Distribution Learning of a Random Spatial Field with a Location-Unaware Mobile Sensor

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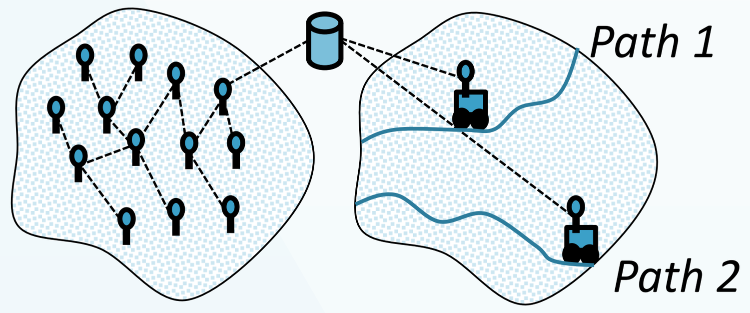
Introduction

- Learning the (statistical) distribution of physical fields from observed values is a fundamental task in applications like, environment monitoring, smart city and pollution control.



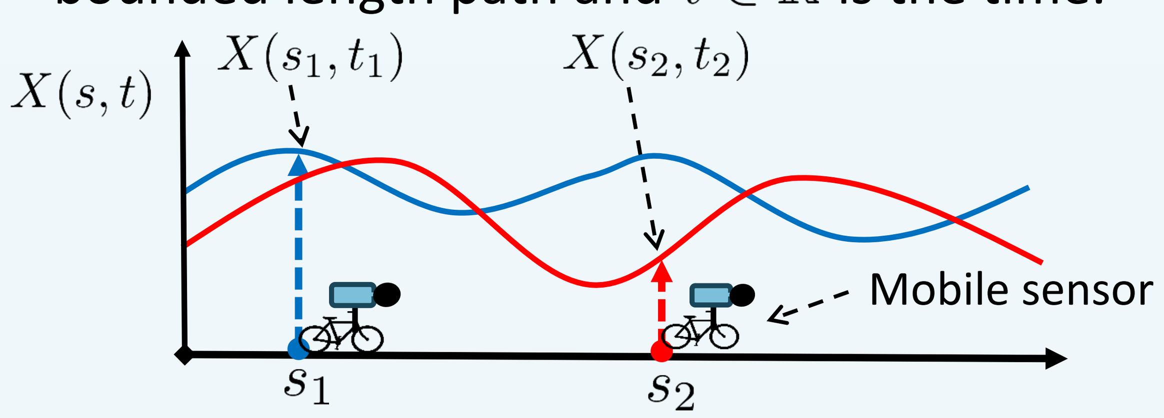
-fixed sensor -mobile sensor -remote processing node

Spatial field sensing: (a) with fixed grid of sensors. (b) with mobile sensors.

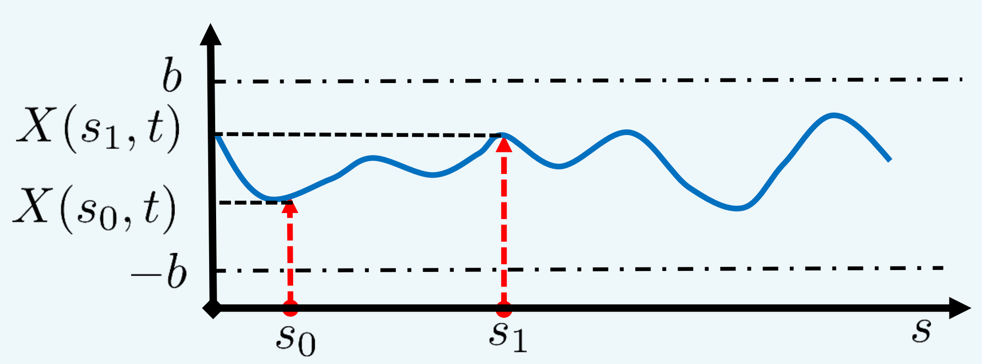
- Mobile sensing is a way to increase spatial sampling density however their location uncertainty is a challenge.
- Since localization methods (like GPS and wireless outdoor localization) require extra energy and hardware, we aim to learn the statistical distribution of spatial fields as a function of space using location unaware mobile sensors.

