Symbol Detection in CDMA-OFDM Coexistence

G V V Sharma 06407010
Aswani Kumar Ravi 06307R13
Nagraj T 06307040
Narendar P 06307037

Department of Electrical Engineering
IIT Bombay
http://www.ee.iitb.ac.in/student/aswanikumar/
Problem Statement

- One user transmits the QPSK symbol $b$ using CDMA (spreading code). Another user transmits a complex vector $c$ using OFDM. Both use the same channel (carrier frequency).
- The transmitted OFDM signal

$$c(t) = \sum_{m=0}^{M-1} c_m e^{j2\pi mt/T} \quad 0 \leq t \leq T \quad (1)$$

where $c_m$ are the complex data symbols being transmitted.
- The signal transmitted by the CDMA user

$$s(t) = \frac{b}{\sqrt{N}} \sum_{n=0}^{N-1} s_n p(t - nT_c) \quad 0 \leq t \leq T \quad (2)$$

where $b$ is the transmitted symbol, $s_n$ the spreading code, $T_c$ the chip duration and $p(t)$ the pulse shaping function.
- OFDM demodulator output, assuming that the noise is AWGN

$$y = br + c + \sigma n$$

where $r$ is the DFT of $s_n$ and $c = \{c\}_{m=1}^{M-1}$. The noise $n$, is complex circularly Gaussian with covariance matrix $2I$.
- We need to detect $b$ and $c$. 
Let the vector $w$ be the linear filter used to demodulate the symbol $b$.
Proposed Solution

- Let the vector $\mathbf{w}$ be the linear filter used to demodulate the symbol $b$.
- The decision metric

$$D = \mathbf{w}^H \mathbf{y} = b\mathbf{w}^H \mathbf{r} + \mathbf{w}^H \mathbf{c} + \sigma \mathbf{w}^H \mathbf{n}$$

where $\mathbf{w}^H$ is the conjugate transpose of the vector $\mathbf{w}$.

An estimate of the symbol $\hat{b}$ is then obtained as

$$\hat{b} = \text{sgn}(\text{Re}\{D\}) + i\text{sgn}(\text{Im}\{D\})$$

The challenge is to evolve an adaptive algorithm for updating the filter $\mathbf{w}$.

We would like to exploit the Gaussian nature of the additive noise to do this.
Proposed Solution

- Let the vector $\mathbf{w}$ be the linear filter used to demodulate the symbol $b$.
- The decision metric

$$D = \mathbf{w}^H \mathbf{y}$$

$$= b \mathbf{w}^H \mathbf{r} + \mathbf{w}^H \mathbf{c} + \sigma \mathbf{w}^H \mathbf{n}$$ (3)

where $\mathbf{w}^H$ is the conjugate transpose of the vector $\mathbf{w}$.
- An estimate of the symbol $b$ is then obtained as

$$\hat{b} = \text{sgn}(\text{Re}\{D\}) + i \text{sgn}(\text{Im}\{D\})$$
Let the vector $\mathbf{w}$ be the linear filter used to demodulate the symbol $b$.

The decision metric

$$D = \mathbf{w}^H \mathbf{y}$$

$$= b \mathbf{w}^H \mathbf{r} + \mathbf{w}^H \mathbf{c} + \sigma \mathbf{w}^H \mathbf{n}$$

where $\mathbf{w}^H$ is the conjugate transpose of the vector $\mathbf{w}$.

An estimate of the symbol $b$ is then obtained as

$$\hat{b} = \text{sgn}(\text{Re}\{D\}) + i \text{sgn}(\text{Im}\{D\})$$

The challenge is to evolve an adaptive algorithm for updating the filter $\mathbf{w}$. 

We would like to exploit the Gaussian nature of the additive noise to do this.
Proposed Solution

- Let the vector $\mathbf{w}$ be the linear filter used to demodulate the symbol $b$.
- The decision metric

$$D = \mathbf{w}^H \mathbf{y} = b \mathbf{w}^H \mathbf{r} + \mathbf{w}^H \mathbf{c} + \sigma \mathbf{w}^H \mathbf{n}$$ (3)

where $\mathbf{w}^H$ is the conjugate transpose of the vector $\mathbf{w}$.
- An estimate of the symbol $b$ is then obtained as

$$\hat{b} = \text{sgn}(\text{Re}\{D\}) + i \text{sgn}(\text{Im}\{D\})$$

- The challenge is to evolve an adaptive algorithm for updating the filter $\mathbf{w}$.
- We would like to exploit the Gaussian nature of the additive noise to do this.
The Adaptive Algorithm: MCPOE

- The mean and variance of the real part of $D$, conditioned on $b$ are

$$\mu_D = \text{Re} \left\{ w^H (br + c) \right\}$$
$$\sigma_D^2 = \sigma^2 ||w||^2$$  (4)

- If $b^+$ and $b^-$ are obtained from $b$ by making its real part $+1$ and $-1$ respectively, the conditional probability of error is

$$P_{e|b} = \frac{1}{2} Q \left( \frac{\mu_D^+}{\sigma_D} \right) + \frac{1}{2} Q \left( \frac{\mu_D^-}{\sigma_D} \right)$$  (5)

where $\mu_D^+$ and $\mu_D^-$ are obtained from $\mu_D$ by substituting $b$ by $b^+$ and $b^-$ respectively.

- Using the gradient descent approach,

$$w^{(i+1)} = w^{(i)} - \lambda \nabla P_{e|b}^{(i)}$$  (6)

where

$$\nabla P_{e|b} = \frac{1}{\sqrt{2\pi}} \exp \left( - \frac{\mu_D^2}{\sigma_D^2} \right) \frac{||w||^2 \left( b^- r + c \right) - \mu_D^- w}{2\sigma ||w||^3}$$
$$- \frac{1}{\sqrt{2\pi}} \exp \left( - \frac{\mu_D^2}{\sigma_D^2} \right) \frac{||w||^2 \left( b^+ r + c \right) - \mu_D^+ w}{2\sigma ||w||^3}$$  (7)
