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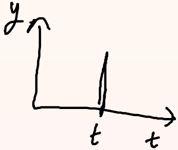
– convolution continuous time
– robustness of using feedback.

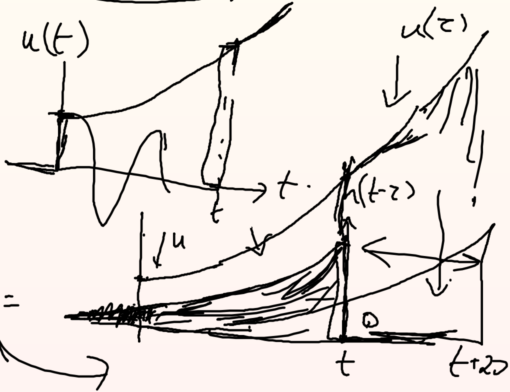
$$y(t) = u * h\ (t)$$

weighing (of u)
function

$$= \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau.$$



$y(t) =$

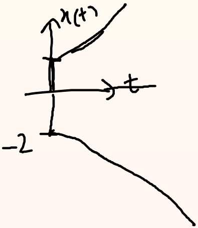


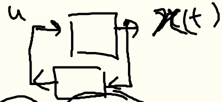
$$\dot{x} = 3x + u$$

$$\frac{d}{dt} x = 3\, x(t)$$

$$x(0) = 1.$$

$$x(t) = x(0)\, e^{3t}$$





$$u = -5x.$$

$$\dot{x} = 3x - 5x$$

$$= -2x$$

$$x(t) = x(0)\, e^{-2t}$$

$$u(t) = -5\, x(0)\, e^{-2t}$$

$$x(0) = 1.$$

$$u(t) = -5 e^{-2t}$$

$$-5e^{-2t} \rightarrow \boxed{\dot{x} = 3x + u} \rightarrow x(t)$$

nominal

perturbation

(a) $\dot{x} = 3.01x + u$

(b) $x(0) = 1.01$

"Robust"

$x(0) \to 1.$

$\dot{x} = 3x - 5e^{-2t}$

$\mathcal{L}(f(t)) = F(s)$

$\mathcal{L}(x(t)) = X(s)$

$\mathcal{L}(f') = sF(s) - f(0)$

$$sX(s) - x(0) = 3X(s) - \frac{5}{s+2}$$

$$(s-3)X(s) = 1 - \frac{5}{s+2} = \frac{s-3}{s+2}$$

$$X(s) = \frac{\cancel{(s-3)}}{(s+2)\cancel{(s-3)}} = \frac{1}{s+2}$$

$u(t) = -5e^{-2t}.$

$$\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) = e^{-2t} = x(t)$$

$\dot{x} = 3.01x + u. = 3.01x \boxed{-5e^{-2t}}$

$x(0) \to$

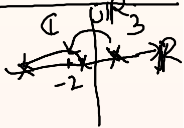
$1 \to 1.01$

$sX(s) - 3.01X(s) = \underline{x(0)} - \frac{5}{s+2}$

$(s-3.01)X(s) = \frac{s-3}{s+2}.$

Robustness of "closed loop" feedback.

$X(s) = \frac{s-3}{(s-3.01)(s+2)} = \frac{a}{s-3.01} + \frac{b}{s+2}.$



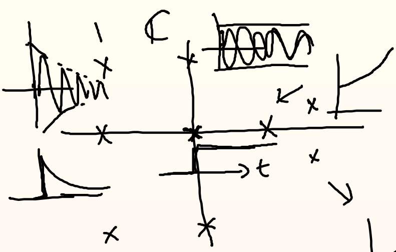
$a \approx 0$

$b \approx 1.$

$x(t) = \underline{a e^{3.01t}} + b e^{-2t}$

Had you used $u = -5x$ $\dot{x} = 3.01x + u$

check $3 \to 3.01$



$$\mathcal{L}(f(t)) = \frac{1}{s+a}$$

$$= \frac{(ps+q)}{s^2+bs+c}$$