Spatial field

- The field of interest is $X(s,t)$ where $s \in \mathcal{P}$ is a bounded length path and $t \in \mathbb{R}$ is the time.



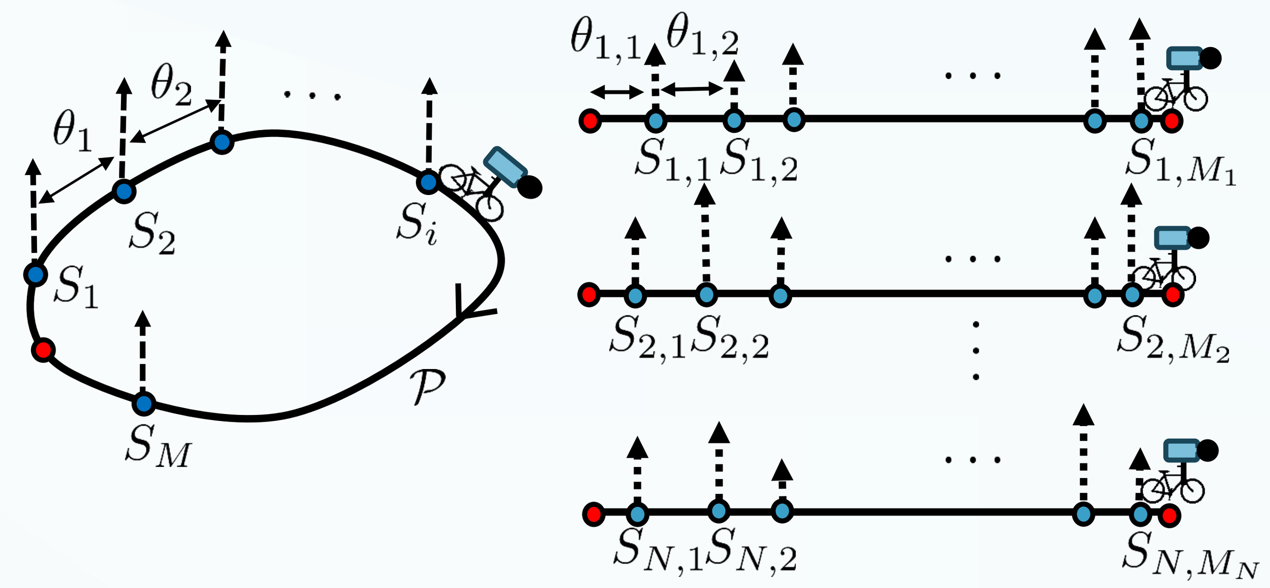
- The field is assumed to be time varying, bounded and Lipschitz continuous.



$|X(s_1,t) - X(s_0,t)| \leq \alpha |s_1 - s_0|$

Sensing model

- The spatial field is sampled using a location unaware mobile sensor.
- N trials of the mobile sensing are performed.

