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on

Waveguide Resonators

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Abstract

Waveguide resonators are used as high-Q bandpass filtering elements in waveguide based microwave circuits. Q values of thousands can be conveniently obtained, which is impossible in case of resonant circuits based on lumped elements. Different types of rectangular and cylindrical resonators will be described along with the key parameters used to measure the goodness of a resonator. Different variations of rectangular and cylindrical resonators will also be introduced.

I. Introduction

Resonant circuits are circuits, which offers a high impedance or low impedance (for parallel and series resonance respectively) to the source at a particular frequency of operation. The frequency at which the resonant circuit has a very high or low impedance is called its *resonant frequency*.The *frequency selectivity* property of resonant circuits are exploited in building filter circuits.

Resonant circuits can be built either using lumped elements or distributed elements. Fig 1 shows two types of resonant circuits using lumped elements. In lumped elements resonant circuit, capacitors store the electric energy and the inductors store the magnetic energy, while the resistance shows up as loss.During resonance, transfer of energy takes place between inductors and capacitors.

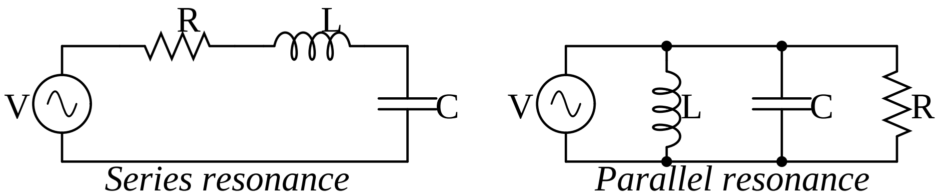


Fig. 1. Series and Parallel resonant circuits comprising lumped elements

Another type of resonant circuits is the distributed resonant circuit, which utilizes an open or shorted transmission line. The resonance occurs in the form of standing waves due to superposition of the forward and reverse traveling waves.Even in the distributed resonant circuit, energy transfer happens every quarter-cycle. As the resonant circuit needs a standing wave along the transmission line, its dimensions are comparable with the wavelength,λ. We will see that any form of transmission line of suitable lenghts can be used as a resonator. When the transmission line used is a waveguide, the resulting resonator is called a *cavity resonator* and the resonator is called a *strip resonator* when a microstrip is used as the transmission line.

A. Loaded and Unloaded Q

Quality factor, Q is very extensively used to asses a resonator. Q-factor signifies the sharpness in the frequency response of a resonator. Higher the Q of a resonator sharper will be its frequency response Q has two mathematical representations given by (1) and (2)

$$Q = \omega \frac{(average\ \ energy\ \ stored)}{(energy\ \ loss/second)} \tag{1}$$

$$Q = \frac{f_0}{\Delta f} \tag{2}$$

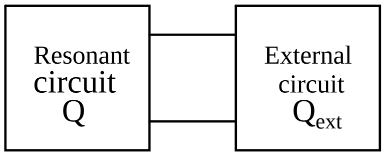


Fig. 2. A resonant circuit loaded by an external circuit

where Δf is the difference in the frequency where the magnitude falls to 3dB and f_0 is the centre frequency. Δf is also one criteria to define *bandwidth*.

The Q defined by (1) and (2) refers to the resonant cavity when it is not connected to any load and hence this is known as *"unloaded Q "*. When the resonant cavity is connected to an external circuitry with its Q, Q_{ext} as shown in Fig 2, the Q of the overall circuitry, Q_L is

$$\frac{1}{Q_L} = \frac{1}{Q} + \frac{1}{Q_{ext}} \tag{3}$$

This method to find the overall Q of a resonant circuit can be used in general to any form of resonator, may it be a waveguide resonator, a strip resonator or a lumped element resonator.

Lumped element resonators have several limitations over waveguide resonators as following.

1) Values of Q that can be achieved by lumped elements is usually limited to hundreds, which prevents their use in very narrowband and accurate tuning and spectrum analysis applications. Q values of several tens of thousands are achievable with waveguide resonators.
2) Lumped element resonant circuits are usually limited to 10GHz as the capacitance and inductance values required to get very high resonant frequency becomes too small to be fabricated. [1] describes a lumped element resonator based bandstop filter using 20-finger interdigital capacitors and inductors made of superconducting films. They could achieve a minimum bandwidth of 90 MHz with stopband suppression of 50 dB and resonant frequency being $\approx 5GHz$. This rules out the use of them at frequencies above 7-10 GHz E.g., X-band (8-12 GHz) and K-band (18-27 GHz) radar systems.
3) Simplicity in construction of waveguide resonators (which, we will shortly study) is an added advantage. Ruggedness is usually considered to be an added advantage, however, the lumped resonators built using strip capacitance and inductance are also rugged and hence ruggedness is not an issue with lumped element resonators.
4) Power dissipation capability is far lesser than waveguide resonators. This can be felt intuitively, lumped element resonators have a very small surface area and hence power dissipation is less whereas, waveguide resonators have a larger surface area and hence can dissipate more power.

II. Distributed Transmission Line Resonators

As we saw in the previous section that resonators can also be built using shorted or open sections of a transmission line. This apparently means that waveguides can also be used for this purpose as they are also a form of transmission line. Simple $\lambda/2$ sections of transmission lines are a good point to start our discussion about waveguide resonators a.k.a cavity resonator.

A. Short-circuited $\lambda/2$ line [2]

Let us consider a short circuited transmission line as shown in Fig 3 of length l and the distributed parameters R, L, and C per unit length. $l = \lambda_0/2$ at the resonant frequency $f = f_0$. For small changes in frequency around the resonant frequency $f = f_0 + \Delta f$, $\beta l = 2\pi f l/c = \pi\omega/\omega_0 = \pi + \pi\Delta\omega/\omega_0$, since at f_0, $\beta l = \pi$.

