = [j - \mu]^+$, is given as

$$R_{r_i r_j} = E \left[ r_i r_j^H \right] =
\begin{bmatrix}
R_{r_i (l), (p)} & \cdots & R_{r_i (l), (j-1)} \\
\vdots & \ddots & \vdots \\
R_{r_i (j-1), (p)} & \cdots & R_{r_i (j-1), (j-1)}
\end{bmatrix}.$$  \hspace{1cm} (N.15)
The component matrix $R_{r_{i}^{(\theta)}r_{j}^{(\phi)}} \in \mathbb{C}^{N\times N}$, $\theta \in [l, i - 1]$, $\phi \in [p, j - 1]$ using Table 4.1 is given by

$$
R_{r_{i}^{(\theta)}r_{j}^{(\phi)}} = E \left[ \left( H_{\theta,i} t_{\theta} + u_{i}^{(\theta)} \right) \left( H_{\phi,j} t_{\phi} + u_{j}^{(\phi)} \right)^{H} \right] = H_{\theta,i} R_{t_{\theta}u_{i}^{(\theta)}} H_{\phi,j}^{H} + \{ \phi - i \}^{+} R_{u_{i}^{(\theta)}t_{\phi}u_{j}^{(\phi)}} H_{\phi,j}^{H} + \{ \theta - j \}^{+} H_{\theta,i} R_{t_{\theta}u_{j}^{(\phi)}}, \quad \text{(N.16)}
$$

where $\{ x \}^{+}$ is the unit step sequence given by

$$
\{ x \}^{+} = \begin{cases} 0, & \text{if } x < 0. \\ 1, & \text{if } x \geq 0. \end{cases} \quad \text{(N.17)}
$$

This is used in (N.16) to take care of the uncorrelatedness of $u_{\eta}^{(\kappa)}$ with $t_{\nu}$ when $\nu < \eta$. Here, $\eta = i$ or $j$, $\kappa = \theta$ or $\phi$ and $\nu = \theta$ or $\phi$.

Now $R_{u_{\eta}^{(\kappa)}t_{\nu}} \in \mathbb{C}^{N\times N}$ is given by

$$
R_{u_{\eta}^{(\kappa)}t_{\nu}} = R_{u_{\eta}^{(\kappa)}r_{\nu}} F_{\nu}^{H} = \left[ R_{u_{\eta}^{(\kappa)}r_{\nu}} \cdots R_{u_{\eta}^{(\kappa)}r_{(\nu-1)}} \right] F_{\nu}^{H} \quad \text{(N.18)}
$$

where $q = [\nu - \mu]^{+}$. The component matrices, $R_{u_{\eta}^{(\kappa)}r_{\nu}}$, $\zeta \in [q, \nu - 1]$, are given by

$$
R_{u_{\eta}^{(\kappa)}r_{\nu}} = E \left[ u_{\eta}^{(\kappa)} r_{\nu}^{(\zeta)} H \right] = E \left[ u_{\eta}^{(\kappa)} \left( H_{\zeta,\nu} t_{\zeta} + u_{\nu}^{(\zeta)} \right)^{H} \right] = R_{u_{\eta}^{(\kappa)}t_{\zeta}} H_{\zeta,\nu}^{H}. \quad \text{(N.19)}
$$

Now again $R_{u_{\eta}^{(\kappa)}t_{\zeta}}$ can be found like how we have done in (N.18).
Appendix O

Proof of Claim 14: Precoder of E-MMSED-L

We start with the MSE as defined in (2.3), modifying it to suit E-MMSED as

\[
J'_D \triangleq E \|s - (h_{K+1}t)_{stack}\|^2 = E \|s - (h_{K+1}t)_{stack}\]H\|s - (h_{K+1}t)_{stack}\] (O.1)
\]

where \(s = [s, \cdots, s]^T \in \mathbb{C}^{K_1 \times 1}\) and \((h_{K+1}t)_{stack} \in \mathbb{C}^{K_1 \times 1}\) is a vector stacked with \(h_{K+1}t, K_1\) times. Also,

\[
h_{K+1} = \left[ h_{1,K+1} \cdots h_{K,K+1} \right] \in \mathbb{C}^{1 \times KN}
\]

with\[
h_{i,K+1} = [h_{i,1,K+1}, \cdots, h_{i,N,K+1}], \ i \in [1, K].
\]

Assuming the precoder as \(F\) we have \(t = Fr_{av}\) and we can expand (O.1) as

\[
J'_D = \sum_{i=1}^{K_1} E [s - h_{K+1}t]^H [s - h_{K+1}t]
\]

\[
= K_1 \left[ 1 - 2\Re \text{Tr} (h_{K+1}R_{s't}) + \text{Tr} (h_{K+1}^H h_{K+1}R_t) \right]
\]

where

\[
R_{s't} = E [st]^H = h_0^H F^HR_{s't_0} \text{ from (4.48)}
\]

with \(R_{s't_0} = \sqrt{\frac{p_0}{K_0}}\) (O.4)
\[ \mathbf{R}_t = E[tt^H] = \frac{p_0}{K_0} \mathbf{Fh}_0^H \mathbf{F}^H + \frac{F}{K_0} \sum_{k=0}^{K_0-1} \sum_{l=0}^{K_0-1} E[u^{(k)}u^{(l)H}] \mathbf{F}^H \quad \text{from (4.48)} \]
\[ = \frac{p_0}{K_0} \mathbf{Fh}_0^H \mathbf{F}^H + \sum_{k=0}^{K_0-1} \sum_{l=0}^{K_0-1} I_{KN} \delta (k - l) \mathbf{F}^H \]
\[ = \frac{p_0}{K_0} \mathbf{Fh}_0^H \mathbf{F}^H + \frac{\sigma_n^2}{K_0} \mathbf{F} \mathbf{F}^H. \quad \text{(O.5)} \]

Now including the power constraint \( C'(\mathbf{F}) = E[tt^H] \leq p_1/K_1 \) into (O.3), we make the Lagrangian as
\[ \mathcal{L}' = J_D' + \lambda' \left[ \text{Tr}(\mathbf{R}_t) - \frac{p_1}{K_1} \right] \]
\[ = K_1 - 2K_1 \sqrt{\frac{p_0}{K_0} \Re \text{Tr}(\mathbf{h}_{K+1}^H \mathbf{h}_0^H \mathbf{F}^H)} + K_1 \text{Tr} \left( \mathbf{h}_{K+1}^H \mathbf{h}_{K+1} \frac{p_0}{K_0} \mathbf{Fh}_0^H \mathbf{F}^H + \frac{\sigma_n^2}{K_0} \mathbf{F} \mathbf{F}^H \right) \]
\[ + \lambda' \left[ \text{Tr} \left( \frac{p_0}{K_0} \mathbf{Fh}_0^H \mathbf{F}^H + \frac{\sigma_n^2}{K_0} \mathbf{F} \mathbf{F}^H \right) - \frac{p_1}{K_1} \right] \quad \text{(O.6)} \]
\[ = K_1 - 2K_1 \left[ \frac{p_0}{K_0} \Re \sum_{k=1}^{K} \sum_{l=1}^{N} \mathbf{h}_{0,k,l}^* \mathbf{h}_{k,l}^H \mathbf{f}_{kl} + \frac{K_1 p_0}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,k,l}|^2 |h_{k,l,K+1}|^2 |\mathbf{f}_{kl}|^2 \right. \]
\[ + \frac{K_1 \sigma_n^2}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} |\mathbf{f}_{kl}|^2 \]
\[ + \lambda' \left[ \frac{p_0}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,k,l}|^2 |\mathbf{f}_{kl}|^2 + \frac{K_1 \sigma_n^2}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} |\mathbf{f}_{kl}|^2 - \frac{p_1}{K_1} \right] \]

using the diagonal properties of \( \mathbf{F} \). As it is not possible to get a closed form solution to \( \mathbf{f}_{kl} \) from this, we adopt a different method similar to the one used in [30] to get it. In this method, we find the optimum precoder \( \hat{\mathbf{F}} \) by differentiating the Lagrangian in (O.6) w.r.t. \( \mathbf{F}^* \) and equating it to zero like how it is done for cooperative case. Then we take only the diagonal elements from it and zeroise others in the precoder obtained. Hence differentiating (O.6) w.r.t. \( \mathbf{F}^* \) and \( \lambda' \) we get
\[ \nabla_{\mathbf{F}^*} \mathcal{L}' = -K_1 \sqrt{\frac{p_0}{K_0}} \mathbf{h}_{K+1}^H \mathbf{h}_0^H \]
\[ + K_1 \mathbf{h}_{K+1}^H \mathbf{h}_{K+1} \frac{p_0}{K_0} \mathbf{Fh}_0^H + \frac{K_1 \sigma_n^2}{K_0} \mathbf{F} + \lambda' \left[ \frac{p_0}{K_0} \mathbf{Fh}_0^H + \frac{\sigma_n^2}{K_0} \mathbf{F} - \frac{p_1}{K_1} \right] \quad \text{(O.7)} \]
and
\[ \frac{\partial \mathcal{L}'}{\partial \lambda'} = \frac{p_0}{K_0} \mathbf{Fh}_0^H + \frac{\sigma_n^2}{K_0} \mathbf{F} - \frac{p_1}{K_1} \quad \text{(O.8)} \]
respectively. Equating (O.7) and (O.8) to zero, we get
\[
\mathbf{F} = \left[ K_1 \mathbf{h}_{K+1}^H \mathbf{h}_{K+1} + \lambda \mathbf{I}_{K_N} \right]^{-1} K_1 \sqrt{\frac{p_0}{K_0} \mathbf{h}_{K+1}^H \mathbf{h}_{K+1}^H \left[ \frac{p_0}{K_0} \mathbf{h}_0 \mathbf{h}_0^H + \frac{\sigma_u^2}{K_0} \mathbf{I}_{K_N} \right]^{-1}} \tag{O.9}
\]
and
\[
\frac{p_1}{K_1} = \frac{p_0}{K_0} \mathbf{F} \mathbf{h}_0^H + \frac{\sigma_u^2}{K_0} \mathbf{F} \tag{O.10}
\]
respectively. Applying the matrix identity, \([\mathbf{A} + \mathbf{BCD}]^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1} \mathbf{B} [\mathbf{C}^{-1} + \mathbf{DA}^{-1} \mathbf{B}]^{-1} \mathbf{DA}^{-1}\), to the square-bracketed terms in (O.9) with \(\mathbf{A} = \lambda \mathbf{I}_{K_N}\), \(\mathbf{B} = \mathbf{h}_{K+1}^H\), \(\mathbf{C} = K_1\), and \(\mathbf{D} = \mathbf{h}_{K+1}\), in the first square-bracketed term and with \(\mathbf{A} = \frac{(\sigma_u^2/K_0) \mathbf{I}_{K_N}}{p_0/K_0}, \mathbf{B} = \mathbf{h}_0, \mathbf{C} = \frac{p_0}{K_0}, \text{and} \mathbf{D} = \mathbf{h}_0^H\), in the second square-bracketed term we get
\[
\mathbf{F} = \left[ \frac{\mathbf{I}_{K_N}}{\lambda'} - \frac{\mathbf{h}_{K+1}^H}{\lambda'} \left( \frac{1}{K_1} + \frac{\|\mathbf{h}_{K+1}\|^2}{\lambda'} \right)^{-1} \frac{\mathbf{h}_{K+1}}{\lambda'} \right] K_1 \sqrt{\frac{p_0}{K_0} \mathbf{h}_{K+1}^H \mathbf{h}_{K+1}^H}
\[
- \frac{K_0 \mathbf{h}_0}{\sigma_u^2} - \frac{K_0 \|\mathbf{h}_0\|^2}{\sigma_u^2} \left( \frac{p_0}{K_0} + \frac{\|\mathbf{h}_0\|^2}{\sigma_u^2} \right) \frac{K_0 \mathbf{h}_0^H}{\lambda'}
\[
= \sqrt{\frac{p_0}{K_0} K_1} \mathbf{h}_{K+1}^H \mathbf{h}_{K+1}^H
\[
\left( \lambda' + K_1 \|\mathbf{h}_{K+1}\|^2 \right) \left( K_0 \sigma_u^2 + p_0 K_0 \|\mathbf{h}_0\|^2 \right). \tag{O.11}
\]
Now as the relays do not cooperate amongst themselves, we consider \(\mathbf{F}\) to be diagonal and hence the diagonal element pertaining to relay \(\text{R}_{im}\) is given by
\[
f_{im} = \sqrt{p_0 K_1} \mathbf{h}_{i,m,K+1}^* \mathbf{h}_{0,1,i,m} \left( \lambda' + K_1 \|\mathbf{h}_{K+1}\|^2 \right) \left( \sigma_u^2 + p_0 \|\mathbf{h}_0\|^2 \right). \tag{O.12}
\]
Now we can simplify (O.10) with the diagonal properties of \(\mathbf{F}\) as
\[
\frac{p_1}{K_1} = \frac{p_0}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,1,k,l}|^2 |f_{l}|^2 + \frac{\sigma_u^2}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} |f_{kl}|^2
\[
= \frac{1}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} \left( p_0 |h_{0,1,k,l}|^2 + \sigma_u^2 \right) |f_{l}|^2 \tag{O.13}
\]
Substituting (O.12) into (O.13), we get
\[
\frac{p_1}{K_1} = \frac{1}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} \left( p_0 |h_{0,1,k,l}|^2 + \sigma_u^2 \right) |f_{l}|^2
\[
= \frac{1}{K_0} \sum_{k=1}^{K} \sum_{l=1}^{N} \left( p_0 |h_{0,1,k,l}|^2 + \sigma_u^2 \right) \frac{p_0 K_0 K_1^2 |h_{i,K+1}|^2 |h_{0,l}|^2}{\left( \lambda' + K_1 \|\mathbf{h}_{K+1}\|^2 \right)^2 \left( \sigma_u^2 + p_0 \|\mathbf{h}_0\|^2 \right)^2}.
\]
which leads to
\[
\frac{1}{\lambda' + K_1 \|h_{K+1}\|^2} = \frac{p_{0}^{\frac{1}{2}} (\sigma_u^2 + p_0 \|h_0\|^2)}{K_1 \left[ p_0 K_1 \sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,1,k,l}|^2 |h_{k,l,K+1,1}|^2 \left( p_0 |h_{0,1,k,l}|^2 + \sigma_u^2 \right) \right]^{\frac{1}{2}}}.
\]