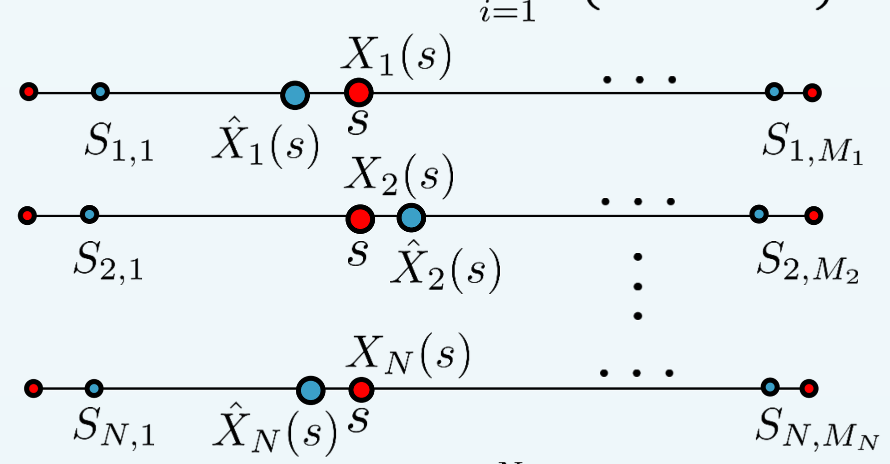


- The unknown sampling locations $S_1, S_2, \ldots, S_M$ are modeled using an unknown renewal process.
- The random variables $\theta_1, \theta_2, \ldots$ are independent and identically distributed.
- It is assumed that $0 < \theta \leq \frac{\lambda}{n}$ and $\mathbb{E}(\theta) = \frac{1}{n}$ where $\lambda > 1$ is finite.
- average sampling rate is n per meter and λ represents maximum sensor speed.

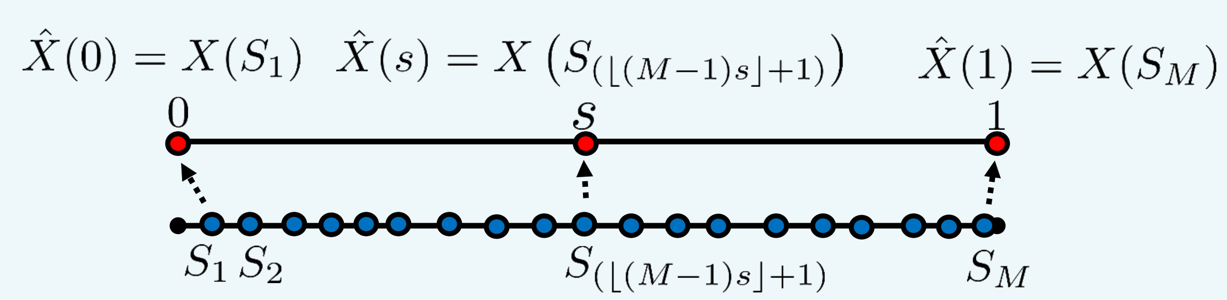
Distribution Learning method

- We wish to estimate the spatial field distribution at location s, $F_{X(s)}(x) = \mathbb{P}(X(s) \leq x)$ using samples from the location unaware mobile sensor.

$$F_{X(s)}(x) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}\left\{X_i(s) \leq x\right\}$$



$$F_{\hat{X}(s)}(x) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}\left\{\hat{X}_i(s) \leq x\right\}$$



- The error between $F_{X(s)}$ and its estimate $F_{\hat{X}(s)}(x) = \mathbb{P}(\hat{X}(s) \leq x)$ depends on error in location estimation $|S_{\lfloor (M-1)s+1 \rfloor} - s|$ and smoothness of the field i.e. α.

- The mean squared error between location s and its estimate $S_{\lfloor (M-1)s \rfloor + 1}$ is

$$\mathbb{E}\left[\left|S_{l(M,s)} - s\right|^2\right] \leq ((n+\lambda-1)s(1-s) + C)\frac{\lambda^2}{n^2},$$

and the error in field estimation is

$$\mathbb{P}\left(|X(s) - X(S_{\lfloor (M-1)s \rfloor + 1})| > \varepsilon\right) \leq \frac{\alpha^2}{\varepsilon^2}((n+\lambda-1)s(1-s) + C)\frac{\lambda^2}{n^2}.$$

This upper bound depends on s and has a maximum at $s = 1/2$.