For a shorted line we know that the input impedance is given by

$$Z_{in} = Z_0 tanh \gamma l \tag{4}$$

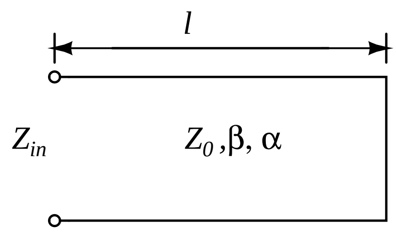


Fig. 3. Short-ciruit transmission line as resonator

putting $\gamma = \alpha + j\beta l$ in (4), we have

$$Z_{in} = Z_0 tanh(\alpha l + j\beta l) = Z_0 \frac{tanh\alpha l + jtan\beta l}{1 + jtan\beta l tanh\alpha l} \tag{5}$$

for small αl, $tanh\alpha l \approx \alpha l$ and $tan\beta l = tan(\pi + \pi\Delta\omega/\omega_0) = tan(\pi\Delta\omega/\omega_0) \approx \pi\Delta\omega/\omega_0$ as $\Delta\omega/\omega_0$ is very small. putting our assumptions together in (5) we have

$$Z_{in} \approx Z_0 \left(\alpha l + j\pi\frac{\Delta\omega}{\omega_0}\right) \tag{6}$$

Now putting

$$Z_0 = \sqrt{\frac{L}{C}}$$

and $\alpha = (R/2)Y_c = (R/2)\sqrt{C/L}$, and $\beta l = \omega_0\sqrt{LC}l = \pi$; $\pi/\omega_0 = l\sqrt{LC}$ in (10) we have

$$Z_{in} = \frac{1}{2}Rl + jlL\Delta\omega \tag{7}$$

We can clearly see from (7) that at resonance ($\Delta\omega = 0$) at which $l = \lambda_0/2$, the input impedance is a pure resistance $R = \frac{1}{2}Rl$, which is same as the response of a series resonant circuit with $R = \frac{1}{2}Rl$ and $L = \frac{1}{2}lL$. The unloaded Q is determined by using (1)

$$Q = \frac{\omega_0 L}{R} = \frac{\beta}{2\alpha}$$

B. Open-circuited $\lambda/2$ line [3]

Let us consider the open-circuited transmission line as shown in Fig 4

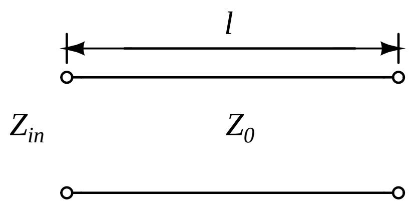


Fig. 4. Open-ciruit transmission line as resonator

The analysis parallels to that of the short-circuited $\lambda/2$ line.

$$Z_{in} = Z_0 coth(\alpha + j\beta)l \tag{8}$$

$$Z_{in} \approx Z_0 \frac{1 + jtan\beta l tanh\alpha l}{tanh\alpha l + jtan\beta l} \tag{9}$$

Using the same set of assumptions for $\omega = \omega_0 + \Delta\omega$, we have

$$Z_{in} \approx Z_0 \left(\alpha l + j\pi \frac{\Delta\omega}{\omega_0} \right)^{-1} \tag{10}$$

This shows that a open-circuited $\lambda/2$ line behaves like a parallel resonant circuit. The unloaded Q is determined by using (1)

$$Q = \omega_0 RC = \frac{\beta}{2\alpha}$$

From our previous discussions, the following observations can be made

1) A short-circuited $\lambda/2$ line and open-circuited line of length equal to odd-multiples of $\lambda/4$ exhibit series resonance.
2) An open-circuited $\lambda/2$ line and short-circuited line of length equal to odd-multiples of $\lambda/4$ exhibit parallel resonance or antiresonance.

III. Waveguide Resonators

Waveguide resonators in its simplest forms are metallic enclosures or cavities. Electric and magnetic energy is stored in this volume thus establishing a resonance condition.The power dissipation is through the surface of the waveguide and the dielectric filling. Coupling energy to the waveguide resonator can be made through a small aperture, a probe or a current loop.

A. Rectangular Resonator

Rectangular resonator is in its simplest form is a rectangular waveguide with shorting plates at both ends as shown in Fig 5. We will start with the lowest mode for a rectangular waveguide, which is TE_{10}.

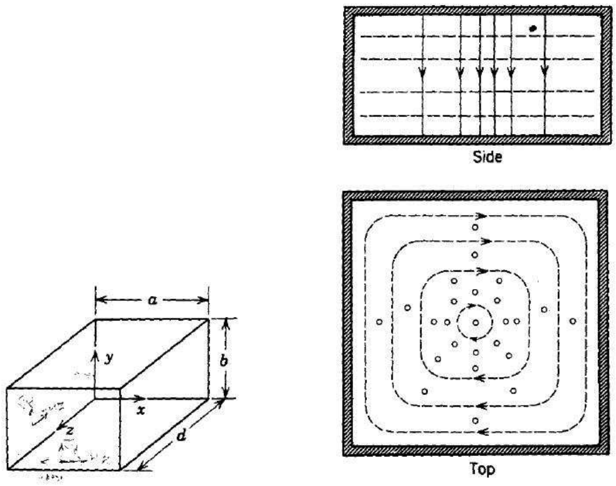


Fig. 5. Rectangular cavity showing the fields for TE_{101} mode [4]

For the analysis the side walls will be considered to be having infinite conductivity and then will be perturbed by adding a finite conductivity for Q calculations[1].