Applying (O.14) into (O.12) we get
\[
f_{im} = \left[ \frac{K_0 p_0}{K_1} \right]^{\frac{1}{2}} \frac{h_{0,1,i,m}^* h_{i,m,K+1,1}^*}{\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,1,k,l}|^2 |h_{k,l,K+1,1}|^2 \left( p_0 |h_{0,1,k,l}|^2 + \sigma_u^2 \right) \left( \frac{K_0 p_0}{K_1} \right)^{\frac{1}{2}}},
\]
which is the same as that in (4.50).
Appendix P

Proof of Claim 15: SNR of E-MMSED-K and L

Using (4.54), $P_S$ in (4.56) can be expanded as

$$P_S = E \sum_{k=K_0}^{K} \left( \sqrt{\frac{p_0}{K_0} s_{K+1} F h_0} \right)^H \left( \sqrt{\frac{p_0}{K_0} s_{K+1} F h_0} \right)$$

$$= \frac{K_1 p_0}{K_0} F H^K h_{K+1} F h_0$$

$$= \frac{K_1 p_0}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,ij}|^2 |h_{ij,K+1}|^2 |f_{ij}|^2 \quad \text{(P.1)}$$

using the diagonal properties of $F$ and $E(|s|^2) = 1$. From (4.55) we can write

$$E |N^{(k)}|^2 = E \left[ \frac{h_{K+1} F}{K_0} \sum_{i=0}^{K_0-1} u^{(i)} \right] \left[ \frac{h_{K+1} F}{K_0} \sum_{j=0}^{K_0-1} u^{(j)} \right]^H$$

$$= \frac{1}{K_0^2} \sum_{i=0}^{K_0-1} \sum_{j=0}^{K_0-1} h_{K+1} F \sigma_u^2 \delta(i-j) I_{KN} F H h_{K+1}^H$$

$$= \frac{\sigma_u^2}{K_0} h_{K+1} F F^H h_{K+1}^H = \frac{\sigma_u^2}{K_0} \sum_{i=0}^{K} \sum_{j=0}^{N} |h_{ij,K+1}|^2 |f_{ij}|^2.$$

Hence from (4.57) $P_N$ can be written as

$$P_N = \frac{\sigma_u^2 K_1}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{ij,K+1}|^2 |f_{ij}|^2. \quad \text{(P.2)}$$
**P.1** E-MMSED-K

Substituting (4.49) into (P.1) we get

\[
P_S = \frac{K_1 p_0}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,ij}|^2 |h_{ij,K+1}|^2 \frac{K_{op1}}{K_1} |h_{0,ij}|^2 |h_{ij,K+1}|^2
\]

\[
|h_{ij,K+1}|^4 \sum_{k=1}^{K} \sum_{l=1}^{N} \left| h_{0,kl}|^2 \right| \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)
\]

\[
= \frac{p_0 p_1}{\sum_{k=1}^{K} \sum_{l=1}^{N} \left| h_{0,ij}|^2 \right| \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)}.
\]

(P.3)

Similarly substituting (4.49) into (P.2) we get

\[
P_N = \frac{\sigma_2^2 K_1}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{ij,K+1}|^2
\]

\[
|h_{ij,K+1}|^4 \sum_{k=1}^{K} \sum_{l=1}^{N} \left| h_{0,kl}|^2 \right| \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)
\]

\[
= \frac{\sigma_2^2 p_1}{\sum_{k=1}^{K} \sum_{l=1}^{N} \left| h_{0,ij}|^2 \right| \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)}.
\]

(P.4)

**P.2** E-MMSED-L

Substituting (4.50) into (P.1) we get

\[
P_S = \frac{K_1 p_0}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,ij}|^2 |h_{ij,K+1}|^2 \frac{K_{op1}}{K_1} |h_{0,ij}|^2 |h_{ij,K+1}|^2
\]

\[
\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,kl}|^2 |h_{kl,K+1}|^2 \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)
\]

\[
= \frac{p_0 p_1}{\sum_{k=1}^{K} \sum_{l=1}^{N} \left| h_{0,ij}|^2 \right| \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)}.
\]

(P.5)

Similarly substituting (4.50) into (P.2) we get

\[
P_N = \frac{\sigma_2^2 K_1}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{ij,K+1}|^2
\]

\[
\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,kl}|^2 |h_{kl,K+1}|^2 \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)
\]

\[
= \frac{\sigma_2^2 p_1}{\sum_{k=1}^{K} \sum_{l=1}^{N} \left| h_{0,ij}|^2 \right| \left( p_0 |h_{0,kl}|^2 + \sigma_n^2 \right)}.
\]

(P.6)
Appendix Q

Proof of Claim 16: Optimum Precoder at $L_k$ for MMSER Scheme When the Relays Do Not Cooperate for $M > 1$ Case

Rewriting (4.79) for convenience here as

$$L_k = \text{Tr} \left( C^H C \right) - 2R \left[ \text{Tr} \left( CR_{sr_k} F_k^H \right) \right] + \text{Tr} \left( F_k^H F_k R_{r_k} \right) + \lambda_k \left[ \text{Tr} \left( F_k^H F_k R_{r_k} \right) - P_k \right].$$

(Q.1)

To simplify (Q.1), we need to find the various terms in it. As the third and fourth terms are similar to that of the single source-destination pair case, we have from (M.3)

$$\text{Tr} \left( F_k^H F_k R_{r_k} \right) = \sum_{j=n}^{k-1} \sum_{i=n}^{k-1} \sum_{l=1}^N f_{kj,l}^* f_{ki,l} \gamma_{kl}^{(i)(j)}$$

(Q.2)

where $f_{ki,l}$, $f_{kj,l}$ and $\gamma_{kl}^{(i)(j)}$ represent the $l$th diagonal elements of $F_{ki}$, $F_{kj}$ and $R_{r_{(j)}}^{(i)}$ respectively. Here, (Q.2) is the same as (M.3), repeated here for convenience. Now, as

$$R_{sr_k} = E \left[ \begin{array}{c} s_1 \\ \vdots \\ s_M \end{array} \right] \begin{bmatrix} r_k^{(n)} & \cdots & r_k^{(k-1)} \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & \vdots \end{bmatrix} \in \mathbb{C}^{M \times N(k-n)},$$

(Q.3)

we get

$$CR_{sr_k} = \begin{bmatrix} \sum_{m=1}^M C_{1m} R_{s_m r_k^{(n)}} & \cdots & \sum_{m=1}^M C_{1m} R_{s_m r_k^{(k-1)}} \\ \vdots & \ddots & \vdots \\ \sum_{m=1}^M C_{Nm} R_{s_m r_k^{(n)}} & \cdots & \sum_{m=1}^M C_{Nm} R_{s_m r_k^{(k-1)}} \end{bmatrix} \in \mathbb{C}^{N \times N(k-n)}.$$

(Q.4)
Hence,

\[
\mathbf{C}_{\text{src}} \mathbf{F}_k^H = \begin{bmatrix}
\sum_{m=1}^{M} c_{1m} \mathbf{R}_{s_m r_k^{(m)}} & \cdots & \sum_{m=1}^{M} c_{1m} \mathbf{R}_{s_m r_k^{(k-1)}} \\
\vdots & \ddots & \vdots \\
\sum_{m=1}^{M} c_{Nm} \mathbf{R}_{s_m r_k^{(m)}} & \cdots & \sum_{m=1}^{M} c_{Nm} \mathbf{R}_{s_m r_k^{(k-1)}}
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_{k,n}^H \\
\vdots \\
\mathbf{F}_{k,k-1}^H
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\sum_{i=n}^{k-1} \sum_{m=1}^{M} c_{1m} \mathbf{R}_{s_m r_k^{(m)}} \mathbf{F}_k^{H, i} \\
\vdots \\
\sum_{i=n}^{k-1} \sum_{m=1}^{M} c_{Nm} \mathbf{R}_{s_m r_k^{(m)}} \mathbf{F}_k^{H, i}
\end{bmatrix} \in \mathbb{C}^{N \times N}. \tag{Q.5}
\]

So,

\[
\text{Tr} \left[ \mathbf{C}_{\text{src}} \mathbf{F}_k^H \right] = \sum_{i=n}^{k-1} \sum_{m=1}^{M} N c_{jm} \gamma_{kj}^{s_m(i)} f_{ki,j}^*, \tag{Q.6}
\]

where \( \gamma_{kj}^{s_m(i)} \) is the \( j \)th element of the row vector \( \mathbf{R}_{s_m r_k^{(i)}} \in \mathbb{C}^{1 \times N} \) and \( f_{ki,j} \) is the \( j \)th diagonal element of \( \mathbf{F}_{k,i} \).

Substituting the values in (Q.2) and (Q.6) into (4.79), \( \mathcal{L}_k \) is obtained as

\[
\mathcal{L}_k = \sum_{m=1}^{M} \sum_{j=1}^{N} |c_{jm}|^2 - 2 \Re \left[ \sum_{i=n}^{k-1} \sum_{m=1}^{M} c_{jm} \gamma_{kj}^{s_m(i)} f_{ki,j}^* \right] + (1 + \lambda_k) \sum_{j=n}^{k-1} \sum_{i=1}^{N} f_{ki,l} f_{ki,l}^* \gamma_{kl}^{(i)(j)} - \lambda_k p_k. \tag{Q.7}
\]

Differentiating (Q.7) w.r.t. the conjugate of the precoder matrix diagonal element, \( f_{kj,l}^* \), \( j \in [n, k - 1], \ l \in [1, N] \) and the Lagrangian multiplier \( \lambda_k \), we get

\[
\frac{\partial \mathcal{L}_k}{\partial f_{kj,l}^*} = -\sum_{m=1}^{M} c_{lm} \gamma_{kl}^{s_n(j)} + (1 + \lambda_k) \sum_{i=n}^{k-1} f_{ki,l} \gamma_{kl}^{(i)(j)}, \ j \in [1, k - 1], \ l \in [1, N] \tag{Q.8}
\]

and

\[
\frac{\partial \mathcal{L}_k}{\partial \lambda_k} = \sum_{j=n}^{k-1} \sum_{i=1}^{N} f_{kj,l}^* f_{ki,l} \gamma_{kl}^{(i)(j)} - p_k \tag{Q.9}
\]

respectively. Equating (Q.8) and (Q.9) to zero, we get

\[
\sum_{i=n}^{k-1} f_{ki,l} \gamma_{kl}^{(i)(j)} = \frac{\sum_{m=1}^{M} c_{lm} \gamma_{kl}^{s_n(j)}}{1 + \lambda_k}, \ j \in [n, k - 1], \ l \in [1, N] \tag{Q.10}
\]

and

\[
p_k = \sum_{j=n}^{k-1} \sum_{i=1}^{N} f_{kj,l}^* f_{ki,l} \gamma_{kl}^{(i)(j)} \tag{Q.11}
\]
respectively. Equation (Q.10) can be written in matrix form as

\[ f_{kl} \Upsilon_{kl} = \frac{1}{1 + \lambda_k} c_l \Upsilon_S^{kl}, \quad l \in [1, N], \]  

(Q.12)

where

\[ c_l = [c_{l1}, \cdots, c_{lM}], \quad f_{kl} = [f_{kn,l} \cdots f_{k,k-1,l}], \]

\[ \Upsilon_{kl} = \begin{bmatrix} \gamma^{(n)(n)}_{kl} & \cdots & \gamma^{(n)(k-1)}_{kl} \\ \vdots & \ddots & \vdots \\ \gamma^{(k-1)(n)}_{kl} & \cdots & \gamma^{(k-1)(k-1)}_{kl} \end{bmatrix} \quad \text{and} \quad \Upsilon_S^{kl} = \begin{bmatrix} \gamma^{s_1(n)}_{kl} & \cdots & \gamma^{s_1(k-1)}_{kl} \\ \vdots & \ddots & \vdots \\ \gamma^{s_M(n)}_{kl} & \cdots & \gamma^{s_M(k-1)}_{kl} \end{bmatrix}. \]

Here, we also observe that \( c_l \) is the \( l \)th row of \( C \) and

\[ \Upsilon_S^{kl} = \begin{bmatrix} \Upsilon_{s_1}^{kl} \\ \vdots \\ \Upsilon_{s_M}^{kl} \end{bmatrix} \quad \text{with} \quad \Upsilon_{s_m}^{kl} = [\gamma^{s_m(n)}_{kl} \cdots \gamma^{s_m(k-1)}_{kl}], \quad m \in [1, M]. \]  

(Q.13)

Now, (Q.11) can be written in matrix form as

\[ p_k = \sum_{l=1}^{N} \text{Tr} \left( f_{kl}^H f_{kl} \Upsilon_{kl} \right). \]  

(Q.14)

From (Q.12), we get

\[ f_{kl} = \frac{1}{1 + \lambda_k} c_l \Upsilon_S^{kl} \Upsilon_{kl}^{-1} \]  

(Q.15)

as \( \Upsilon_{kl} \) is a nonsingular matrix which depends on \( R_{sr_k} \) and \( R_{rk} \). Substituting (Q.15) into (Q.14), we get

\[ p_k = \frac{1}{(1 + \lambda_k)^2} \sum_{l=1}^{N} \text{Tr} \left( \Upsilon_S^{kl} c_l^H c_l \Upsilon_S^{kl} \Upsilon_{kl}^{-1} \right). \]  

(Q.16)

From (Q.15) and (Q.16), the optimum \( f_{kl} \) is given by

\[ \hat{f}_{kl} = \left[ \frac{p_k}{\sum_{l=1}^{N} \text{Tr} \left( \Upsilon_S^{kl} c_l^H c_l \Upsilon_S^{kl} \Upsilon_{kl}^{-1} \right)} \right]^{\frac{1}{2}} c_l \Upsilon_S^{kl} \Upsilon_{kl}^{-1}, \quad l \in [1, N], \]  

(Q.17)

which is the desired result shown in (4.85) and hence the claim follows.
Bibliography


Publications arising out of Ph.D. thesis

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<tr>
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<th>Title</th>
<th>Conference</th>
<th>Authors</th>
<th>Month</th>
<th>Status</th>
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<td>1.</td>
<td>Improved Layering Protocol for Wireless Cooperative Networks</td>
<td>International Conference on Future Computer and Communication (ICFCC)</td>
<td>Pannir S Elamvazhuthi, Parag S Kulkarni, Bikash K Dey</td>
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Abstract—In a radio network consider two layers of relays between a source-destination pair, power loss being same for all the relays in a layer but different from layer to layer. In this scenario, so far systems have considered only the signals reaching from source to first layer, first to second, and so on and not source to second, or first to destination. So, signals from these ‘weak links’, that may be significant are ignored. In this paper, we take into account these signals and use them to advantage.