> *Theorem 1:* For every $x \in \mathbb{R}, s \in [0,1]$ and for any $\varepsilon > 0$,
> $$|F_{X(s)}(x) - F_{\hat{X}(s)}(x)| \leq \varepsilon . \max\left(f_{X(s)}(x)\right) + \frac{\alpha^2}{\varepsilon^2}((n+\lambda-1)s(1-s) + C)\frac{\lambda^2}{n^2}.$$

- The supremum of error in field estimation over $s \in [0,1]$ is

$$\mathbb{P}\left(\sup_{s\in[0,1]} |X(s) - X(S_{\lfloor (M-1)s \rfloor + 1})| > \varepsilon\right) \leq \frac{32}{\beta}\frac{\alpha^2}{\varepsilon^2}(n+\lambda-1)\frac{\lambda^2}{n^2}.$$

> *Theorem 2:* For every $x \in \mathbb{R}, s \in [0,1]$ and for any $\varepsilon > 0$,
> $$\sup_{s\in[0,1]} \left|F_{\hat{X}(s)}(x) - F_{X(s)}(x)\right| \leq \varepsilon . \max\left(f_{X(s)}(x)\right) + \frac{32}{\beta}\frac{\alpha^2}{\varepsilon^2}(n+\lambda-1)\frac{\lambda^2}{n^2}$$

- β is the symmetrisation constant.
- If $\varepsilon = \frac{1}{n^{1/3}}$

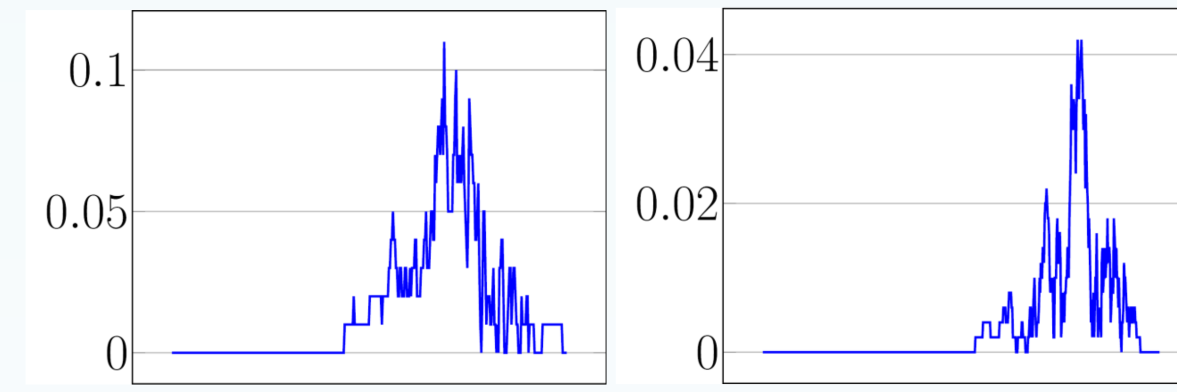
$$|F_{X(s)}(x) - F_{\hat{X}(s)}(x)| \leq (\max\left(f_{X(s)}(x)\right) + \alpha^2 s(1-s))\frac{\lambda^2}{n^{1/3}} + \alpha^2((\lambda-1)s(1-s) + C)\frac{\lambda^2}{n^{4/3}}.$$

Simulation

- To validate our distribution learning method we consider acoustic levels varying along a 1-D path.
- The spatial field is simulated by

$$X(s,t) = \left|1000 + \sum_{r=1}^{10} A_r(t)\cos(2\pi f_r(t) s)\right|.$$

- The sampling locations are modelled by a renewal process where inter-sample intervals are modelled by a Rayleigh distribution with parameter $\frac{1}{n}\sqrt{\frac{2}{\pi}}$.