Assuming TE_{10} mode of excitation with z direction as the direction of propagation and y direction being the orientation of the electric field as shown in Fig 5. The boundary condition that E_y to be zero

[1]by the very definition of Q, any analysis that assumes the losses as zero is unacceptable since the Q will then be infinite

at $z = d$ and $z = 0$ is dictated by the presence of perfect conductors. This is met if the length d is half of guide wavelength.

$$d = \frac{\lambda_g}{2} = \frac{\lambda}{2\sqrt{1 - (\lambda/2a)^2}}$$

The resonant frequency is given by

$$f_0 = \frac{v}{\lambda} = \frac{\sqrt{a^2 + d^2}}{2ad\sqrt{\mu\epsilon}} \tag{11}$$

The last step in (11) was obtained by substituting for λ in terms of d. We have the field equations for the cavity resonator as following, where we have added two waves of the form E_+ and E_- [4]

$$E_y = (E_+ e^{-j\beta z} + E_- e^{j\beta z}) sin\frac{\pi x}{a} \tag{12}$$

$$H_x = -\frac{1}{Z_{TE}}(E_+ e^{-j\beta z} - E_- e^{j\beta z}) sin\frac{\pi x}{a} \tag{13}$$

$$H_z = \frac{j}{\eta}\left(\frac{\lambda}{2a}\right)(E_+ e^{-j\beta z} + E_- e^{j\beta z}) cos\frac{\pi x}{a} \tag{14}$$

To satisfy the boundary conditions that $E_y = 0$ at $z = 0$ and $z = d$ we assume $E_+ = -E_-$ at $z = 0$ since there will be total reflection at the surface of a perfect conductor. At $z = d$, so that $\beta = \pi/d$, we have $E_0 = -2jE_+$:

$$E_y = E_0 sin\frac{\pi x}{a} sin\frac{\pi z}{d} \tag{15}$$

$$H_x = -j\frac{E_0}{\eta}\frac{\lambda}{2d} sin\frac{\pi x}{a} cos\frac{\pi z}{d} \tag{16}$$

$$H_z = j\frac{E_0}{\eta}\left(\frac{\lambda}{2a}\right) cos\frac{\pi x}{a} sin\frac{\pi z}{d} \tag{17}$$

Fig 5 shows the filed pattern according to (15), (16), and (17). We can see that electric field is perpendicular to the top and bottom surface at all points and magnetic field circulates around the displacement current due to time derivative of E_y. We can also see that electric field lines always start at one face and end at the other face of the resonator, hence opposite charges accumulate on the opposite faces and thus a current flows from top to bottom of the resonator through the side walls. The Q of the rectangular resonator is given by [4]

$$Q = \frac{\pi\eta}{4R_s}\left[\frac{2b(a^2 + d^2)^{3/2}}{ad(a^2 + d^2) + 2b(a^3 + d^3)}\right] \tag{18}$$

where R_s is the surface resistivity of the conducting walls. For a cubical resonator i.e., $a = b = d$ we have the quality factor as

$$Q_{cube} = 0.742\frac{\eta}{R_s} \tag{19}$$

Let us consider the resonant frequency of the lumped element circuit consider in the first section i.e., 5GHz. If we build the cubical resonator using copper, the value of $R_s = 0.01845$ [2], $\eta = 377\Omega$ we get

[2] $R_s = \frac{1}{\sigma\delta}$ where σ is the conductivity of copper and δ is the skin depth of copper given by $0.066f^{-1/2}$. σ and δ is related by $\delta = \frac{1}{\sqrt{\pi f \mu\sigma}}$

a Q_{cube} of 15,161. So, for the resonant frequency of 5GHz, we can theoretically get a bandwidth of $\Delta f = \frac{f_0}{Q} \approx 330KHz$! This is ultra narrowband filter which is impossible to attain in lumped filters. Cavity resonators are ideal for very narrowband filters having very high resonant frequency. Remember that Δf refers to half-power bandwidth.

The resonant mode we just studied, which was also depicted in Fig 5 is known as TE_{101} mode as there is 1 half-wavelength variation in $x - direction$ and $z - direction$ and no variation in $y - direction$ but it should also be noted that the naming convention for modes depends on the chosen of direction of propagation. If $y - axis$ was chosen as direction of propagation, then the same mode can as well be called as TM_{110}.

The same analysis can be performed for TE_{mnp} and TM_{mnp} also. The expression for the resonant frequency for these modes in a rectangular resonator is [4]

$$f_0 = \frac{1}{2\pi\sqrt{\mu\epsilon}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \right]^{1/2} \tag{20}$$

From (20) we can observe that different modes with same set of m, n, and p have the same resonant frequency irrespective of the actual field distribution. Modes with different field patterns but having the same resonant frequency is called *degenerate modes*. Following vital observation can be made about rectangular cavities [4]

1) For a given box size, resonant frequency increases with increasing mode. Intuitively the explanation is like this; once the dimension is fixed and some lower order modes are getting excited, if we want to increase the mode number, then more number of waves have to accommodated in the same given dimension. This is possible only if the waves gets shorter (as the dimension is fixed) which means that frequency has to increase.
2) For a given resonant frequency, box dimension has to be increased to accommodate higher modes. The explanation parallels our earlier logic. If we want to put in more waves in a box, we have to increase box dimensions if the waves cannot be made shorter
3) Higher Q can be obtained by going for higher modes for a given resonant frequency. As the box becomes bigger, the volume to surface area ration increases thus reducing the losses. These higher order modes are made use of in *echo boxes*[3]