Here we propose a new protocol and show that under reasonable channel strength (variance greater than or equal to 0.2) of the ‘weak links’, it achieves lower BER than a suitably modified existing one, assuming the same total transmit power. Also the proposed protocol uses only half the number of relays and two thirds of the overall time spent by the existing protocol.

Keywords—wireless cooperative network; MIMO; distributed space-time code; fading channel; layered protocol

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems obtain high reliability and data rate using multiple antennas at the transmitter and receiver. The field of cooperative communication deals with cooperation of single-antenna radio nodes to emulate a MIMO communication system. The enormous potential in this area has drawn considerable attention from the scientific community.

Sendonaris et al. [1] consider a system with one destination and two sources cooperating with each other to achieve better performance. Distributed space-time coding (DSTC), proposed by Laneman and Wornell [2], use a space-time code at relays and achieve higher spectral efficiency than repetition-based schemes. Jing and Hassibi [3] use a system with a layer of single-antenna relays between source and destination and obtain the benefits closer to that of MIMO. Borade et al. [4] use a multihop network with multiple layers of radio nodes to relay information from source to destination wherein the ‘weak links’ between non-consecutive layers of nodes are neglected.

In this paper, we consider a multihop network of single-antenna radio nodes, as shown in Fig. 1, with two layers of relays between source and destination. We adopt the strategy of processing and forwarding at the relays proposed by Jing and Hassibi [3]. However we also make use of the ‘weak links’ between the non-consecutive layers shown by dashed lines in Fig. 1.
II. SYSTEM MODEL AND PREVIOUS WORK

In this section we will articulate the system model first, as the previous work can be explained using this and extended later in section IV.

A. System Model

Our system model consists of a source S, a destination D, and two layers L1 and L2 of N relays each as shown in Fig. 1, where Rij is the jth relay in the layer Li. The channel coefficients for the links S → R_{1j}, R_{1j} → R_{2j}, R_{2j} → D, S → R_{2j}, and R_{1j} → D are denoted by h_{s,1j}, h_{1j,2i}, h_{2i,d}, h_{s,2i}, and h_{1j,d} respectively. The channels are assumed to be Rayleigh fading and quasi static with a coherence interval of T symbol duration. The variances of the channel coefficients are σ^2_k = 1 for 'strong' channels represented by firm lines and σ^2_j < 1 for 'weak' channels represented by dashed lines in Fig. 1.

Each of the relays, R_{ij}, has a matrix \( A_{ij} = [a_{ij}^{(k)}] \) of size \( T \times T \), where \( k \) and \( j \) represent the row and column numbers respectively. These matrices are chosen to be orthogonal with the components \( a_{ij}^{(k)} \) being independent identically distributed (iid) zero mean real Gaussian with variance 1/T. The performance of the system has been found [5] to be the same using simulations with real orthogonal instead of complex unitary matrices considered by Jing and Hassibi [3].

Assume that during the kth phase (\( k = 1, 2, 3 \)), \( u_{ij}^{(k)} \) and \( r_{ij}^{(k)} \) are the noise vectors added with the received vectors \( r_{ij}^{(k)} \) at the relay \( R_{ij} \) and \( r_{ij}^{(k)} \) at D respectively. The superscript denotes the phase and the suffix a particular relay or destination. The components of these noise vectors are assumed to be iid zero mean complex Gaussian with unit variance.

B. Jing Hassibi Scheme

The scheme proposed by Jing and Hassibi [3] considered only S, L1, and D in Fig. 1. The channel variance from S to L1 and L1 to D was assumed to be constant at unity by the authors. The scheme consisted of two phases; in phase 1, S transmits while L1 layer relays R_{1j} receive and in phase 2, these relays post-multiply the received column vectors with \( A_{ij} \) and transmit to D.

At D an ML decoder extracts the signal transmitted and it is proved that this effectively obtains a DSTC and achieves the same diversity as that of a multiple-antenna system with some degradation in BER. Let us call this basic protocol as Jing Hassibi Scheme (JHS).

In the next section we propose a new protocol, analyze, and derive its ML decoder.

III. PROPOSED PROTOCOL

We shall call the proposed protocol as DWS (Destination Weak Signal) to imply that in this protocol the destination D uses the weak signal vector in its decoder received from the L1 relays’ transmissions.

Assume that there are three sequential transmission phases of T symbol duration each, with a total average power \( P \). Various phases of DWS are shown in Fig. 2 and explained as:

- Phase 1: S transmits; L1 and L2 layer relays receive.
- Phase 2: L1 layer relays transmit; L2 layer relays and D receive.
- Phase 3: L2 layer relays transmit and D receives.

Also we assume that R_{1j} has knowledge of \( h_{s,1j} \) and R_{2j} has knowledge of \( h_{s,2j} \) and \( h_{1j,2i}, i,j \in \{1,\ldots,N\} \). Later in section V, we prove that the assumption of R_{2j} having channel knowledge is redundant.

A. First phase

In the first phase, S transmits \( c_1 s(\tau) \) at time \( \tau \), for \( 1 \leq \tau \leq T \), i.e. S transmits \( c_1 s \) during T symbol duration, where \( s = [s(1)\cdots s(T)]^T \in \mathbb{C}^{T \times 1} \), normalized with \( E[s^H s] = 1 \). R_{ij} receives \( r_{ij}^{(1)}(\tau) = c_1 s(\tau) h_{s,1j} + u_{ij}^{(1)}(\tau) \) at time \( \tau \) and in vector form receives the length-T vector

\[
\mathbf{r}_{ij}^{(1)} = c_1 h_{s,ij} + \mathbf{u}_{ij}^{(1)}, \quad i \in \{1, 2\}, \quad j \in \{1, \ldots, N\}.
\]

(1)

The multiplication factor \( c_1 \) is selected to be \( \sqrt{p_1 T} \) so that the average power transmitted by S is \( p_1 T \).

B. Second phase

In this phase, the relays L1 transmit \( c_2 t_{1j}^{2} \) for T symbol duration, where \( t_{1j}^{(2)} = h_{s,1j}^{*} A_{1j} \mathbf{r}_{1j}^{(1)} \). The factor \( c_2 \) is selected to be \( c_2 = \sqrt{p_2/N(1+p_1)} \) so that the average power transmitted is \( p_2 T' \) during this phase. The relays R_{2j} receive \( r_{2j}^{(2)} \) which can be proved [5] to be

\[
r_{2j}^{(2)} = c_1 c_2 \sum_{i=1}^{N} h_{s,1j} h_{1j,2i} A_{ij} s + c_2 \sum_{i=1}^{N} h_{s,1j} h_{1j,2i} A_{ij} u_{ij}^{(1)} + u_{2j}^{(2)}.
\]

(2)

D receives the weak signal vector \( r_{d}^{(2)} \) in phase 2 which can be shown [3] to be

\[
r_{d}^{(2)} = \mathbf{m}_s + \mathbf{u}_d = \mathbf{x}, \quad \text{say}
\]

(3)

where \( \mathbf{m}_s \) is the mean and \( \mathbf{u}_d \) is the noise component of \( \mathbf{x} \) respectively, given by

\[
\mathbf{m}_s = c_1 c_2 \sum_{j=1}^{N} h_{s,1j}^{2} h_{1j,d} A_{1j} s \quad \text{and}
\]

\[
\mathbf{u}_d = c_2 \sum_{j=1}^{N} h_{s,1j} h_{1j,d} A_{ij} u_{ij}^{(1)} + u_{d}^{(2)}.
\]

(4)
C. Third phase

In phase 3, \( R_{2j} \), \( j \in 1, \ldots, N \), transmits \( c_3^{(3)} \), where \( t_{2j}^{(3)} = A_2^r r_{2j} \) and \( r_{2j} \) is a concatenated vector given by

\[
r_{2j} = \begin{bmatrix} h_{1,2j,1}^{(1)} \\ \| h_{1,2j} \| r_{2j}^{(2)} \end{bmatrix}.
\]

We assume that each of the relays \( R_{2j} \) has \( T \times 2T \) matrix \( A_2^r = \{ A_2^{rj} (1) | A_2^{rj} (2) \} / \sqrt{2} \), where \( A_2^{rj} (1) \) and \( A_2^{rj} (2) \) are the sub-matrices comprising of the first and the last \( T \) columns of \( A_2^r \) respectively. These sub-matrices are chosen to be orthogonal and so \( A_2^r \) is orthogonal too. Here

\[
\| h_{1,2j} \| = \sqrt{\| h_{1,2j,1} \|^2 + \cdots + \| h_{N,2j,1} \|^2}
\]

is the norm of the channel coefficient vector \( h_{1,2j} = [ h_{1,2j,1} \ldots h_{N,2j,1} ]^T \). Also, \( c_3 \) is selected [5] to be

\[
c_3 = \frac{1}{8Np_{1,2} + N^2(1 + \frac{1}{p_2} + \frac{1}{p_3}) + (1 + p_1)N\sigma_2^2} \left( 1 + p_1 \right) N\sigma_t^2
\]

and

\[
\| h_{1,2j} \| = \sqrt{\| h_{1,2j,1} \|^2 + \cdots + \| h_{N,2j,1} \|^2}
\]

is the total average transmitted power is \( p_3T \) in phase 3. The received vector at \( D \) can be proved [5] to be

\[
r_{2j}^{(3)} = m_z + u_z = z, \text{ say}
\]

where \( m_z \) is the mean and \( u_z \) is the noise component of \( z \) respectively.

\[
m_z = \frac{c_1 c_3}{\sqrt{2}} N \sum_{i=1}^{N} \sum_{j=1}^{N} \| h_{1,2i} \| \| h_{1,2j} \| h_{1,2j,d} A_2 (1) A_{1,s}
\]

+ \frac{c_1 c_3}{\sqrt{2}} \sum_{j=1}^{N} \| h_{1,2j} \| h_{1,2j,d} A_2 (1) s
\]

and

\[
u_z = \frac{c_3}{\sqrt{2}} \sum_{j=1}^{N} \| h_{1,2j} \| h_{1,2j,d} A_2 (1) u_z
\]

+ \frac{c_3}{\sqrt{2}} \sum_{j=1}^{N} \| h_{1,2j} \| h_{1,2j,d} A_2 (2) u_z
\]

+ \frac{c_3}{\sqrt{2}} \sum_{j=1}^{N} \| h_{1,2j} \| h_{1,2j,d} A_2 (2) u_z
\]

D. ML Decoder

Now let us assume that \( D \) employs ML decoding of the stacked receive vector \( y = \left[ x^T | z^T \right] \). As all the channel coefficients are assumed to be known at \( D \), \( y \) is jointly Gaussian with mean \( m_y \) and covariance matrix \( \Sigma_y \) respectively. Here \( P_s \) and \( P_x \) are the covariance matrices of \( x \) and \( z \) respectively and can be shown [5] to be

\[
P_s = \left[ 1 + \frac{c_2}{2} \sum_{j=1}^{N} \| h_{1,2j} \| \| h_{1,2j,d} \|^2 \right] I_T
\]

and

\[
P_x = \left[ 1 + \frac{c_2}{2} \sum_{j=1}^{N} \| h_{1,2j} \| \| h_{1,2j,d} \|^2 \right] I_T
\]

+ \frac{c_2}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} \| h_{1,2j} \| \| h_{1,2j,d} \|^2 \| h_{1,2k} \| \| h_{1,2k,d} \|^2 \| h_{1,2j,k} \| \| h_{1,2j,k,d} \|^2
\]

respectively. Here \( P_s \) and \( P_x \) are the cross covariance matrices of \( x \) and \( z \) respectively. Here \( P_{xx} \) and \( P_{xz} \) are the cross covariance matrices of \( x \) and \( z \), related as \( P_{xx} = P_{yy} \). Now \( P_{xx} \) is given [5] by

\[
P_{xx} = E \left[ (x - u_x) (x - u_x)^H \right] = E \left[ (x - u_x) (x - u_x)^H \right]
\]

\[
= \frac{c_2}{\sqrt{2}} \sum_{j=1}^{N} \sum_{k=1}^{N} \| h_{1,2j} \| \| h_{1,2j,d} \|^2 \| h_{1,2k} \| \| h_{1,2k,d} \|^2 \| h_{1,2j,k} \| \| h_{1,2j,k,d} \|^2
\]

Now we can write the ML function [6] of \( y \) as

\[
Pr(y|s) = \exp \left[ -\frac{1}{2} \| y - m_y \| P_y^{-1} (y - m_y) \right]
\]

By maximizing this we can obtain the decoded vector as \( \hat{s} = \arg \max_s Pr(y|s) = \arg \min_s \| y' \|^2 \) where \( y' = P_y^{-1} y \).

IV. EXTENDED JING HASSIBI SCHEME (EJHS)

In this section, we extend JHS [3] to have three phases so that the proposed protocol can be compared with this. Ignoring the dashed lines, Fig. 2 shows the various phases of EJHS.

