A Generalized Approach for Finite Precision 5/3 Filter Design

Bhushan Patil, Gaurav Sharma, Pushkar Patwardhan, Vikram Gadre

Department of Electrical Engineering
Indian Institute of Technology, Bombay, Mumbai - 400076, India.
bhushanp@ee.iitb.ac.in

Abstract

In this paper we present a general approach for the design of LeGall's 5/3 linear phase perfect reconstruction filter-bank. Using our generalized approach, we can design the 5/3 filters with the coefficients having different finite wordlengths (precision) - thus this can be seen as a generalization of the LeGall 5/3 filters. We construct design examples with the filters having different wordlengths, and also analyze the frequency characteristics of the designed filters.

1. Introduction

Filter-banks with the filters having finite precision (or wordlength) coefficients are desirable for reducing the cost of implementation and/or the speed of computations [4]-[7]. With finite precision coefficients, the issue of coefficient quantization is totally eliminated, and thus perfect reconstruction can be achieved in practical implementations (something which is impossible or difficult when the filters have infinite precision coefficients).

In [2], short kernel biorthogonal perfect-reconstruction filter-banks are constructed with the filters having finite precision coefficients. The 5/3 filter-bank constructed in [2] (with the analysis lowpass and highpass filters having length of 5 and 3 taps respectively) has been used in the JPEG2000 image compression standard (particularly for the lossless compression mode), The 5/3 analysis filters as designed in [2] are as follows

$H_0(z) = \frac{1}{8}(-1 + 2z^{-1} + 6z^{-2} + 2z^{-3} - z^{-4})$

$H_1(z) = \frac{1}{2}(1 + 2z^{-1} + z^{-2})$

As can be seen, the coefficients of the LeGall 5/3 filters can be expressed in 5 bit precision. It is not easy to modify the polynomial factorization design approach of [2] if we wish to design 5/3 filters with a different precision. In this paper we present a generalized approach for the design of 5/3 filter-banks, so that we can design the 5/3 filters with different precision. The paper is organized as follows: In Section II, we briefly review the perfect-reconstruction conditions for bi-orthogonal filter-banks, and also review the design approach of [2]. In section III, we present our generalized approach for designing 5/3 filters. In section IV we present design examples, and also analyze the frequency responses of the designed finite-precision filters. Section V summarizes the results of the paper.

2. Perfect reconstruction short kernel filter banks

Perfect reconstruction filter banks have been extensively studied [1], [3]. A polynomial factorization approach to the design of perfect reconstruction filter-banks was proposed in [2]. where the filters are derived by a factorization of a symmetric product polynomial. In this section we review the approach of [2].

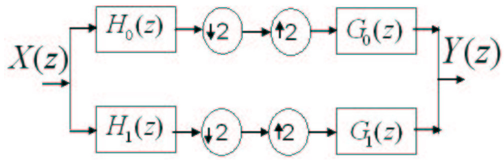

Figure 1: Block diagram of two band filter bank

Fig. 1 shows the two channel filter bank. The output of the filter bank as shown in Fig. 1 can be expressed as

$$Y(z) = 1/2[G_0(z)H_0(z) + G_1(z)H_1(z)]X(z)$$
$$+1/2[G_0(z)H_0(-z) + G_1(z)H_1(-z)]X(-z)$$

where X(z) is the input signal and Y(z) is the reconstructed output signal. Necessary and sufficient conditions for perfect reconstruction are

$$H_0(z)G_0(z) + H_1(z)G_1(z) = Cz^{-D} \quad (1)$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \quad (2)$$

We shall choose $C = 2$ to generalize from [2]. Equation (1) gives the constraint for perfect signal reconstruction and equation (2) gives the constraint for alias cancelation. The constraint in (2) can be satisfied by the following choice of the synthesis filters

$$G_0(z) = H_1(-z), G_1(z) = -H_0(-z) \quad (3)$$

Le Gall's approach [2] consists of designing a polynomial P(z) that satisfies the condition $\Delta z = P(z) - P(-z) = 2z^{-m}$, where $P(z) = H_0(z)H_1(-z)$. Then the polynomial P(z) can be factorized to obtain the analysis filters. The finite precision 5/3 filters are obtained in [2] by factorization of a particular choice of P(z). Our aim in this paper is to construct different sets of 5/3 filters using a different approach.