B. Circular Cylindrical Resonator

Resonators can also be constructed by putting shorting plates at the two ends of a circular waveguide as shown in Fig III-B . The excited mode corresponds to TM_{01} mode in a circular waveguide. Fig 6 shows the electric and magnetic field lines in the circular resonator. We can observe that opposite and equal charges exist on the endplates thus necessitating a current between the endplates. The field equations are

$$E_z = E_0 J_0(kr) \tag{21}$$

$$H_\phi = \frac{jE_0}{\eta} J_1(kr) \tag{22}$$

$$k = \frac{p_{01}}{a} = \frac{2.405}{a} \tag{23}$$

The resonant frequency is

$$f_0 = \frac{k}{2\pi\sqrt{\mu\epsilon}} = \frac{2.405}{2\pi a\sqrt{\mu\epsilon}} \tag{24}$$

[3]echo boxes can refer to any resonant cavity, but usually refers to high-Q, low-loss resonators used to store some part of the energy in transmitted radar pulse, so that receiver can then use it for tuning or calibration purposes

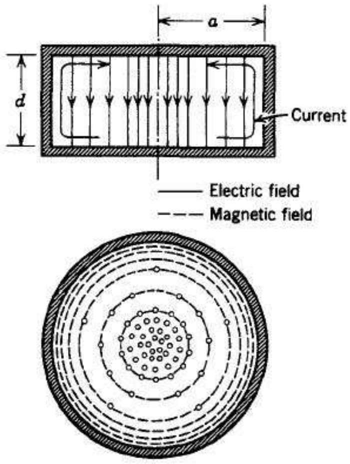


Fig. 6. Electric and magnetic fields inside a circular resonator [4]

Q is given by

$$Q = \frac{\omega_0 U}{W_L} = \frac{\eta}{R_s}\frac{p_{01}}{2(a/d+1)} \tag{25}$$

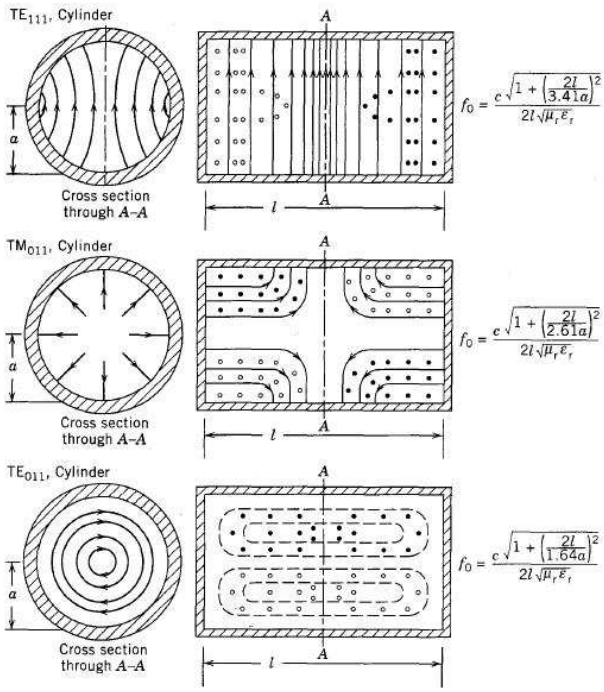


Fig. 7. Different excitation modes in a circular cylindrical resonator [4]

Fig 7 shows different excitation modes possible in a circular resonator along with the expression for resonant frequencies. One important observation about TE_{011} is that the currents in both the cavity and endplates are circumferential and hence good contact is not necessary between the plates and the cavity if one of the plates has to be moved for tuning purposes.

C. Elliptical resonators [5]

These types of resonators have elliptical cross section and are found to have dual-mode resonance.Fig 8 shows cylindrical resonators with elliptical cross-section

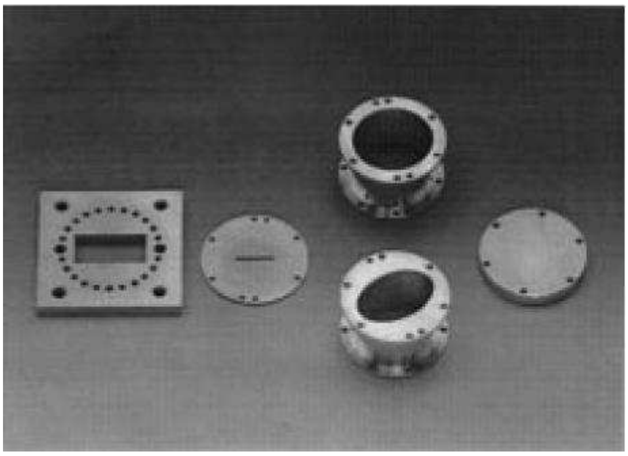


Fig. 8. Elliptical cavity resonators [5]

These resonators exhibit dual mode resonance depending upon the inclination angle of the junction between rectangular and elliptical resonator. Rigorous fullwave solutions for the fields inside the resonator can be obtained from [5].

D. Band Gap Cavity Resonators [6], [7]

Eventhough this type of resonators are not meant for waveguide circuits, they are very interesting to study. These types of cavity resonators use the metal layers of a PCB as the top and bottom surfaces of a waveguide and use a *via fence* as sidewalls as shown in Fig 9 . This setup acts exactly as a cavity resonator.