Phase 1 is similar to that of DWS except that the vector received by \( R_{1j} \) is given by (1) with \( i = 1 \) and \( i \neq 2 \). In phase 2 the relays \( R_{1j} \) transmit \( c_2 t_{2j}^{(2)} = c_2 A_{2,j}^1 u_z \) and the relays \( R_{2j} \), \( 1 \leq j \leq N \) receive \( r_{2j}^{(2)} \) given by

\[
r_{2j}^{(2)} = c_3 c_2 \sum_{i=1}^{N} h_{1,2i} \| h_{1,2j} \| h_{1,2j,d} A_{2,j} (1) s + c_2 \sum_{i=1}^{N} h_{1,2i} \| h_{1,2j} \| h_{1,2j,d} A_{2,j} (2) u_z
\]

In phase 3, the relays \( R_{2j} \) transmit the vector \( c_3 t_{2j}^{(3)} = c_3 A_{2,j}^r r_{2j}^{(2)} \). The factors \( c_2 \) and \( c_3 \) are selected to be \( \sqrt{p_2 / [N(1 + p_1)]} \) and \( \sqrt{p_3 / [N(1 + p_2)]} \) so that the transmitted powers in phases 2 and 3 are \( p_2T \) and \( p_3T \) respectively. In phase 3 the received vector at \( D \) can be shown [5] to be

\[
r_{2j}^{(3)} = m_z + u_z = z, \text{ say}
\]

where

\[
m_z = c_1 c_3 \sum_{i=1}^{N} h_{1,2i} \| h_{1,2j} \| h_{1,2j,d} A_{2,j} (1) s
\]

\[
u_z = c_2 c_3 \sum_{i=1}^{N} h_{1,2i} \| h_{1,2j} \| h_{1,2j,d} A_{2,j} (2) u_z
\]

+ \frac{c_3}{\sqrt{2}} \sum_{j=1}^{N} \| h_{1,2j} \| h_{1,2j,d} A_{2,j} (2) u_z
\]

\[
u_z = c_2 c_3 \sum_{i=1}^{N} h_{1,2i} \| h_{1,2j} \| h_{1,2j,d} A_{2,j} (2) u_z
\]

+ \frac{c_3}{\sqrt{2}} \sum_{j=1}^{N} \| h_{1,2j} \| h_{1,2j,d} A_{2,j} (2) u_z
\]
Also it can be proved [5] that $\mathbf{P}_z$, the covariance matrix of $\mathbf{z}$, is given by

$$
\mathbf{P}_z = \left[ 1 + \sigma_2^2 \sum_{j=1}^{N} |h_{j1}|^2 \right] \mathbf{I}_T 
+ \sigma_2^2 \sum_{j=1}^{N} \sum_{k=1}^{N} h_{j1j2} h_{j2j3} h_{j3j4} \mathbf{A}_{j1} \mathbf{A}_{j2}^T.
$$

(17)

The decoded vector is then $\hat{s} = \arg\max_{\mathbf{z}} \Pr(\mathbf{z}|s) = \arg\min_{\mathbf{z'}} \|\mathbf{z'}\|^2$ where $\mathbf{z'} = \mathbf{P}_z^{-1}(\mathbf{z} - \mathbf{m}_s)$.

V. SIMULATIONS AND RESULTS

We ran simulations for 10,000 data packets each of size $T = 5$ symbol duration and number of relays in each layer $N = 5$. The components of the signal vector $\mathbf{s} \in \Omega = \{s_1, \ldots, s_{8024}\}$ are given by $s(k) = s_{1j}(k) + j s_{2j}(k)$, $1 \leq k \leq 5$. We have selected $s_{1j}(k)$ and $s_{2j}(k)$ equally likely from the 2-PAM signal set $1/\sqrt{10} \{-1, 1\}$.

A. Optimum Power Allocation

Power is to be allocated to various transmissions to minimize a parameter, such as pairwise error probability (PEP). Derivation of PEP was found to be complicated for both DWS and EJHS as they involve complex expressions. Jing and Hassibi [3] have proved that the optimum power allocation obtained by minimizing PEP also maximizes receive SNR at D for HJS. Hence in this paper we have selected receive SNR as the parameter to be maximized and expect that this gives near optimum power allocation.

We derived receive SNRs for DWS and EJHS in Appendices A and B as shown in (A5) and (B1) respectively. We also maximized the SNR of EJHS and proved that the optimum power allocations are $p_1^{\text{opt}} = p_2^{\text{opt}} = p_3^{\text{opt}} = P/3$ as shown in equation (B2). For DWS, the optimum points have been found by a fine computer search with the constraints $p_1 + p_2 + p_3 = P$ and $p_1, p_2, p_3 \geq 0$ for various values of $P$ and $\sigma^2$. A representative 3 dimensional plot of receive SNR for $\sigma^2 = 0.2$ when $P = 18$ dB is shown in Fig. 3.

The optimum power allocations for the three phases vary with $\sigma^2$ and $P$ for DWS. Figure 4 shows the plots of $p_1^{\text{opt}}, p_2^{\text{opt}},$ and $p_3^{\text{opt}}$ for $\sigma^2 = 0.05, 0.1, 0.2,$ and 0.5 while varying $P$. It is interesting to note that the power allocated to third phase is zero for DWS for all values of $P$ and $\sigma^2$. Hence as mentioned in section III the channel knowledge of $R_{j2}$ is redundant and DWS uses only two thirds of the total time spent by EJHS. In the next subsection we will compare the BER performance of DWS and EJHS.

B. BER Plots

We have allocated optimum powers $p_1^{\text{opt}}, p_2^{\text{opt}},$ and $p_3^{\text{opt}}$ in the three phases and used the ML decoders derived in section III and IV for generating BER plots. SNR is varied by varying $P$, as the noise variance is assumed to be constant at unity.

Figure 5 shows the BER plots of DWS for $\sigma^2 = 0.1$ to 1 and EJHS.

Fig. 5. Comparison of BER plots of DWS and EJHS.
that the proposed protocol DWS performs better than EJHS with increase in $\sigma^2_2$. Also we observe that for $\sigma^2_2 = 0.2$ the performance of DWS is better when $P \leq 10$ dB and for $\sigma^2_2 = 0.3$ when $P \leq 21$ dB. For all other higher values of $\sigma^2_2$, DWS performs better for all $P$ in the given range.

To summarize, when $\sigma^2_2 \geq 0.2$ the proposed protocol, DWS performs better than EJHS. Also DWS uses only half the number of relays and two thirds of the time spent by EJHS. Though the diversity obtained for EJHS is higher, the performance is below that of DWS in terms of BER. Diversity of EJHS is better because DWS does not transmit in third phase and uses only half of the relays used by EJHS. Also the performance of DWS is better because the relays of DWS use the receive channel knowledge $h_{s,i}$ while transmitting. An interesting result that emerges is that the second set of relays $R_2$ uses only half the number of relays and two thirds of the time spent by $R_1$.

In second phase can be written as

$$P_s^{(2)} = E \left[ \frac{1}{2} \sum_{i=1}^{N} \left| h_{i,j} \right|^2 \sum_{j=1}^{N} \left| h_{s,i} \right|^2 + E \left[ \left| \mathbf{u}_d^{(2)} \right|^2 \right] \right].$$

As the channel coefficients are all independent and zero mean, unless $i = j$, the above equations simplify to

$$P_s^{(2)} = \frac{c^2}{2} \sum_{j=1}^{N} \left[ \left| h_{i,j} \right|^2 \sum_{j=1}^{N} \left| h_{s,i} \right|^2 + E \left[ \left| \mathbf{u}_d^{(2)} \right|^2 \right] \right].$$

Similarly, $P_s^{(3)}$ and $P_n^{(3)}$ can be derived from (9) and (10) as

$$P_s^{(3)} = c^2 \sum_{j=1}^{N} \left[ \left| h_{i,j} \right|^2 \sum_{j=1}^{N} \left| h_{s,i} \right|^2 + E \left[ \left| \mathbf{u}_d^{(2)} \right|^2 \right] \right]$$

$$P_n^{(3)} = \frac{3}{2} \sum_{j=1}^{N} \left[ \left| h_{i,j} \right|^2 \sum_{j=1}^{N} \left| h_{s,i} \right|^2 + E \left[ \left| \mathbf{u}_d^{(2)} \right|^2 \right] \right].$$

The receive SNR is then

$$\text{SNR}_{DWS} = \frac{P_s^{(2)} + P_s^{(3)}}{P_n^{(2)} + P_n^{(3)}}$$

$$= \frac{16Nc^2\sigma^2_2 + 2N^2c^2_2(3\sigma^2_1 + \sigma^2_2) + 8N^2c^2_2\sigma^2_2}{2NTc^2\sigma^2_2 + 4T + N^2c^2_2T + N^2c^2_2T + Nc^2_2\sigma^2_2}.$$  \hspace{1cm} (A5)

Substituting in the above equation the values of $c_1$, $c_2$, and $c_3$ from section III we can simplify $\text{SNR}_{DWS}$ as

$$\text{SNR}_{EJHS} = \frac{P_s^{(2)}}{P_n^{(2)}}$$

$$= \frac{c^2_1N^2\sigma^2_2}{c^2_1N^2\sigma^2_2 + c^2_2NT + c^2_2 + T \sigma^2_3 \sum_{j=1}^{N} \left[ \left| h_{i,j} \right|^2 \sum_{j=1}^{N} \left| h_{s,i} \right|^2 + E \left[ \left| \mathbf{u}_d^{(2)} \right|^2 \right] \right] \sum_{j=1}^{N} \left[ \left| h_{i,j} \right|^2 \sum_{j=1}^{N} \left| h_{s,i} \right|^2 + E \left[ \left| \mathbf{u}_d^{(2)} \right|^2 \right] \right]}. \hspace{1cm} (A5)$$

B. RECEIVE SNR AT D - EJHS

As in Appendix A we can derive the receive SNR using the received vector $z$ shown in (15) as

$$\text{SNR}_{EJHS} = \frac{P_s^{(2)}}{P_n^{(2)}}$$

$$= \frac{c^2_1N^2\sigma^2_2}{c^2_1N^2\sigma^2_2 + c^2_2NT + T \sigma^2_3}$$

$$= \frac{1}{\frac{c^2_1N^2\sigma^2_2}{c^2_1N^2\sigma^2_2 + c^2_2NT + T \sigma^2_3} + 1 + p_1 + p_2 + p_3}. \hspace{1cm} (B1)$$

Substituting $p_1 = P - p_2 - p_3$ in (B1), applying the constraints $p_1, p_2, p_3 \geq 0$, and using the partial derivatives \[7\], it can be derived that $\text{SNR}_{EJHS}$ attains the maximum value of $P^2/9(3 + 3P + P^2)$ when $p_1 = p_2 = p_3 = \frac{1}{3}$. \hspace{1cm} (B2)
An MMSE Strategy at Relays with partial CSI for a Multi-layer Relay Network

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EDICS: WIN-APPL, WIN-CONT

Abstract

We consider a relay network with a single source-destination pair and multiple layers of relays between them. We assume that these layers sequentially relay the signal transmitted by the source to the destination. Unlike existing work, we also assume that the destination and all the forward layers present between the transmitting layer and the destination receive signals during every transmission phase. We optimally combine these signals, say \( \mu \) of them, using a precoder at each relay layer for onward transmission. We obtain this precoding matrix by minimizing the mean-squared error (MSE) at the relays, and do not require channel-state-information (CSI) of the forward channels at the relays unlike existing systems that minimize MSE at the destination and require CSI of the forward channels at the relays. Our closed-form solution for this matrix is valid for any \( K \) number of layers, whereas minimizing MSE at the destination does not have closed-form solution for \( K > 1 \). For \( K > 1 \), we enhance an existing scheme to obtain a sub-optimal closed-form precoder solution and use it for comparison. We show using simulations that our scheme approaches the bit-error-rate (BER) performance of this scheme, when \( \mu \) is increased, even with partial CSI.

Index Terms

MMSE, channel-state-information, multi-layer relay network, relay precoder, amplify-and-forward.

I. INTRODUCTION

Ing and Hassibi [1] proved that a spatially distributed network of single-antenna radio nodes can emulate a multiple-input multiple-output (MIMO) communication system. They showed that this creates a distributed space-time code and achieves the same diversity as that of a MIMO system at high total transmitted power. These authors also extended it to include multiple-antenna nodes in [2] and [3].

When multiple radio nodes are present, the effectiveness of cooperative communication [4], [5] can be increased by arranging these nodes into a layered architecture to relay information. Pottie and Kaiser [6] showed how a distributed and layered signal processing architecture can overcome the energy and bandwidth constraints in many wireless sensor network applications.

A relay can use a simple forwarding technique called amplify-and-forward (AF) [7], in which it amplifies what is received and transmits. Other well-known relaying methods include decode-and-forward (DF) (e.g., [8]), coded cooperation (e.g., [9]), compress-and-forward (e.g., [10], [11]), and partial-decode-and-forward (PDF) (e.g., [12]). Chiu et al. [13] designed the precoder for not only the relay but also the source, while using the PDF strategy at the relays. They used orthogonal space-time block coding [14] in both the source and the relay transmissions. Though other relaying protocols can perform better, the AF protocol is widely considered interesting because of its simplicity of implementation. In this paper, we restrict our attention to the AF protocol. In an AF multi-layer relay system, the performance of the system depends on the precoders used at the relays, and we consider the problem of designing such precoders.
Our system model consists of a set of single-antenna radio nodes grouped into $K$ layers of relays, $L_k$, $k \in [1, K]$, between a source and a destination (we will call the source S or $L_0$ and the destination D or $L_{K+1}$ in the sequel) as shown in Fig. 1. Earlier work involving multiple layers of relays use only the signal reaching at a particular layer $L_k$ from the preceding layer $L_{k-1}$ to construct the signals transmitted from $L_k$. On the contrary, in this paper, we construct the transmit signals at $L_k$ using signals that have reached from $L_j$, $j \in [0, k-1]$, although the signal from $L_j$, $j \in [0, k-2]$ reaches $L_k$ with lower power compared to that from $L_{k-1}$. Thus, we take advantage of the broadcast nature of the wireless medium by utilizing overhead signals. We call these low power overhead signals as leaked signals.

We will call the channel states of the channels from $L_{k-j}$ to $L_k$, available at $L_k$, to be backward CSI and the channel states of the channels from $L_k$ to $L_{k+j}$ as forward CSI. Here $j > 0$. Forward and backward CSI are together called the global CSI, and just the backward CSI is referred to as partial CSI. We note here that obtaining forward CSI requires either sending the estimated channels from the receiving nodes over reliable feedback channels, or direct estimation of these channels from the backward transmission (in a time-division duplex system). The feasibility of this depends on the application scenario.