3. Generalized approach to design of 5/3 filters

Our approach for designing 5/3 filters consists of starting with a parametrised set of symmetric (linear phase) functions, and then deriving constraints on the parameters which are imposed by the PR equations. Thus, we finally arrive at an independent set of parameters, which can be independently varied to construct different cases of 5/3 filters.

By replacing z by -z in equation(1) and (2), we get the following set of equations, which can be viewed as the constraints required for perfect-reconstruction.

$$H_0(z)G_0(z) + H_1(z)G_1(z) = 2z^{-D} \tag{4}$$

$$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0 \tag{5}$$

$$H_0(z)G_0(-z) + H_1(z)G_1(-z) = 0 \tag{6}$$

Assuming symmetry/linear phase, we can express the filters as follows:

$$H_0(z) = a_0 + a_1z^{-1} + a_2z^{-2} + a_1z^{-3} + a_0z^{-4} \tag{7}$$

$$H_1(z) = b_0 + b_1z^{-1} + b_0z^{-2} \tag{8}$$

$$G_0(z) = c_0 + c_1z^{-1} + c_0z^{-2} \tag{9}$$

$$G_1(z) = d_0 + d_1z^{-1} + d_2z^{-2} + d_1z^{-3} + d_0z^{-4}. \tag{10}$$

We note that we do not assume the relation of equation-3.

Now eliminating $G_1(z)$ from equation (4) and (5), we get

$$H_0(z)G_0(z) - \frac{H_1(z)H_0(-z)G_0(z)}{H_1(-z)} = 2z^{-D}$$

or

$$G_0(z) \times [od(H_0(z)H_1(-z))] = z^{-D}H_1(-z) \tag{11}$$

where od() denotes only the odd power terms of the given polynomial.

Substituting equations (7)-(10) in the above equation, we obtain

$$(c_0 + c_1z^{-1} + c_0z^{-2})[od(a_0 + a_1z^{-1} + a_2z^{-2} + a_1z^{-3} + a_0z^{-4})(b_0 - b_1z^{-1} + b_0z^{-2})]$$
$$= z^{-n}(b_0 + b_1z^{-1} + b_2z^{-2})$$

or,

$$(c_0 + c_1z^{-1} + c_0z^{-2})[z^{-1}(a_1b0 - a_0b_1) + z^{-3}(2a_1b_0 - a_2b_1) + z^{-5}(a_!b_0 - a_0b_1)]$$
$$= z^{-n}(b_0 - b_1z^{-1} + b_2z^{-2})$$

Simplification and comparison of coefficients results in the following possibility:
$D = 3$
$a_1b_0 = a_0b_1$
and, $(2a_1b_0 - a_2b_1)c_0 = b_0$
also, $(2a_1b_0 - a_2b_1)c_1 = -b_1$.
These equations further results

$$\frac{b_1}{b_0} = \frac{-c_1}{c_0} = \frac{a_1}{a_0} = k(say) \tag{12}$$

and, $a_2 = \frac{2a_1b_0}{b_1} - \frac{b_0}{b_1c_0}$ which results in

$$a_2 = 2a_0 - \frac{1}{kc_0}. \tag{13}$$

To obtain the relation between coefficients of $G_1(z)$, eliminate $H_0(z)$ from equation (4) and (6) to obtain

$$G_1(z)H_1(z) - \frac{G_1(-z)H_1(z)G_0(z)}{G_0(-z)} = 2z^{-3}$$

or,

$$(G_1(z)G_0(-z) - G_0(z)G_1(-z))H_1(z)$$
$$= 2z^{-3}(G_0(-z))$$

or,

$$H_1(z)od[G_1(z)G_0(-z)] = z^{-3}G_0(-z) \tag{14}$$

Now putting the values of polynomials from equation (7)-(10) in the above equations we get

$$(b_0 + b_1z^{-1} + b_0z^{-2})[od(d_0 + d_1z^{-1} + d_2z^{-2} + d_1z^{-3} + d_0z^{-4})(c0 - c_1z^{-1} + c_0z^{-2})]$$
$$= z^{-3}(c_0 - c_1z^{-1} + c_0z^{-2})$$

or,

$$(b_0 + b_1z^{-1} + b_0z^{-2})[z^{-1}(d_1c_0 - d_0c_1) + z^{-3}(2d_1c_0 - d_2c_1) + z^{-5}(d_1c_0 - d_0c_1)]$$
$$= z^{-3}(c_0 - c_1z^{-1} + c_0z^{-2}).$$