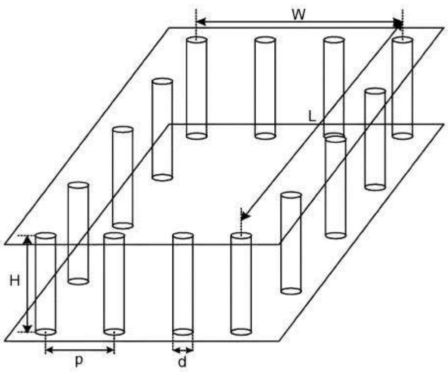


Fig. 9. Cavity resonators using via fence as sidewalls [6]

IV. Excitation of Cavity Resonators

Cavity resonators will be useful as circuit elements only if they can be coupled to other circuit elements. Energy has to be fed into the cavity and taken out from it for it to be useful as a filtering element. Excitation techniques used for launching waves into the waveguide can be used for coupling or exciting waves in a resonator as well. Following are the common methods used for coupling

1) A probe or an antenna oriented in the direction of electric field as shown in Fig 10. The position of the probe in the waveguide is chosen according to the coupling required and the impedance matching. We know that the voltage V will be zero near the short plates and current will be maximum at that position. As we move away from the shorting plates V and I will show a sinusoidal variation

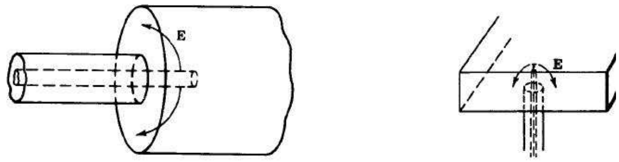


Fig. 10. A probe used to excite waveguides and cavity resonators [4]

throughout the resonator. This also means that the impedance also varies as we move along the resonator. Impedance matching can be obtained by placing the probe at the appropriate position.

2) A current carrying loop oriented in the plane normal to the magnetic field as shown in Fig 11.

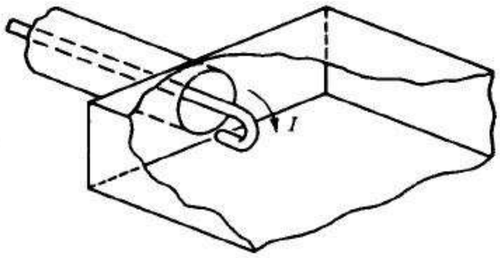


Fig. 11. A loop used to excite waveguides and cavity resonators [4]

3) A small slit or an aperture in the resonator connecting it to waveguide circuit as shown in Fig 12. A slit in the transverse plane acts as inductive impedance and a slit along the broadside acts as a capacitive impedance. Hence they are sometimes called capacitive and inductive *iris*. The impedance offered by the slits depends upon the slit size and shape. For critical coupling the impedance of the aperture is [3]

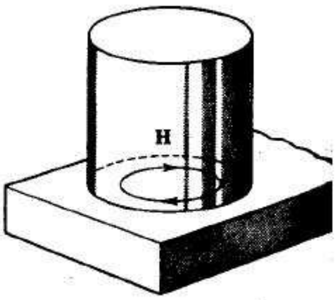


Fig. 12. Apperture used to excite waveguides and cavity resonators [4]

$$X_L = Z_0\sqrt{\frac{\pi k_0 \omega_{res}}{2Q\beta^2 c}} \tag{26}$$

where k_0 is the cutoff wavenumber. The normalized inductive suseptance for the aperture is given by [3]

$$B = \frac{-ab}{2\beta\alpha_m} \tag{27}$$

where a and b are the dimensions of the waveguide. The value of α_m can be obtained from Table 1.

Aperture shape	α_e	α_m
Round hole	$\frac{2r_0^3}{3}$	$\frac{4r_0^3}{3}$
Rectangular hole	$\frac{\pi l d^2}{16}$	$\frac{\pi l d^2}{16}$

Table 1: Electric and Magnetic Polarizations [3]

A proper selection of excitation technique depends on the allowable insertion loss, deviation in the resonant frequency, bandwidth, return loss and Q. [8] examines the 3 different excitation technique for a via fence cavity resonator at 60 GHz. They have examined slot excitation with shorting vias, slot excitation using a $\lambda_g/4$ open stub, and probe excitation.Fig 13 summarizes their findings

	slot excitation with a shorting via	slot excitation with an open stub	probe excitation
Resonant Frequency (f_{res})	59.4 GHz	59.2 GHz	59.8 GHz
Insertion Loss (S21)	1.28 dB	0.84 dB	0.95 dB
Return Loss (S11)	18 dB	20.59 dB	22.3 dB
Bandwidth (BW)	1.18 %	1.5 %	1.8 %
Simulated Unloaded Q (Q_u)	360	367	355
Measured External Q (Q_{ext})	73.23	60.52	49.80

Fig. 13. Comparison of excitation techniques for cavity resonator at 60GHz [8]

V. Perturbations in Cavity Resonators

Perturbations are in general any form of irregularity in the cavity. These perturbations alter either the electric field or the magnetic field. This changes the capacitance or the inductance of the cavity thus altering the resonant frequency. Perturbations can also occur in the form of surface roughness, which alters the losses present thus altering the Q of the cavity. Theoretically the perturbation analysis is done by noting the change in the resonant frequency with changes in its volume [4].

$$\frac{\Delta\omega}{\omega_0} = \frac{\int_{\Delta V}(\mu H^2 - \epsilon E^2)dV}{\int_V(\mu H^2 + \epsilon E^2)dV} = \frac{\Delta U_H - \Delta U_E}{U} \tag{28}$$

where ΔU_H is the magnetic energy removed, ΔU_E the electrical energy removed, and U is the total energy stored. (28) is the perturbation analysis equation for *shape perturbations*,[9] gives a detailed perturbation analysis for some specific forms of shape and material perturbations.