Ding et al. [15] employed AF strategy at the relays, designed a unitary precoder, and achieved maximum diversity gain. The closed form expression of the precoder was extended to $M$-ary signals of larger constellation size in [16]. Gomadam and Jafar [17] obtained relay precoders by maximizing the receive signal-to-noise ratio (SNR) at the destination. Many authors [18]–[23] have considered minimizing the MSE at the destination (MMSE or MMSED) to obtain relay precoders using global CSI. This global optimization is challenging, and authors of [17] and [23] considered two layers of relays and obtained the precoder by iterative techniques, while the authors of [18]–[22] derived the optimal precoders in closed-form for a single layer of relays. The disadvantage in an iterative technique is that at any iteration, only an approximate solution is found. For real-time computation at the relays, the quality of the solution then depends on the processing speed of the relays. Further, to the best of our knowledge, iterative solutions are also available only for up to two layers of relays, and their generalization to more layers or incorporating leaked signals is challenging under the MMSED criterion.

In some systems, the relays may be able to exchange information amongst themselves before transmission. In such a system, the relays are said to be cooperative. Otherwise, the precoder matrix would be diagonal and the relays are said to be non-cooperative. All the literature discussed in the previous paragraph, showed the efficacy of the derived precoders when the relays are non-cooperative except [19], in which the authors derived the precoders for cooperative relays. For either cooperative and non-cooperative relays, to our knowledge, existing literature provides minimum MSE (MMSE) design of AF precoders:

- in closed form only when there is a single layer of relays, and in the form of iterative numerical solution when there are two layers,
• assuming that global CSI is available with the relays, and  
• without considering leaked signals.

All the above concerns are addressed in this paper. Unlike [18]–[23], which adopted MMSED, we minimize MSE at the relays (MMSER) and obtain relay precoder matrices. We show that the MMSER strategy makes the optimal precoder design a layer-wise optimization as opposed to a global optimization. This yields optimal precoders in closed form for arbitrary number of relay layers even when leaked signals are used, and it requires partial CSI, i.e., only the backward CSI. In the absence of optimum MMSED precoder solution for more than two layers of relays, we have proposed some suboptimal MMSED precoding schemes, and we show using simulation that despite the lack of forward CSI, our MMSER precoding strategy outperforms/approaches the performance of these MMSED strategies by using more leaked signals.

A. Contribution

Our contributions in this paper are:

• For the multi-layer relay network shown in Fig. 1, we propose a novel MMSER relaying strategy, which does not require forward CSI in the transmitting nodes.

• We obtain closed form solutions for the optimum MMSER relay precoders for both cooperative and non-cooperative relays for arbitrary number of layers of relays while also including leaked signals.

• We enhance the MMSED strategies (though these enhancements may not be optimal) proposed in [18], [19], and [21] to work in this multi-layer network for meaningful comparison with MMSER.

• We show using simulations that combining more number of leaked signals improves the performance of MMSER, which outperforms/approaches that of MMSED schemes that use global CSI.

B. Notation and Organization

$I_N$ denotes the $N \times N$ identity matrix and $i_N$ is the vector of $N$ elements $[1, 1, \cdots, 1]^T$. For $x \in \mathbb{Z}$, $[x]^+$ denotes zero if $x \leq 0$ and $x$ if $x > 0$. $\xi \sim \mathcal{CN}(0, \sigma^2)$ represents a circularly symmetric complex Gaussian random variable with real and imaginary parts having mean 0 and variance $\sigma^2/2$. $\text{diag}[x_1, \cdots, x_N]$ is a diagonal matrix with diagonal elements $x_i$, $i \in [1, N]$.

The remainder of this paper is organized as follows. In Section II, we state the problem and introduce the MMSER strategy. Thereafter, in Section III, the MMSER strategy is presented in detail and the precoders for the relay layers are derived. Section IV gives details on how we extend and enhance the MMSED strategy, so that the BER performance of MMSER can be meaningfully compared. In Section V, we describe the equalizer that is used at destination D for MMSER and MMSED schemes to decode the received vector. In Section VI, we present simulation results. Finally, in Section VII, we summarize and conclude the paper.

II. SYSTEM MODEL AND THE PROPOSED SCHEME

In our system model shown in Fig. 1, $R_{i,m}$ denotes the $m$th relay in the $i$th layer and we assume that it is half-duplex, $\forall i, m$. We use $\ell$ and $h$ to represent the links and the channel coefficients respectively, with the first two subscripts denoting the transmitter and the next two the receiver. Therefore, the channel coefficients of the links, $\ell_{0,i,m}$ from $S \to R_{i,m}$, $\ell_{i,m,j,n}$ from $R_{i,m} \to R_{j,n}$, and $\ell_{j,n,K+1}$ from $R_{j,n} \to D$ are denoted as $h_{0,i,m}$, $h_{i,m,j,n}$, and $h_{j,n,K+1}$ respectively. Let $h_{0,i,j} \in \mathbb{C}^{N \times 1}$, $h_{i,j} \in \mathbb{C}^{N \times N}$, and $h_{j,K+1} \in \mathbb{C}^{1 \times N}$ represent the vectors/matrices of channel coefficients from S to $L_i$, $L_j$ to $L_j$, and $L_j$ to D respectively. We assume that the channels are Rayleigh fading and quasi static.

Let $L$ be the set of all links and $L_{i,j} \subset L$, $i < j$ be the set of links from $L_i$ to $L_j$. As an example, for $i \in [1, K-1]$, $j \in [i+1, K]$, the link set $L_{i,j}$ is given by

$$L_{i,j} = \{\ell_{i,1,j,1}, \cdots, \ell_{i,N,j,N}\}.$$  

Let us define the length of the link $\ell_{i,m,j,n}$ as $\rho_{ij} \triangleq j-i$, or simply by $\rho$ when the layers are understood from the context. All the links in link set $L_{i,j}$ have the same length $\rho_{ij} = j-i$.

Now, let $\mathcal{L}_\rho$ be a class of subsets $L_{i,j}$ of $L$ with length $\leq \rho$. Clearly, $\mathcal{L}_j \subset \mathcal{L}_j$, if $i < j$. As an example, link class $\mathcal{L}_2$ is given by

$$\mathcal{L}_2 = \{L_{0,1}, L_{0,2}, L_{1,2}, L_{1,3}, \cdots, L_{K-1,K+1}, L_{K,K+1}\}.$$
TABLE I
RECEIVED AND TRANSMITTED VECTORS - RELAY LAYERS

<table>
<thead>
<tr>
<th>Layer</th>
<th>Received vector</th>
<th>Transmitted vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lₖ</td>
<td>(r_k^{(0)} = h_{0,k}t_0 + u_k^{(0)})</td>
<td>(t_k = F_kr_k), where</td>
</tr>
<tr>
<td>1 ≤ k</td>
<td>(r_k^{(i)} = h_{i,k}t_i + u_k^{(i)})</td>
<td>(r_k^T = [r_k^{(1)T}, \ldots, r_k^{(k-1)T}])</td>
</tr>
<tr>
<td>≤ K</td>
<td>∀ (i \in [n, k-1]) and (i ≠ 0).</td>
<td>with (n = [k - µ]^+).</td>
</tr>
</tbody>
</table>

The proposed MMSER-µ strategy

Let us consider the \(K+1\) hop network shown in Fig. 1. In the MMSER-µ (\(µ \in [1, K+1]\)) strategy, S transmits in phase 0, and L₁, L₂, · · · , Lₚ receive and store for later use; L₁ transmits in phase 1, and L₂, · · · , Lₚ+1 receive and store for later use and so on till phase K, when D receives from Lₚ.K. To be specific, in MMSER-µ, the relays and D store all signals received through the link sets in \(\mathcal{L}_µ^+\) for later use. For \(µ_1 < µ_2\), MMSER-µ₂ would use more number of leaked signals, and thus is expected to perform better than MMSER-µ₁.

We assume synchronous reception and transmission at the relay nodes and all noise signals added at the receiver front-ends are complex zero-mean independent and identically distributed (i.i.d.) Gaussian random variables with variance \(σ_u^2\).

Let us denote the transmitted, received, and noise signals by \(t, r, \) and \(u\) respectively with subscript and superscript on them denoting layer and phase respectively. Let the signal transmitted by S be \(t_0 = √p_0s\) with an average power of \(p_0\) Watts, where \(s \in \mathbb{C}\) is a unit variance constellation point. In various phases, the layer Lₖ would have received \(k-n\) vectors each of size \(N\) from previous transmissions, starting from phase \(n\) till phase \(k-1\), where \(n = [k-µ]^+\). All these vectors are stacked together as given in Table I to form the overall received vector \(r_k \in \mathbb{C}^{(k-n)N×1}\).

For example, let us take \(K = 8\) and \(µ = 3\). Here, \(n = [k-3]^+ = 0\) for \(k = 2\) and \(n = [k-3]^+ = 4\) for \(k = 7\). Hence, L₂ and L₇ would have the overall received vectors

\[
r_2 = \begin{bmatrix} r_2^{(0)} \\ r_2^{(1)} \end{bmatrix} \quad \text{and} \quad r_7 = \begin{bmatrix} r_7^{(4)} \\ r_7^{(5)} \\ r_7^{(6)} \end{bmatrix}
\]

respectively. Also \(r_2 \in \mathbb{C}^{2N×1}\) and \(r_7 \in \mathbb{C}^{3N×1}\).

The stacked received vector is transmitted by Lₖ relays, after precoding with \(F_k\) in phase \(k\) as shown in Table I. The precoder matrix \(F_k \in \mathbb{C}^{N×(k-n)N}\) at Lₖ is given by

\[
F_k = [F_{k,n}, \ldots, F_{k,k-1}], \quad (1)
\]

where \(F_{k,i} \in \mathbb{C}^{N×N}\) with \(n ≤ i ≤ k-1\) and \(n = [k-µ]^+\). The precoder submatrices \(F_{k,i}\) can be selected to be non-diagonal or diagonal depending upon whether the relays would cooperate or not respectively.

In phases \(i = K-µ+1\) to \(i = K\), D receives \(r_{K+1}^{(i)} = h_{i,K+1}t_i + u_{K+1}^{(i)}\). If \(K-µ+1 = 0\), then it receives \(r_{K+1}^{(0)} = h_{0,K+1}t_0 + u_{K+1}^{(0)}\) from S directly in phase 0.

Now, we define the cost function at the relay layer Lₖ to be the MSE

\[
J_k \triangleq E \left[ ||s - t_k||^2 \right], \quad (2)
\]

where \(s = [s, \ldots, s]^T \in \mathbb{C}^{N×1}\), \(t_k = F_kr_k\) as given in Table I and \(E[.]\) is the expectation operator. The relays then transmit a scaled version of the solution to meet the layer-wise power constraint. The cost function given in (2) is motivated by the following facts:

- Somewhat similar to the principle behind regenerative relaying (DF), the relays are desired to transmit a signal that is close to the source symbol or its scaled version.
- The otherwise complex optimization problem (with MMSED criterion) is replaced by a smaller layer-wise optimization problem which, as we will see, yields a solution in closed form.
- The requirement of only backward CSI gives an added practical advantage.

Now, our aim is to minimize \(J_k\) under the power constraint \(E [ t_k^H t_k ] ≤ p_k\) and find the precoder matrices \(F_k, \forall k \in [1, K]\).
III. MMSER PRECODER

Khajehnouri and Sayed [18] minimized the MSE

\[ J_D = E \left[ |s - h_{1,2}t_1|^2 \right] \]

at the destination and found the precoder matrix for L₁, when K = 1 with no power constraint. Here, \( h_{1,2} \in \mathbb{C}^{1 \times N} \) and \( t_1 \in \mathbb{C}^{N \times 1} \). Krishna et al. [19] derived a non-diagonal precoder matrix for cooperative relays with average power constraint \( p_1 \). To compare with their results, these authors modified the \( i^{th} \) diagonal element of the precoder matrix to restrain power in the Khajehnouri-Sayed scheme [18] as

\[ f_{1,i} = \frac{p_1^2 h_{0,1,i}^* h_{1,i,2}}{|h_{1,i,2}|^2 \left[ \sum_{j=1}^{N} |h_{0,1,j}|^2 \left( p_0 |h_{0,1,j}|^2 + \sigma_u^2 \right) \right]^{\frac{1}{2}}} \]  

(3)

for non-cooperative relays in their equation (24). Lee et al. [20] obtained \( f_{1,i} \) using constrained optimization [24] for non-cooperative relays as

\[ f_{1,i} = \frac{p_1^2 h_{0,1,i}^* h_{1,i,2}}{\left[ \sum_{j=1}^{N} |h_{0,1,j}|^2 \left| h_{1,i,2} \right|^2 \left( p_0 |h_{0,1,j}|^2 + \sigma_u^2 \right) \right]^{\frac{1}{2}}} \]  

(4)

where we have removed the uncertainty channel terms to match the scope of this paper. In both (3) and (4), we have changed symbol notation to be consistent with this paper.

Let us call these non-cooperative systems, which use (3) and (4), as MMSED-Khajehnouri/ Krishna (MMSED-KK) and MMSED-Lee (MMSED-L) respectively. We also call the cooperative system proposed by Krishna et al. [19] as MMSED-Krishna (MMSED-K). We note that, both MMSED-KK and MMSED-L require \( h_{1,i,2} \) or forward CSI at the relay \( R_{1,i} \), \( i \in [1, N] \) from (3) and (4) respectively.

In our strategy, we minimize the MSE at the relays resulting in precoders that do not depend on forward CSI, and use leaked signals to improve the performance. Let us now derive the precoders for MMSER-\( \mu \) for any \( K \).

A. MMSER precoder matrix at \( L_k \), \( k \in [1, K] \)

In our proposed MMSER scheme, each layer obtains an estimate of the signal vector \( s \). This estimate or a scaled version of this estimate can be transmitted, subject to the sum transmit power constraint for each layer of relays.