Comparing the coefficients of the above identity we get the following $d_1c_0 - d_0c_1 = 0$
$b_0(2d_1c_0 - d_2c_1) = c_0$
$b_1(2d_1c_0 - d_2c_1) = -c_1$
which further results in

$$\frac{-d_1}{d_0} = \frac{-c_1}{c_0} = k \tag{15}$$

and, $d_2 = \frac{2d_1c_0}{c_1} - \frac{c_0}{c_1b_0}$ or,

$$d_2 = 2d_0 + \frac{1}{kb_0}. \tag{16}$$

Using the relation between the coefficients found from equations (12)-(15),we express every filter in terms of it's z^0 coefficient.
Finally the filter bank is given as

$$H_0(z) = a_0 + a_0 k z^{-1} + \left(2a_0 - \frac{1}{kc_0}\right) z^{-2} + a_0 k z^{-3} + a_0 z^{-4} \tag{17}$$

$$H_1(z) = b_0 + b_0 k z^{-1} + b_0 z^{-2} \tag{18}$$

$$G_0(z) = c_0 - c_0 k z^{-1} + c_0 z^{-2} \tag{19}$$

$$G_1(z) = d_0 - d_0 k z^{-1} + \left(2d_0 + \frac{1}{kd_0}\right) z^{-2} - d_0 k z^{-3} + d_0 z^{-4}. \tag{20}$$

The above filter bank is derived from equations (11) and (14) which are derived from the necessary and sufficient conditions for perfect reconstruction. If all the conditions obtained from equation (17) to equation (20) are put back in equation (1), then it produces another restriction for filterbank design

$$d_0 = -\frac{a_0 c_0}{b_0}. \tag{21}$$

The above restriction leaves 4 degrees of freedom in the design.The system determined by equation(17) to equation (20), alongwith the restriction as given in equation(21), gives a general four variable(a_0,a_1,$b_0$andc_0) system for the 5/3 filter bank.

4. Design Examples of finite-precision 5/3 filters

As shown in section III, we can derive different sets of 5/3 filters by choosing different values for the parameters in equation(17) to (20). In this section, we construct design examples of 5/3 filters by choosing the parameters in (17)-(20) such that the filter coefficients have finite precision (with different wordlengths, based on the choice of the independent parameters).

The following table shows the design examples of finite wordlength filters. Also shown is the choice of the parameters in (17)-(20) which has been used to construct the filters.

Sr.	Bits	Parameters	Filter Coefficient
1	2-bit	$a_0 = -0.5$ $a_1 = 0.5$ $b_0 = -0.5$ $c_0 = 0.5$	$H_0 = [-.5\ .5\ 1\ .5\ -.5]$ $H_1 = [-.5\ .5\ -.5]$ $G_0 = [.5\ .5\ .5]$ $G_1 = [-.5\ -.5\ 1\ -.5\ -.5]$
2	3-bit	$a_0 = -0.25$ $a_1 = 0.5$ $b_0 = -0.5$ $c_0 = 0.5$	$H_0 = [-.25\ .5\ .5\ .5\ -.25]$ $H_1 = [-.5\ 1\ -.5]$ $G_0 = [.5\ 1\ .5]$ $G_1 = [-.25\ -.5\ .5\ -.5\ -.25]$
3	4-bit	$a_0 = 0.25$ $a_1 = -0.5$ $b_0 = -0.5$ $c_0 = -0.5$	$H_0 = [.25\ -.5\ -.5\ -.5\ .25]$ $H_1 = [-.5\ 1\ -.5]$ $G_0 = [-.5\ -1\ -.5]$ $G_1 = [-.25\ -.5\ .5\ -.5\ .25]$
4	LeGall 5-bit	$a_0 = -0.125$ $a_1 = 0.25$ $b_0 = 0.5$ $c_0 = 0.5$	$H_0 = [-1/8\ 2/8\ 6/8\ 2/8\ -1/8]$ $H_1 = [1/2\ -1\ 1/2]$ $G_0 = [1/2\ 1\ 1/2]$ $G_1 = [-1/8\ -2/8\ 6/8\ -2/8\ -1/8]$

We now plot the normalised frequency responses of the filters in the above Table. The goal is to compare the frequency responses when the wordlength of the filter-coefficients is varied. Figure 2, 3, 4 and 5 shows the comparison of the frequency responses of the various wordlength filters (H0, H1, G0, G1 respectively)with standard Le Gall's filters.