VI. Waveguide resonators as Filters

Most widely used application of waveguide resonators are as filtering elements. Cavity resonators are also used as dummy loads wherein their resonance and antiresonance impedance is made use of. We will look into some interesting filtering applications. We will also look into some latest trends involving micro machining techniques.

The frequency selectivity of the cavity resonators can be used in filtering by stacking several of them together. Bandpass filters are constructed by coupling several cavities. The number of cavities to be coupled depends on the stopband attenuation and the bandwidth. As already mentioned Q values of several thousands can be obtained with such cavity filters.

We will start with ridged waveguide filters. Even though they are not exactly cavity based filters, the individual ridges can be considered as cavity themselves and the successive ridges can be thought of as coupled cavities. Ridged waveguides are essential slow wave structures[4]. Ridged waveguides involve

[4]slow wave structures are those in which the phase velocity along the direction of the axis is less than the velocity of light. Spirals or helixes being one example

periodic structures for filtering action. Compact filters have been built using such ridged waveguides for X-band. [10] describes a X-band ridged waveguide bandpass filter with an integrated lowpass filter for improved spurious response. These filters are sometimes called combline filters also.

Another form of widely used cavity resonator is the rectangular or circular coaxial cavity resonator. The structure is same as uniform cavity except for the rod in the center. This rod is usually used to tune the cavity resonator by changing its height and width. The center rod has an associated capacitance w.r.t to the resonator walls. So, by changing its width and height, capacitance changes and thus changing the resonant frequency. One thing to be noted is that when the dimensions of the tuning rod is altered, Q factor is also altered as the volume inside the cavity changes. Several variations of these have also been implemented. One important variation being the center conductor periodically loaded with dielectric discs [11] achieving very high values of Q. Fig 14 shows a filter using combline structures involving coaxial resonators

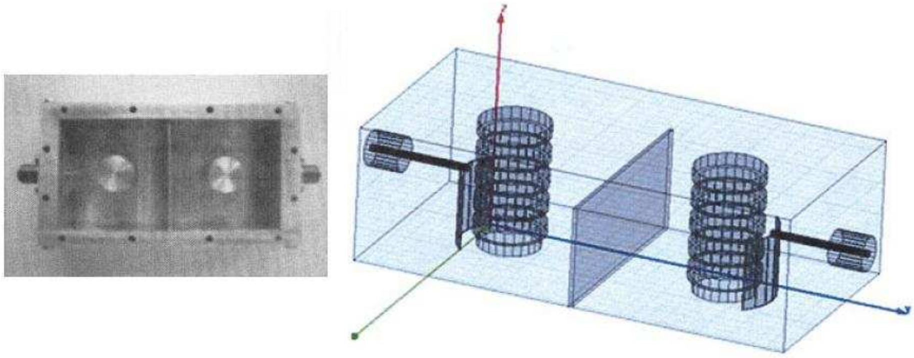


Fig. 14. Combline Bandpass filters using coaxial resonators [10]

Dielectric loading of cavity resonators is sometimes used, which has many advantages. By placing a dielectric resonator inside the cavity, the guide wavelength is brought down thus leading to compact configuration. The important consideration here is the Q of the dielectric material should be very high given by

$$Q_{diel} = \frac{1}{tan\delta}$$

where $tan\delta$ is the loss tangent of the dielectric material. Apart from the increase in Q factor and decrease in size, another major advantage is the insensitivity of the resonator for the perturbations in cavity walls. These observations reported in [12], [13] can be explained intuitively. At very high frequencies, the field lines are concentrated in the *puck* region and hence the conductor loss is reduced. Here again we can see the importance of using a dielectric of high Q. The dielectric resonator is so placed inside the metal cavity so that it is near to the metal walls only at 4 points and any perturbations in the cavity walls that are far from it does not really affect the cavity.

We will now study filter structures made up of co-axial cavities. These are widely used in commercially available products. [14] outlines the procedure to design bandstop filters using coupling of cavity resonators. The procedure starts with the generation of short circuit admittance parameters $y_{21}(s)$ and $y_{22}(s)$ from the prototype polynomials [14]

$$S_{11}(s) = \frac{\frac{P(s)}{\epsilon}}{E(s)}; S_{21}(s) = \frac{\frac{F(s)}{\epsilon_R}}{E(s)}$$

as

$$y_{21}(s) = \frac{y_{21n}(s)}{y_d(s)} = \frac{\left(\frac{F(s)}{\epsilon_R}\right)}{m_1(s)}$$

$$y_{22}(s) = \frac{y_{22n}(s)}{y_d(s)} = \frac{n_1(s)}{m_1(s)} \tag{29}$$

for even-degree networks and

$$y_{21}(s) = \frac{\left(\frac{F(s)}{\epsilon_R}\right)}{n_1(s)}$$
$$y_{22}(s) = \frac{m_1(s)}{n_1(s)} \tag{30}$$

for odd-degree networks where

$$m_1(s) = Re(e_0 + p_0) + jIm(e_1 + p_1)s + Re(e_2 + p_2)s^2 + \cdots$$
$$n_1(s) = jIm(e_0 + p_0) + Re(e_1 + p_1)s + jIm(e_2 + p_2)s^2 + \cdots$$

where e_i and p_i are the complex coefficients of $E(s)$ and $P(s)/\epsilon$ respectively. Once $y_{21n}(s), y_{22n}(s)$ and $y_d(s)$ are determined,the coupling matrix synthesis is done as outlined in [15]. Direct source-load coupling coefficient M_{SL} is obtained

$$jM_{SL} = \left.\frac{y_{21}(s)}{y_d(s)}\right|_{s=j\infty} = \left.\frac{\frac{jF(s)}{\epsilon_R}}{y_d(s)}\right|_{s=j\infty} \tag{31}$$