Expanding the expression of the MSE given in (2), we get

\[ J_k(\mathbf{F}_k) = E \left[ (s - \mathbf{F}_k r_k) \right]^H (s - \mathbf{F}_k r_k) \]  

(5)

Now, the estimate is obtained by finding the optimum \( \mathbf{F}_k \) given by \( \hat{\mathbf{F}}_k = \arg \min_{\mathbf{F}_k} J_k(\mathbf{F}_k) \), subject to the constraint \( E \left[ t_k^H t_k \right] \leq p_k \). We write the constraint function as

\[ C_k(\mathbf{F}_k) = E \left[ t_k^H t_k \right] - p_k \leq 0 \]  

(6)

and let \( D = \text{domain}(\mathbf{F}_k) \cap \text{domain}(C_k) \). This problem is an MMSE estimation problem with a convex constraint on the estimate. It is a convex optimization problem with a unique solution as discussed in [25].

Now, given the optimization variable \( \mathbf{F}_k \in \mathbb{C}^{N \times (n-k)N} \), the cost function \( J_k : \mathbb{C}^{N \times (n-k)N} \to \mathbb{R} \), and the inequality constraint function \( C_k : \mathbb{C}^{N \times (n-k)N} \to \mathbb{R} \), we define the Lagrangian \( L_k : \mathbb{C}^{N \times (n-k)N} \times \mathbb{R} \to \mathbb{R} \) as

\[ L_k(\mathbf{F}_k, \lambda_k) = J_k(\mathbf{F}_k) + \lambda_k C_k(\mathbf{F}_k) \]  

(7)

where \( \lambda_k \geq 0 \) is the Lagrange multiplier and the domain of \( L_k = D \times \mathbb{R} \).

Claim 1. (1) When the relays cooperate, the optimum precoder matrix \( \hat{\mathbf{F}}_k \) is given by

\[ \hat{\mathbf{F}}_k = \frac{p_k^\frac{1}{2}}{\left[ \text{Tr} \left[ \mathbf{R}_{sr_k}^H \mathbf{R}_{sr_k} \mathbf{R}_{r_k}^{-1} \right] \right]^\frac{1}{2}} \mathbf{R}_{sr_k} \mathbf{R}_{r_k}^{-1} \]  

(8)
where
\[ \mathbf{R}_{sr_k} = E[\mathbf{s}\mathbf{r}^H_k] \text{ and } \mathbf{R}_{r_k} = E[\mathbf{r}_k\mathbf{r}^H_k]. \]

(2) When the relays do not cooperate, the optimum precoder vector \( \mathbf{f}_{k,l} \) is given by
\[
\hat{\mathbf{f}}_{k,l} = \frac{\frac{1}{p_k}}{\sum_{l=1}^{N} \text{Tr} (\mathbf{Y}^s_{kl} \mathbf{Y}^{-1}_{kl})} \mathbf{Y}^s_{kl} \mathbf{Y}^{-1}_{kl}, \tag{9}
\]
where
\[
\mathbf{f}_{k,l} = [f_{k,n,l}, \ldots, f_{k,k-1,l}],
\]
\[
\mathbf{Y}^s_{kl} = [\gamma_s^{(n)}_{kl}, \ldots, \gamma_s^{(k-1)}_{kl}],
\]
and
\[
\mathbf{Y}_{kl} = \begin{bmatrix}
\gamma_{(n)(n)}_{kl} & \cdots & \gamma_{(n)(k-1)}_{kl} \\
\vdots & \ddots & \vdots \\
\gamma_{(k-1)(n)}_{kl} & \cdots & \gamma_{(k-1)(k-1)}_{kl}
\end{bmatrix}, \quad l \in [1, N]. \tag{12}
\]

Here \( f_{k,i,l} \), \( \gamma_s^{(j)}_{kl} \), and \( \gamma^{(i,j)}_{kl} \) represent the \( l^\text{th} \) diagonal elements of \( \mathbf{F}_{k,i} \), \( \mathbf{R}_{sr_k}^{(i)} = E[\mathbf{s}\mathbf{r}^H_k] \), and \( \mathbf{R}_{r_k}^{(i)} = E[\mathbf{r}_k\mathbf{r}^H_k] \) respectively.

Proof: See Appendix A.

We notice the similarity of (8) and (9), where \( \mathbf{R}_{sr_k} \) and \( \mathbf{R}_{r_k} \) are analogous to \( \mathbf{Y}^s_{kl} \) and \( \mathbf{Y}_{kl} \) respectively, except for the extra summing operator in the denominator in (9).

For the non-cooperative case, the optimum precoder \( \mathbf{F}_k \), is made from the optimum vectors, \( \mathbf{f}_{k,l} \), by noting that these vectors give the \( l^\text{th} \) diagonal elements of all submatrices, \( \mathbf{F}_{k,i}, \quad i \in [n,k-1], \quad n = [k-\mu]^+ \), that make up the precoder.

The sum transmit power constraint at each layer allows for optimal allocation of power among the relays depending on the quality of the estimate at each relay. Specializing equation (8) to layer 1, we get
\[
\hat{\mathbf{F}}_1 = \frac{\frac{1}{p_1}}{N \left( p_0 \| \mathbf{h}_{0,1} \|^2 + \sigma^2 \right)} \mathbf{i}_N \mathbf{h}_{0,1}^H \tag{13}
\]
as \( \mathbf{R}_{sr_k} = \frac{1}{p_0} \mathbf{i}_N \mathbf{h}_{0,1}^H \) and \( \mathbf{R}_{r_k} = p_0 \mathbf{h}_{0,1} \mathbf{h}_{0,1}^H + \sigma^2 \mathbf{I}_N \) from Table I. From (13), we can observe that the relays with better backward channels are allocated more power.

B. Information required for MMSER precoders

From (8) and (9), we can see that the precoders of MMSER depend on two correlation matrices \( \mathbf{R}_{sr_k} \) and \( \mathbf{R}_{r_k} \). From Table I, we see that, these matrices depend on \( \mathbf{H}_{i,k}, \forall i \in [n,k-1], \quad i \neq 0 \) (if \( i = 0 \), then on \( \mathbf{h}_{0,k} \)), and the precoder matrices \( \hat{\mathbf{F}}_i \) for \( i \in [n,k-1] \). Since \( \hat{\mathbf{F}}_1 \) (see equation(13)) depends only on backward CSI at L1, the information required to construct \( \hat{\mathbf{F}}_k \) at L_k is also the backward CSI at L_k. Therefore, MMSER-\( \mu \) does not require forward CSI.

IV. EXTENSION OF MMSED SCHEMES

Derivation of precoders for MMSED schemes, MMSED-KK and MMSED-L (equations (3) and (4) give the diagonal elements of the precoders of these schemes for \( K = 1 \), is complicated when \( K > 1 \). The system in [23] has two layers, i.e., \( K = 2 \), and the precoders are obtained iteratively. In this Section, we enhance the MMSED strategies (though these enhancements may not be optimal) proposed in [18], [19], and [21] to obtain closed-form solutions to precoders in a multi-layer network for meaningful comparison with the MMSER-\( \mu \) system proposed in this paper. We call these systems E-MMSED, for Enhanced-MMSED systems, and find their precoders to be dependent on global CSI as shown in (17) and (18).
A. E-MMSED Strategy

For a meaningful comparison, we consider the total number of phases as \( K + 1 \), and the total power transmitted as \( P \) to be the same as that used by MMSER. We assume: \( S \) transmits \( K_0 \) times in as many phases; all the relays average their received signals and transmit \( K_1 = K - K_0 + 1 \) times in as many phases. Thus, \( D \) would have a vector of \( K_1 \) received signals.

\( S \) transmits \( t'_0 = \sqrt{p_0} s \) repeatedly \( K_0 \) times from phase zero to phase \( K_0 - 1 \), and the relays follow suit transmitting a signal vector \( t \), which is explained later, from phase \( K_0 \) to \( K \). For each of these transmissions, when \( S \) transmits or the relays transmit, the channel does not vary as we assume a slow varying channel. Hence, the average power \( p'_0 \) can be equally divided in various phases when \( S \) transmits and \( p'_r \) when the relays transmit. Thus, \( p_0 = p_0/K_0 \) Watts and \( p'_r = p_r/K_1 \) Watts respectively, with \( p_0 + p_r = P \), the total power available.

The relays in all the layers would receive in phase \( k \), \( k \in [0, K_0 - 1] \), a vector \( r^{(k)}(k) \in \mathbb{C}^{KN \times 1} \) given by

\[
r^{(k)} = t'_0 h_0 + u^{(k)}, \quad k \in [0, K_0 - 1],
\]

where \( h_0 = \begin{bmatrix} h_{0,1} \\ \vdots \\ h_{0,K} \end{bmatrix} \) and \( u^{(k)} = \begin{bmatrix} u^{(k)}_1 \\ \vdots \\ u^{(k)}_K \end{bmatrix} \)

with \( u^{(k)}_i = [u^{(k)}_{i,1} \cdots u^{(k)}_{i,N}]^T, \quad i \in [1, K] \).

We assume that all the relays transmit together in their transmission phases, as though they are in a single layer. We also assume that the noise is uncorrelated, i.e.,

\[
E \left[ u^{(k)}(k)u^{(l)H} \right] = \sigma^2_u \delta(k - l) I_{KN},
\]

where \( \delta(m) = 1 \) when \( m = 0 \) and \( \delta(m) = 0 \) when \( m \neq 0 \) is the Kronecker delta function.

The relays average all the signals received and repeat transmission of the signal \( t = Fr_{av} \), in phases \( K_0 \) to \( K \), where

\[
r_{av} = \frac{1}{K_0} \sum_{i=0}^{K_0-1} r^{(i)} = t'_0 h_0 + \frac{1}{K_0} \sum_{i=0}^{K_0-1} u^{(i)}
\]

from (14). As \( F \in \mathbb{C}^{KN \times KN} \) is a diagonal precoder matrix, let us define it as \( F \triangleq \text{diag}[f_{1,1}, \ldots, f_{1,N}, f_{2,1}, \ldots, f_{2,N}, \ldots, f_{K,1}, \ldots, f_{K,N}] \), where \( f_{i,m} \) is the multiplying factor of the relay \( R_{i,m} \).

From (15), \( t = Fr_{av} \) becomes

\[
t = t'_0 Fh_0 + \frac{F}{K_0} \sum_{i=0}^{K_0-1} u^{(i)},
\]

which is transmitted \( K_1 \) times, so that the total number of phases would be \( K + 1 \), the same as that of MMSER.

Now, we will derive precoders for enhanced MMSED-KK (E-MMSED-KK) and enhanced MMSED-L (E-MMSED-L) schemes.

1) Precoder of E-MMSED-KK: We take (3) and replace \( p_0 \), the power transmitted by \( S \), with \( p_0/K_0 \) and \( p_1 \), the power transmitted by the relays, with \( p_r/K_1 \), as we allocate fractions of powers to them due to their multiple transmissions. Further, the noise variance \( \sigma^2_u \) is replaced by \( \sigma^2_u/K_0 \), since the noise variance at each of the relays \( R_{i,m}, \quad i \in [1, K], \quad m \in [1, N] \) after averaging over the \( K_0 \) phases (in (15)) is \( \sigma^2_u/K_0 \). Therefore, we get \( f_{i,m}, \quad i \in [1, K] \) and \( m \in [1, N] \), a diagonal element of the precoder of E-MMSED-KK as

\[
f_{i,m} = \frac{p_{i,m}K_1}{K_0} \frac{h_{0,i,m}^* h_{i,m,K+1}^2}{\left( \sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,k,l}|^2 + \sigma^2_u \right)^{1/2}}.
\]
2) Precoder of E-MMSED-L: Similarly, to get the diagonal elements of the precoder of E-MMSED-L, we replace the signal power transmitted and noise variance as in E-MMSED-KK into (4) to get its diagonal element $f_{i,m}$ as

$$f_{i,m} = \left[ \frac{p_i K_0}{K_1} \right]^\frac{1}{2} h_{i,m,K+1}^r \left( \sum_{k=1}^{K} \sum_{i=1}^{N} |h_{0,k,i}|^2 |h_{k,i,K+1}|^2 \left( p_0 |h_{0,k,i}|^2 + \sigma^2_0 \right) \right]^\frac{1}{2}. \tag{18}$$

We note that in both (17) and (18), we have a double summation in the denominator instead of a single summation, when $K$ is greater than 1. Also, we see that in both cases, the relay $R_{i,m}$ needs forward CSI $h_{i,m,K+1}$.

B. Selection of $K_0$

Let us now select the best $K_0$ and use it while comparing its performance with MMSER-$\mu$ system. As it is hard to derive BER, we obtain SNR at $D$ for any $K_0$ and attempt to select the value of $K_0$ that maximizes it.

In phase $k$, $k \in [K_0,K]$, $D$ receives a scalar

$$r_{K+1}^{(k)} = h_{K+1} r + u_{K+1}^{(k)}, \tag{19}$$

where $h_{K+1} = [h_{1,K+1}, \ldots, h_{K,K+1}] \in \mathbb{C}^{1 \times KN}$. Substituting (16) into (19), we get

$$r_{K+1}^{(k)} = h_{K+1} \left[ t_0 F h_0 + \frac{K_0-1}{K_0} \sum_{i=0}^{K_0-1} u^{(i)} \right] + u_{K+1}^{(k)} - r_{sig}^{(k)} + r_{noi}^{(k)} + u_{K+1}^{(k)},$$

where

$$r_{sig}^{(k)} = \left[ \frac{p_0}{K_0} \right]^\frac{1}{2} h_{K+1} r_0 h_0 s$$

are respectively, the signal and noise components of the received signal in phase $k$, without considering the noise that is added at $D$. We do not take the noise added at $D$ with these components, as it is not considered while deriving optimum precoder in MMSED. Now, we concatenate these components in $D$ into vectors as $r_{sig} = [r_{sig}^{(K_0)} \ldots r_{sig}^{(K+1)}]^T$ and $r_{noi} = [r_{noi}^{(K_0)} \ldots r_{noi}^{(K+1)}]^T$.