(a) for N=100 and n=100 *(b) for N=100 and n=1000*

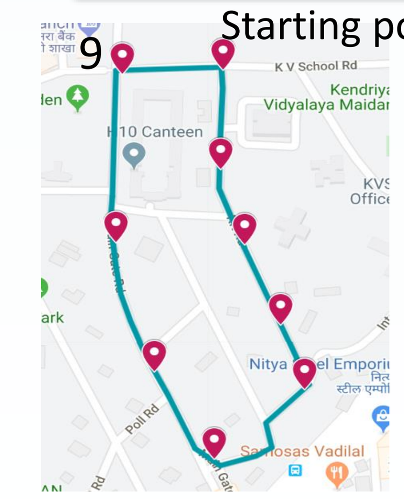
$\left|\hat{F}_{X(s)}(\frac{1}{2}) - \hat{F}_{\hat{X}(s)}(\frac{1}{2})\right|$

Experiment

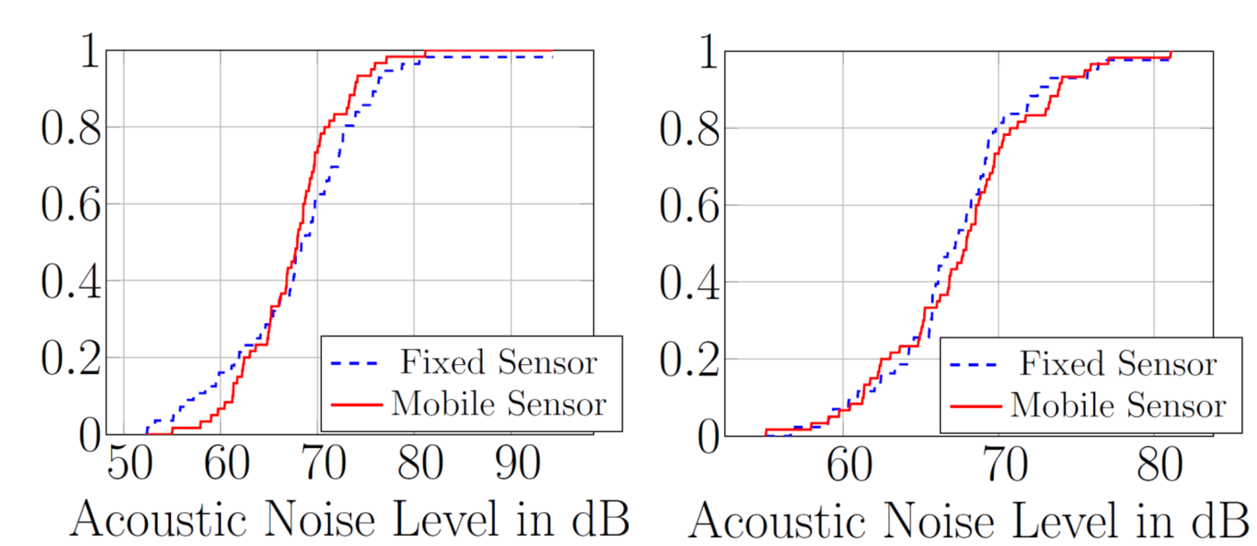
- Acoustic noise levels are measured along the path shown in the map using the sound meter by BAFX products (Model no: BAFX3608).

Specifications of sound meter

Range: 30-130 dB	*Sampling rate: 1 per sec*
Memory: 4700 readings	*Accuracy: ± 1.5 dB*



Dataset is collected on the path shown in the map.
Dataset 1: 43 trials of mobile sensing by walk, Samples of static sensors
Dataset 2: 43 trials of mobile sensing using a bicycle , Samples of static sensor



(a) location 9 using dataset 2 *(b) location 9 using dataset 1*

$\hat{F}_{X(s)}(s)$ vs $\hat{F}_{\hat{X}(s)}(s)$

Conclusion

- We proposed a data-driven method for learning the statistical distribution of a Lipschitz continuous spatial field along a path.
- We showed that the accuracy of the distribution-learning method increases with the spatial sampling rate of the mobile sensor.

Acknowledgements

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