One such example of a BS filter based on chebyshev polynomial is presented here. Following the procedure as outlined the coupling matrix is obtained [14]

$$\mathbf{M} = \begin{bmatrix} 0 & 0.5109 & 0 & 0 & 0 & 1 \\ 1.5109 & 0 & 0.9118 & 0 & 0.9465 & 0 \\ 0 & 0.9118 & 0 & 0.7985 & 0 & 0 \\ 0 & 0 & 0.7985 & 0 & 0.9118 & 0 \\ 0 & 0.9465 & 0 & 0.9118 & 0 & 1.5109 \\ 1 & 0 & 0 & 0 & 1.5109 & 0 \end{bmatrix}$$

The coupling matrix is then translated to a cavity filter as shown in Fig 15 . We can see from the coupling matrix $\mathbf{M}$ that all the coupling coefficients are positive implying an inductive coupling. These inductive coupling are realized as slits.

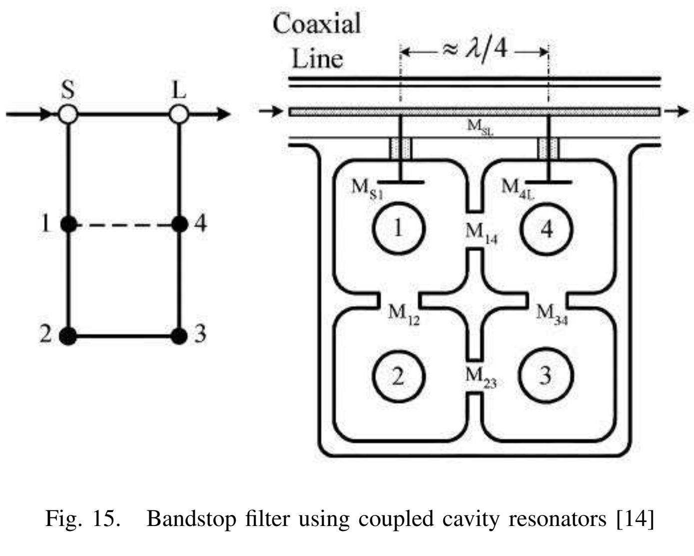


Fig. 15. Bandstop filter using coupled cavity resonators [14]

Another example is a 4-2 BS filter with asymmetric frequency response with 22dB return loss and 2 transmission zeroes at $s = +j1.3127$ and $s = j1.8082$. The coupling diagram and realization is as shown in Fig 16

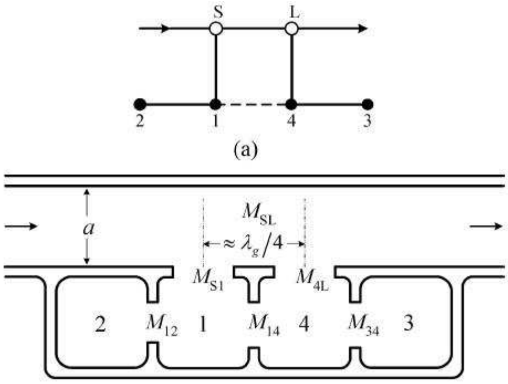


Fig. 16. A 4-2 Bandstop filter using coupled cavity resonators [14]

The frequency response of such a filter is shown in Fig 17. A couple of other examples involving

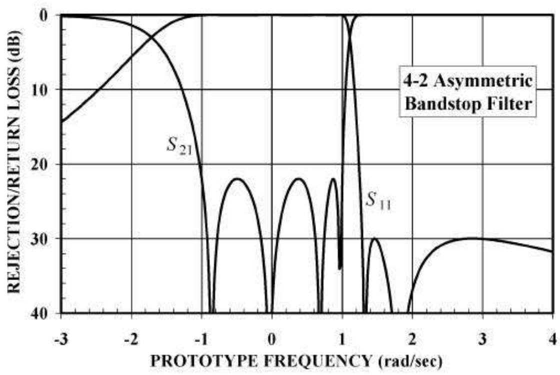
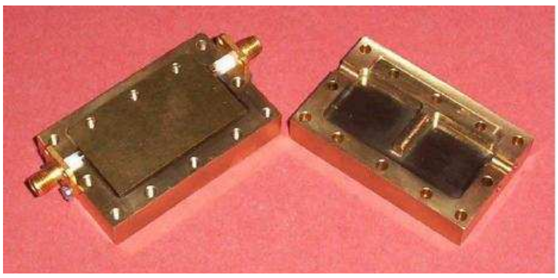


Fig. 17. Frequency response of the 4-2 BS filter[14]

dual-band BS filters are given in [14].

We know that cavity dimension should be $\lambda_g/2$ for it to resonate but [16] describes a new variant called folded waveguide resonator. The waveguide is bifurcated by a metal plate with slots at the ends which, allows the standing waves to fold along the bifurcation. This reduces the formfactor of the resonator to half of the regular waveguide resonator. Fig 18 shows one such example involving the folded waveguide resonator [16].



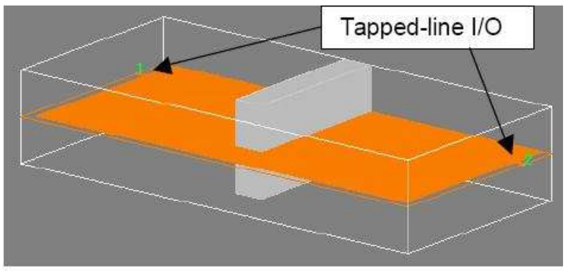


Fig. 18. Folded waveguide resonator centered at 4.665GHz and Q = 650 [16]

Q factor can be further increased in coaxial coupled resonators by rounding the edges. This rounding essentially reduces the path length taken by the current thus reducing the losses. But the rounding also reduces the volume of the cavity, which might bring down the Q. [17] has analysed the effects of rounding and has reported an increase of 5% in Q values. Fig 19 shows such a cavity filter.