The signal and noise powers from these vectors are defined as

$$P_S \triangleq E \left[ r_{sig}^{H} r_{sig} \right] = E \left[ \sum_{k=K_0}^{K} \left| r_{sig}^{(k)} \right|^2 \right] \tag{21}$$

and

$$P_N \triangleq E \left[ r_{noi}^{H} r_{noi} \right] = E \left[ \sum_{k=K_0}^{K} \left| r_{noi}^{(k)} \right|^2 \right] \tag{22}$$

respectively.

Claim 2. The ratio $P_S/P_N$ is given by

$$P_S = \frac{p_0 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,i,j}|^4}{\sigma^2_0 \sum_{k=1}^{K} \sum_{i=1}^{N} |h_{0,k,i}|^2}, \tag{23}$$

and

$$P_S = \frac{p_0 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{i,j}|^4 |h_{i,j,K+1}|^4}{\sigma^2_0 \sum_{k=1}^{K} \sum_{i=1}^{N} |h_{0,k,i}|^2 |h_{k,i,K+1}|^4}. \tag{24}$$

for E-MMSED-KK and E-MMSED-L respectively.

Proof: See Appendix B.

From Claim 2, i.e., from (23) and (24), we see that we can select any $K_0$ from $[1,K]$ for both E-MMSED-KK and E-MMSED-L and these ratios do not vary. This also reflects in the BER plots shown in Section VI in Fig. 4 that for different values of $K_0$, BER does not change.
V. DECODER AT THE DESTINATION

The signal received at the destination is given by

$$r_D = \begin{bmatrix} r_{D_1} \\ \vdots \\ r_{D_m} \end{bmatrix},$$

(25)

where \(m\) is the total number of signals \(D\) has received in as many number of phases. For MMSER-\(\mu\), \(m = \mu\), \(r_{D_1} = r_{0}^{(K+1-\mu)}\), and \(r_{D_m} = r_{0}^{(K)}\). For E-MMSED schemes, \(m = K_1 = K - K_0 + 1\) as seen in section IV, \(r_{D_1} = r_{K+1},\) and \(r_{D_m} = r_{K+1}\). Let us write the decoded signal to be

$$\hat{s} = f_D r_D$$

(26)

where \(f_D \in \mathbb{C}^{1 \times m}\) is the decoder vector to be obtained by minimizing the MSE, \(J_D \triangleq E|s - \hat{s}|^2\). The optimum \(f_D\) can be obtained as

$$\hat{f}_D = \mathbf{R}_{s'r_D} \mathbf{R}_r^{-1},$$

(27)

where

$$\mathbf{R}_{s'r_D} = E[sD^H] \in \mathbb{C}^{1 \times m} = [R_{s'r_{D_1}} \ldots R_{s'r_{D_m}}]$$

(28)

and

$$\mathbf{R}_r = E[r_D r_D^H] \in \mathbb{C}^{m \times m} = \begin{bmatrix} R_{r_{D_1}r_{D_1}} \ldots R_{r_{D_1}r_{D_m}} \\ \vdots \\ R_{r_{D_m}r_{D_1}} \ldots R_{r_{D_m}r_{D_m}} \end{bmatrix}. $$

(29)

We note that, prime (') is used over \(s\) in correlation matrix \(\mathbf{R}_{s'r_D}\) to identify it to be a scalar unlike in (A2).

We use the decoder found in (27) in our simulations for both MMSER-\(\mu\) and E-MMSED systems in Section VI.

VI. SIMULATION RESULTS

In this Section, we compare the BER performance of the proposed MMSER scheme with existing schemes and our enhanced E-MMSED schemes. We run Monte-Carlo simulations for both the cooperative and non-cooperative relays cases. First, we study the behavior of MMSER, E-MMSED-KK, and E-MMSED-L by varying the following parameters: total number of layers, \(K\), number \(K_0\) of transmissions by \(S\), and power \(p_0\) allocated to \(S\). This is used to analyze the performance of MMSER and MMSED schemes and select the best values of \(K_0\) and \(p_0\) for E-MMSED in the simulations.

A. Simulation parameters

We select \(s\) from the Gray coded quadrature phase shift keying constellation with unit variance and use MMSE decoders at \(D\), derived in Section V. In all the plots, we use SNR = 10\log (1/\sigma_u^2) on the \(x\)-axis, where \(\sigma_u^2\) is the variance of the noise added at the receivers.

For the simulations, we incorporate the signal-power path loss as in [26]; i.e., if \(d_{ij}\) is the distance from \(L_i\) to \(L_j\), then the channel coefficient \(h_{i,m,j,n}\) of the link \(\ell_{i,m,j,n}\) is given by

$$h_{i,m,j,n} = \frac{\xi_{i,m,j,n}}{d_{ij}^\beta},$$

where \(\{\xi_{i,m,j,n} \in \mathbb{C} : i, j \in [1,K], m, n \in [1,N]\}\) is an i.i.d. collection of random variables with \(\xi_{i,m,j,n} \sim \mathcal{N}(0, 1)\) and \(\beta\) is the path loss exponent. Hence, the variance of the channel coefficient \(h_{i,m,j,n}\) is given by \(1/d_{ij}^{2\beta}\).

Let us assume that the layers are equi-spaced, and hence the distance between any two layers \(L_i\) and \(L_j\) is \(d_{ij} = (j-i)d/(K+1)\), where \(d\) is the distance between \(S\) and \(D\). Let \(d = 1\). Then, the channel variance of the link \(h_{i,m,j,n}\) is \(\text{Var}[h_{i,m,j,n}] = 1/d_{ij}^{2\beta} = (K+1)^\beta/\rho_{ij}^{\beta}\), where \(\rho_{ij} = j-i\), and \(\beta = 2\). Though this special spatial...
structure is not required for operating our strategy, we use it for the simulations. More general scenarios can be considered, as we do not restrict the distance or the channel variance of the links in any manner, while deriving the optimum MMSER precoders. In all the simulations, whenever we need to increase the number of layers $K$, we do it by inserting them between $S$ and $D$ keeping the distance $d_{0,K+1} = d$ constant.

For $K = 1$, Jing and Hassibi [1] proved that Jing-Hassibi Scheme (JHS) achieves maximum SNR at $D$ when power is equally divided between $S$ and the relays; i.e., $p_0 = p_1 = P/2$. For $K = 2$, we extended JHS (EJHS) [27] and proved that EJHS achieves maximum SNR at $D$ when power is equally divided amongst $S$, $L_1$ and $L_2$; i.e., $p_0 = p_1 = p_2 = P/3$. Extending this for any $K$, we use $p_0 = P/(K + 1)$ in all simulations for EJHS.

B. Summary of results

– Figs. 2 and 3 show respectively how the performance of MMSER-1 worsens and that of MMSER-2 improves as $K$ increases.

– Figs. 4 and 5 show performance of E-MMSED-KK and L as a function of $K_0$ and $p_0$. This is used to select the best $K_0$ and $p_0$ for comparison with MMSER-$\mu$.

– Fig. 6 shows a comparison of BERs of MMSER-1 and MMSER-2 with MMSED for the single-layer case ($K = 1$), when the relays cooperate.

– Figs. 7 and 8 show that the BER performance of MMSER outperforms that of E-MMSED-KK and approaches that of E-MMSED-L when $\mu$ is increased from 1 to $K + 1$ with $K = 3$ and 4 respectively, when the relays do not cooperate.

![Fig. 2. Plots of SNR and BER of MMSER-1 for varying $K$, the number of layers with total power $P = 1$ Watt.](image)

C. Usefulness of leaked signals, $\mu > 1$

Fig. 2 shows SNR at $D$ and BER plots of MMSER-1 when $K$ is varied. It can be observed that as $K$ increases, the SNR at $D$ decreases and BER of MMSER-1 increases. Fig. 3 shows BER plots of MMSER-2 for varying $K$. Unlike MMSER-1, the performance of MMSER-2 improves as the number of layers increases. This is because MMSER-2 uses leaked signals.

D. Selection of $K_0$ and $p_0$ for E-MMSED-KK and L

Fig. 4 shows the performance of E-MMSED-KK and E-MMSED-L, for various values of the number $K_0$ of transmissions of $S$. As was shown in (23) and (24), the BER plots also corroborate the fact that the performance does not vary with $K_0$. Hence, we use $K_0 = 1$ in all the simulations of E-MMSED.

Another parameter that needs to be fixed is the power allocated to $S$ $p_0$, and the relays $p_r = P - p_0$. We find these using simulations as shown in Fig. 5, where we have used $K = 4$, $N = 2$, and $P = 1$ Watt. For E-MMSED-KK, it can be seen that 20% of total power $P$ or $p_0 = P/(K + 1) = P/5$ achieves low BER for SNR $\leq 10$ dB and almost same BER for SNR $>10$ dB than other power allocations. Similarly, $p_0 = P/2$ or 50% of total power achieves lowest BER for E-MMSED-L. Hence, we use these values for $p_0$ in all subsequent simulations for E-MMSED.
Fig. 3. BER plots of MMSER-2 for varying $K$, the number of layers with total power $P = 1$ Watt. Unlike MMSER-1, BER performance improves as $K$ is increased due to the use of leaked signals.

Fig. 4. E-MMSED-KK and E-MMSED-L showing same BER performance for varying $K_0$. Total power used in simulations is $P = 1$ Watt.

Fig. 5. Search for optimum power allocation for MMSED. Shows that when power is equally distributed to S and layers, BER performance of E-MMSED-KK is the best and when 50% of power is allocated to S, E-MMSED-L attains best BER performance. Total power used in simulations is $P = 1$ Watt.
E. Comparison: Single layer case

Fig. 6 shows plots of BER in a single layer network, when relays cooperate. We have used $P = 2$ Watts for these simulations. We compare the performance of MMSER-1, MMSER-2, MMSED-K, and MMSED-K with leaked signals. For MMSED-K, we have generated the plot using the equation (20) derived by Krishna et al. in [19]. We have also incorporated leaked signals in the MMSED-K scheme to obtain the MMSED-K with leak scheme.

MMSED schemes use global CSI optimally while MMSER-µ schemes use backward CSI alone. When there is only one relay ($N = 1$), MMSER-1 performs exactly same as that of MMSED. The performance of MMSER-2 and MMSED-K with leaked signals are also identical. This is a case where there is no advantage with forward CSI in the MMSED schemes. However, the use of leaked signals helps MMSER-2 and MMSED-K with leaked signals perform better than the MMSER-1 and MMSED-K schemes.

When there are 2 relays, the MMSED scheme performance improves significantly because forward CSI can be used to achieve beamforming gain in the transmission from the relay layer to the destination. In this case, MMSER-2 is able to bridge a significant part of the gap between MMSER-1 and MMSED using the leaked signal from S at D. The MMSED-K scheme with leaked signals further improves upon the MMSED-K scheme.

![Cooperative relays performance comparison](image)

Fig. 6. Cooperative relays performance comparison. Performances of MMSER-1 and MMSED-K are the same when $N = 1$. Similarly, the performance of MMSED-K ‘with leak’ is the same as that of MMSER-2. For $N = 2$, the performance of MMSED-K is better than MMSER-1 and MMSER-2 schemes. Total power used in simulations is $P = 2$ Watts.

F. Comparison: Multi-layer case

Finally, we consider 3 and 4 layer systems with number of relays in each layer to be $N = 2$ and a total transmitted power of $P = 1$ Watt for comparing BER performance of the proposed system MMSER, with E-MMSED systems. Ideally, we would like to compare the MMSER scheme with the MMSED scheme. However, the MMSED solution is known only for the one layer case (in closed-form) and two layer case (as an iterative solution). Therefore, we cannot compare with MMSED when the number of layers ($K$) is 3 or 4. In Figs. 7 and 8, we show the BER plots of EJHS, MMSER-µ, $\mu = 1$ to 5, E-MMSED-KK and E-MMSED-L when $K = 3$ and 4 respectively.

In both Figs. 7 and 8, we observe that there is an advantage of 6 dB of SNR at $\text{BER} = 10^{-1}$ for MMSER-2 over MMSER-1. MMSER-3 has an advantage of 2 dB of SNR at $\text{BER} = 10^{-2}$ than MMSER-2. Among the two E-MMSED schemes developed, the E-MMSED-L scheme uses global CSI more effectively. Our MMSER-µ scheme performs better than E-MMSED-KK scheme even though we do not use forward CSI, and worse compared to the E-MMSED-L scheme. MMSER-µ approaches the performance of E-MMSED-L using leaked signals. For example, at $\text{BER} = 10^{-3}$, the advantage of E-MMSED-L over MMSER comes down from 2 to 1 dB of SNR, when $\mu$ is increased from 4 to 5. The lack of forward CSI in MMSER-µ is compensated using leaked signals at the relays.

The comparisons for the multi-layer case in Figs. 7 and 8 also show much more gain (than in the single-layer case) using global CSI in terms of the gap between MMSER-1 and the E-MMSED-L schemes. Therefore, the
gain from global CSI seems to increase when the number of layers increases. However, using more leaked signals reduces this gap significantly. In Figs. 7 and 8, MMSER-4 and MMSER-5 seem to approach the E-MMSED-L scheme respectively.

Fig. 7. BER plots of EJHS, E-MMSED, and MMSER-µ, µ ∈ [1 – 4]. Performance of MMSER-µ is better than E-MMSED-KK when µ ≥ 2 and it approaches that of E-MMSED-L when µ is increased. Total power used in simulations is P = 1 Watt.

Fig. 8. BER plots of EJHS, E-MMSED, and MMSER-µ, µ ∈ [1 – 5]. Performance of MMSER-µ is better than E-MMSED-KK when µ ≥ 2 and it approaches that of E-MMSED-L when µ is increased. Total power used in simulations is P = 1 Watt.

VII. SUMMARY AND CONCLUSION

In this paper, we have considered the AF relaying protocol for a multi-layer cooperative system, and proposed a precoder design method MMSER which minimizes the MSE at each relay instead of the MSE at the destination that is considered in earlier works. Whereas MMSED is a difficult optimization problem using global CSI and even an iterative numerical solution is not available for more than two layers (to the best of our knowledge), our approach needs only backward CSI and results in layer-wise optimization that yields closed form solution for any number of relay layers even when leaked signals are considered. Since MMSED precoder solutions are not available for more than two layers, for comparison, we have proposed E-MMSED schemes, that are suboptimal, for multiple layers.
These schemes may be of independent interest. Though MMSE uses a suboptimal cost function, its performance is shown to exceed/approach the performance of the proposed E-MMSE schemes. We believe that our MMSE schemes provide an interesting method for AF precoder design for multi-layer relay system.