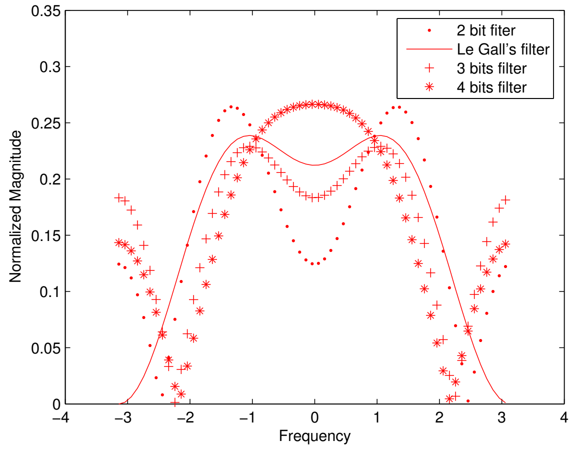

Figure 2: frequency responses of H0 filters at various wordlengths

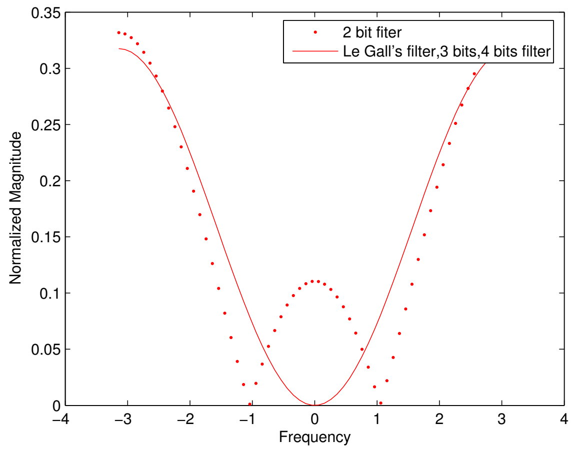

Figure 3: frequency responses of H1 filters at various wordlengths

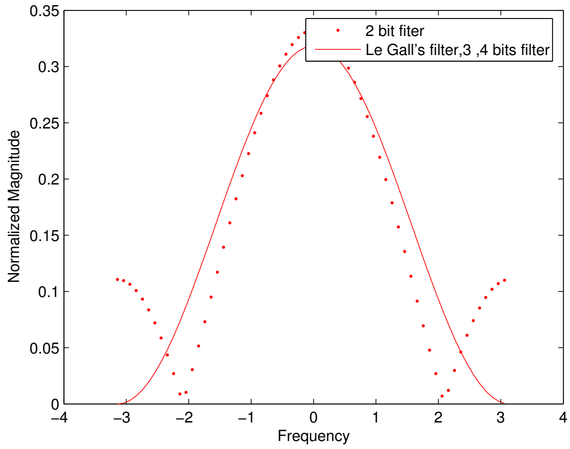

Figure 4: frequency responses of G0 filters at various wordlengths

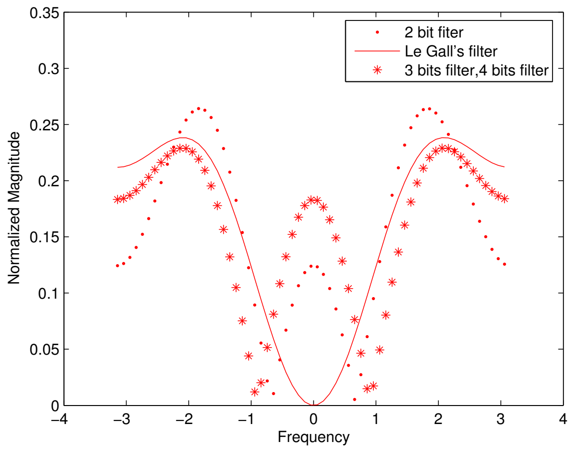

Figure 5: frequency responses of G1 filters at various wordlengths

5. Summary and conclusion

In this paper, we have presented a generalized approach for designing 5/3 filters with finite precision coefficients. We used this approach to design 5/3 filters with different coefficient wordlengths. We compared the frequency responses of the filters, with the goal of studying the effect of the coefficient wordlength on the frequency response. Figures 2,3,4 and 5 shows the evolution of the frequency response as we move from 2-bit representation to 5-bit representation. This constitute a preliminary study of the dependence of frequency response and wordlength in a perfect reconstruction system.

6. References

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