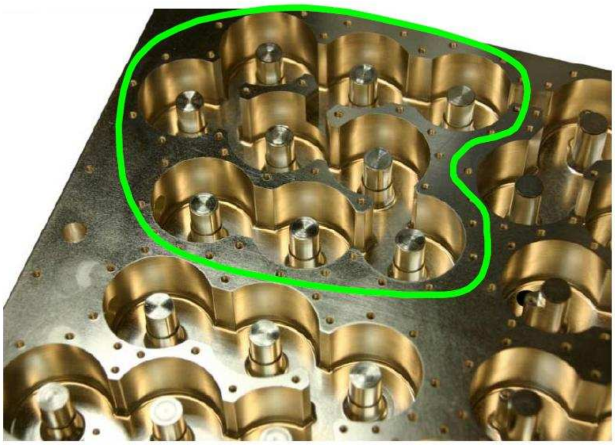


Fig. 19. Cavity resonator filter with rounded edges [17]

VII. Latest Trends

The major advancement in the recent times in the field of fabrication of cavity resonators has been micromachining technique known as LIGA.One of the main factors that reduces the Q of the cavity is the conductor loss. This can be reduced by plating the surface with high conductivity metal like silver or gold. However with such plating, surface roughness plays an important role in deciding the Q. At low frequencies, the skin depth will be very large compared to the roughness and it wont alter the Q. But at very high frequencies, roughness forms a substantial part of the skin depth and a major fraction of the surface current follows the roughness profile thus increasing the conductor losses. Hence at very high frequencies the surface should be highly smooth to keep the conductor losses to minimum possible.

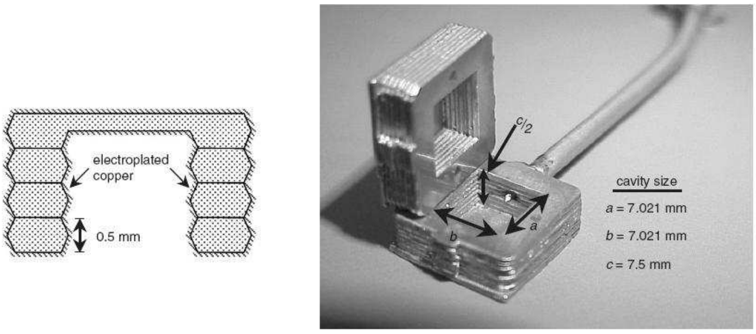


Fig. 20. Cavity resonator using micromachined splitblock technique [18]

[18] describes a new technique using micromachining on silicon wafers and then bonding different plates together to form a cavity resonator as shown in Fig 20. The same paper claims that a Q of 4550 was obtained at 29.325 GHz. So achieving smoothness is the key to obtain high values of Q. LIGA is a technique used in fabrication of MEMS which has been used to form cavities with very high degree of surface smoothness. The technique uses X-rays in the photolithographic process to etch patterns [19]. Fig 21 shows a SEM image of a cavity formed using such a technique.The unloaded Q claimed by the paper is 2684.2 ± 7.2 at 24.0026 GHz.

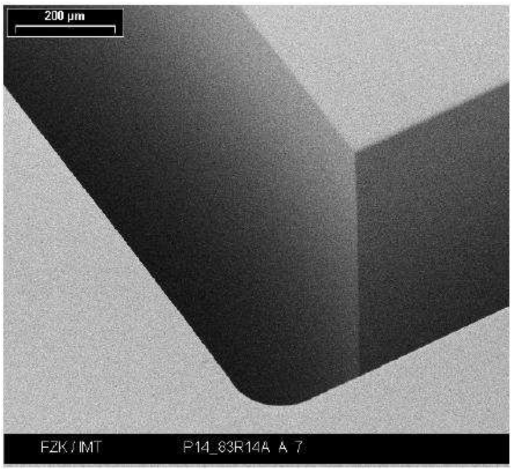


Fig. 21. Edges of a cavity resonator using LIGA technique [19]

VIII. Conclusions

We saw that cavity resonators are the best choice when very high values of Q is needed. We also saw some examples of cavity resonators where Q of over 2500 was obtained at K-band and higher. These values of Q is impossible to obtain with lumped element based resonators for two reasons. Primary reason being that at very high frequency, the inductors and capacitors required are too small to be physically realisable[5]. The second reason being the very low values of Q for these components. We saw that the Q of the resonator is lesser than the lowest Q of the component. Inductors with Q over 100 is not available in very high frequency region [6]. We also saw that SAW filters can also be very attractive option because of their compact profile but they have very limited frequency of operation [7]. SAW filters have also limited power dissipation capabilities as already mentioned.

Thus we can say that Cavity resonators are the best possible option above C-band. With proper design very low insertion loss can be obtained. This high value of Q makes them a good choice for diplexers. Very high isolation can be obtained between ports using such cavity resonators

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[5] Inductor manufactures like Murata and Coil craft have inductances as low as 0.5nH

[6] Coil craft and Murata manufactures very high-Q inductors which can have a maximum value of 100 and even this has a downward trend in GHz region

[7] IDT has SAW filters upto GPS frequency of 1575.42MHz with a bandwidth of 2MHz and an insertion loss of 1.8dB(max)

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