We found the evaluation of the achieved MSE at the destination for our MMSE schemes challenging, and thus resorted to simulation based study. Analytical performance evaluation remains a valuable future work.

APPENDIX A

PROOF OF CLAIM 1: OPTIMUM PRECODER AT L_k

Expanding (5), we get

\[ J_k(\mathbf{F}_k) = E \left[ s^H s - 2 \Re \left( \mathbf{r}_k^H \mathbf{F}_k^H s \right) + \mathbf{r}_k^H \mathbf{F}_k \mathbf{r}_k \right] \]

\[ = N - 2 \Re \left( \text{Tr} \left[ \mathbf{F}_k^H \mathbf{R}_{srk} \right] \right) + \text{Tr} \left[ \mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} \right], \quad (A1) \]

where \( \Re(.) \) is the real-part and \( \text{Tr}[.] \) denotes the trace operator. To arrive at (A1), we have used \( \Re(\cdot) \) as it is a scalar and the cyclic properties of the \( \text{Tr}(.\cdot) \) function namely, \( \text{Tr}(ABC) = \text{Tr}(BCA) \).

Here, the correlation matrices \( \mathbf{R}_{srk} \) and \( \mathbf{R}_{rk} \) are given by

\[ \mathbf{R}_{srk} = E \left[ \mathbf{s}\mathbf{r}_k^H \right] = E \left[ \mathbf{s} \left( \mathbf{r}_k^{(n)H}, \cdots, \mathbf{r}_k^{(k-1)H} \right) \right] \]

\[ = \begin{bmatrix} \mathbf{R}_{srk}^{(n)} & \cdots & \mathbf{R}_{srk}^{(k-1)} \end{bmatrix} \quad (A2) \]

and

\[ \mathbf{R}_{rk} = E \left[ \mathbf{r}_k\mathbf{r}_k^H \right] = \begin{bmatrix} \mathbf{R}_{rk}^{(n)} & \cdots & \mathbf{R}_{rk}^{(k-1)} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{rk}^{(k-1)} & \cdots & \mathbf{R}_{rk}^{(k-1)} \end{bmatrix} \quad (A3) \]

respectively, where \( n = [k - \mu]^+ \). Also, \( \mathbf{R}_{srk}^{(i)} \) and \( \mathbf{R}_{rk}^{(i)} \), \( i,j \in [n,k-1] \), are given by

\[ \mathbf{R}_{srk}^{(i)} = E \left[ \mathbf{s}\mathbf{r}_k^{(i)H} \right] \quad \text{and} \quad \mathbf{R}_{rk}^{(i)} = E \left[ \mathbf{r}_k^{(i)H}\mathbf{r}_k \right] \]

respectively, which can be found using Table I. Now, expanding (6), we get

\[ C_k(\mathbf{F}_k) = \text{Tr} \left( \mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} \right) - p_k. \quad (A4) \]

Substituting (A1) and (A4) into (7), we get

\[ \mathcal{L}_k(\mathbf{F}_k, \lambda_k) = N - 2 \Re \left( \text{Tr} \left[ \mathbf{F}_k^H \mathbf{R}_{srk} \right] \right) + \text{Tr} \left( \mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} \right) \]

\[ + \lambda_k \left[ \text{Tr} \left( \mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} \right) - p_k \right]. \quad (A5) \]

Now, we will derive \( \hat{\mathbf{F}}_k \) for the case when the relays are cooperative.

1) Cooperative relays: Here, each of the sub-matrices \( \mathbf{F}_{k,i} \in \mathbb{C}^{N \times N} \) shown in (1) are nondiagonal as mentioned earlier. Differentiating (A5) w.r.t. \( \mathbf{F}_k^H [28] \) and using complementary slackness [24] yield

\[ \nabla \mathcal{F}_k \mathcal{L}_k = - \mathbf{R}_{srk} + \mathbf{F}_k \mathbf{R}_{rk} + \lambda_k \mathbf{F}_k \mathbf{R}_{rk} \quad (A6) \]

and

\[ \lambda_k \left[ \text{Tr} \left( \mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} \right) - p_k \right] = 0, \quad (A7) \]

respectively. Equating (A6) to zero, we get

\[ \mathbf{F}_k = \frac{1}{1 + \lambda_k} \mathbf{R}_{srk} \mathbf{R}_{rk}^{-1}. \quad (A8) \]

From (A7), we have either \( \lambda_k = 0 \) or

\[ p_k = \text{Tr} \left( \mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} \right). \quad (A9) \]
If the unconstrained estimate $\mathbf{F}_k = \mathbf{R}_{sr_k} \mathbf{R}^{-1}_{rk}$ satisfies the power constraint, then it is also the solution to the MMSE problem with the power constraint, and $\lambda_k = 0$. Otherwise, $\lambda_k > 0$, and substituting (A8) into (A9) and rearranging, we get

$$
(1 + \lambda_k)^2 = \frac{1}{p_k} \text{Tr} \left[ \mathbf{R}^{H}_{sr_k} \mathbf{R}_{sr_k} \mathbf{R}^{-1}_{rk} \right]
$$

as $(\mathbf{R}^{-1}_{rk})^H = \mathbf{R}_{rk}^{-1}$ and $\text{Tr}(\mathbf{ABC}) = \text{Tr}(\mathbf{BCA})$. Substituting (A10) into (A8), we get the optimum precoder as shown in (8). This is simply the scaled version of the unconstrained MMSE estimate scaled such that the power constraint is met with equality.

Even when $\lambda_k = 0$, i.e., when the unconstrained estimate has lower power, we amplify the estimate to use the full sum transmit power for each layer. This is because it is optimal to transmit using the full power for optimal estimation performance at the next layer. Therefore, we always use the precoder specified by (8).

2) Non-cooperative relays: Here each of the sub-matrices $\mathbf{F}_{k,i} \in \mathbb{C}^{N \times N}$ shown in (1) are constrained to be diagonal. To simplify (A5), let us consider

$$
\mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} = \begin{bmatrix}
\mathbf{F}_k^H & \cdots & \mathbf{F}_k^H \\
\vdots & \ddots & \vdots \\
\mathbf{F}_k^H & \cdots & \mathbf{F}_k^H \\
\end{bmatrix} \begin{bmatrix}
\mathbf{F}_{k,n} & \cdots & \mathbf{F}_{k,k-1} \\
\mathbf{F}_{k,n,n} & \cdots & \mathbf{F}_{k,n,k-1} \\
\vdots & \ddots & \vdots \\
\mathbf{F}_{k,k-1,n} & \cdots & \mathbf{F}_{k,k-1,k-1} \\
\end{bmatrix} \mathbf{R}_{rk},
$$

Substituting $\mathbf{R}_{rk}$ from (A3) into (A11) we get

$$
\mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} = \sum_{i=1}^{k-1} \mathbf{F}_{k,i}^H \mathbf{F}_{k,i} \mathbf{R}_{rk(i,j')},
$$

Taking the trace of equation (A12), we get

$$
\text{Tr} \left( \mathbf{F}_k^H \mathbf{F}_k \mathbf{R}_{rk} \right) = \sum_{j=1}^{k-1} \sum_{i=1}^{k-1} \sum_{n=1}^{N} \text{Tr} \left( \mathbf{F}_{k,i}^H \mathbf{F}_{k,i} \mathbf{R}_{rk(i,j')} \mathbf{R}_{rk(i,j')}^H \right),
$$

where $f_{k,i,l}$, $\mathbf{f}_{k,j,l}$, and $\Gamma_{kl}(i,j)$ represent the $i$th diagonal elements of $\mathbf{F}_{k,i}$, $\mathbf{F}_{k,j}$ and $\mathbf{R}_{rk(i,j)}$ respectively. To arrive at (A13), we used the fact that the matrices $\mathbf{F}_{k,i}$, $i \in [n, k - 1]$ are diagonal. Finally, to simplify (A5) we find $\mathbf{F}_k^H \mathbf{R}_{sr_k}$ as

$$
\mathbf{F}_k^H \mathbf{R}_{sr_k} = \begin{bmatrix}
\mathbf{F}_k^H & \cdots & \mathbf{F}_k^H \\
\vdots & \ddots & \vdots \\
\mathbf{F}_k^H & \cdots & \mathbf{F}_k^H \\
\end{bmatrix} \begin{bmatrix}
\mathbf{R}_{sr_k(n)} & \cdots & \mathbf{R}_{sr_k(k-1)} \\
\vdots & \ddots & \vdots \\
\mathbf{R}_{sr_k(n)} & \cdots & \mathbf{R}_{sr_k(k-1)} \\
\end{bmatrix},
$$

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Taking the trace of equation (A14), we get

\[
\text{Tr} \left( F^H_k R_{sr_k} \right) = \sum_{j=n}^{k-1} \text{Tr} \left( F^H_{k,j} R_{sr_k}^{(j)} \right)
\]

\[
= \sum_{j=n}^{k-1} N \sum_{l=1}^{N} f^*_{k,j,l} \gamma_{kl}^{s(j)} ,
\]

(A15)

where \( \gamma_{k_l}^{s(j)} \) is the \( l \)th diagonal element of the correlation matrix \( R_{sr_k}^{(j)} \). From (A13) and (A15), (A5) becomes

\[
L_k = N - 2\Re \left[ \sum_{j=n}^{k-1} N \sum_{l=1}^{N} f^*_{k,j,l} \gamma_{kl}^{s(j)} \right] + (1 + \lambda_k) \sum_{j=n}^{k-1} N \sum_{l=1}^{N} f^*_{k,j,l} f_{k,j,l} \gamma_{kl}^{s(j)} - \lambda_k p_k .
\]

(A16)

Differentiating (A16) w.r.t. the conjugate of the precoder matrix diagonal element, \( f^*_{k,j,l} \), \( j \in [n, k-1] \), \( l \in [1, N] \) and simplifying, we get

\[
\sum_{i=n}^{k-1} f_{k,i,l} \gamma_{kl}^{(i)(j)} = \frac{\gamma_{kl}^{s(j)}}{1 + \lambda_k} ,
\]

(A17)

From complementary slackness, we get \( \lambda_k = 0 \) or

\[
p_k = \sum_{j=n}^{k-1} N \sum_{i=n}^{k-1} f^*_{k,j,l} f_{k,i,l} \gamma_{kl}^{(i)(j)}
\]

\[
= \sum_{l=1}^{N} \text{Tr} \left( F^H_k f_{k,l} \Upsilon_{kl} \right) .
\]

(A18)

Equation (A17) can be written in matrix form as

\[
f_{k,l} \Upsilon_{kl} = \frac{1}{1 + \lambda_k} \Theta_{kl}^s , \quad l \in [1, N],
\]

(A19)

where \( f_{k,l} \), \( \Theta_{kl}^s \), and \( \Upsilon_{kl} \) are as defined in (10), (11), and (12) respectively. From (A19), we get

\[
f_{k,l} = \frac{1}{1 + \lambda_k} \Theta_{kl}^s \Upsilon_{kl}^{-1}
\]

(A20)

as \( \Upsilon_{kl} \) is a nonsingular matrix which depends on \( R_{sr_k} \) and \( R_{rs_k} \). Proceeding (as in the cooperative relays case) with (A18) and (A20), we get the required equation (9).

\[\square\]

APPENDIX B

PROOF OF CLAIM 2: SNR OF E-MMSED

Using (20), \( P_S \) in (21) can be expanded as

\[
P_S = \frac{p_0 K_1}{K_0} h_0^H F^H h_{K+1}^H h_{K+1} F h_0 \text{ where } K_1 = K - K_0 + 1
\]

\[
= \frac{p_0 K_1}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,i,j}|^2 |h_{i,j,K+1}|^2 |f_{ij}|^2
\]

(A21)
using the diagonal properties of $F$ and $E(|s|^2) = 1$. Similarly, from (20) we can write

$$E|_{n_{(k)}^{(l)}}|^2 = E\left[\frac{h_{K+1}F}{K_0} \sum_{i=0}^{K-1} u^{(i)} \right]^H \left[\frac{h_{K+1}F}{K_0} \sum_{j=0}^{K-1} u^{(j)} \right]$$

\[\text{Hence from (22), } P_N \text{ can be written as} \]

$$P_N = \frac{\sigma^2 K_1}{K_0} \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{i,j,K+1}|^2 |f_{i,j}|^2. \quad (A22)$$

\[\text{A. E-MMSED-KK} \]

Substituting (17) into (A21), we get

$$P_S = \frac{p_0 p_1 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,i,j}|^4}{\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{k,i,l}|^2 |h_{k,i,l,K+1}|^2 \left( p_0 |h_{0,k,l}|^2 + \sigma^2 \right)} \quad (A23)$$

Similarly, substituting (17) into (A22), we get

$$P_N = \frac{\sigma^2 p_1 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,i,j}|^2}{\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{k,i,l}|^2 |h_{k,i,l,K+1}|^2 \left( p_0 |h_{0,k,l}|^2 + \sigma^2 \right)} \quad (A24)$$

Dividing (A23) by (A24), we get equation (23).

\[\text{B. E-MMSED-L} \]

Substituting (18) into (A21), we get

$$P_S = \frac{p_0 p_1 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,i,j}|^4 |h_{i,j,K+1}|^4}{\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,k,l}|^2 |h_{k,l,K+1}|^2 \left( p_0 |h_{0,k,l}|^2 + \sigma^2 \right)} \quad (A25)$$

Similarly, substituting (18) into (A22), we get

$$P_N = \frac{\sigma^2 p_1 \sum_{i=1}^{K} \sum_{j=1}^{N} |h_{0,i,j}|^2 |h_{i,j,K+1}|^4}{\sum_{k=1}^{K} \sum_{l=1}^{N} |h_{0,k,l}|^2 |h_{k,l,K+1}|^2 \left( p_0 |h_{0,k,l}|^2 + \sigma^2 \right)} \quad (A26)$$

Dividing (A25) by (A26), we get equation (24).

\[\text{REFERENCES} \]


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