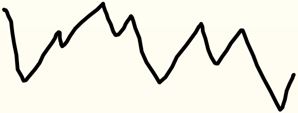

EE113: other closely related topics

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This lecture: a glimpse of inter-relations between

- # Kirchhoff's current laws, # Kirchhoff's voltage laws (for planar/more-general circuits?)
- Tellegen's theorem: topology of circuit (already) implies conservation of power
- Laplacian matrix L: window to a world of graph theory
- Hairy ball theorem: (in)ability to 'smoothly comb' hair on a fully haired ball
- Euler's formula (convex polyhedron):
  #Faces - #Edges + #Vertices = 2 (or more generally (for toruses)?)
- Types of equilibrium points (and their index) (on a sphere/other 'manifolds')

Inverted/regular pendulum:

$\sin\theta \approx \theta$.

manifolds

Linearized model: about the equilibrium point: local analysis

Globally: Torus → double pendul

Shoulder-joint: sphere (globally)

locally plane

$(0, 360°)$

Torus

hinge

hinge

encoder → angle sensor

$(0, +360)$

actuator

double inverted pen

Laplacian matrix

~~ball & sod~~

$$\begin{array}{c|c} & e_1 \quad e_2 \quad \cdots \quad e_5 \\ \hline p & \\ q & \\ r & \\ s & \end{array} \begin{bmatrix} +1 & & \\ & 0 & \\ -1 & & \end{bmatrix}$$

node-edge incidence matrix

undirected

$\in \{0, 1, -1\}$

directed graph

Laplacian matrix of the (undirected graph) $L = AA^T$
(A is node-edge incidence matrix)
Same as $D - A_{\text{adjacency}}$: degree matrix - 'Adjacency matrix' of the undirected graph!
L gives number of (spanning) trees: Kirchhoff's matrix tree theorem
Neat link with multi-agent systems

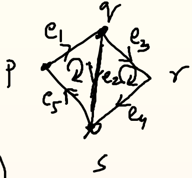
directed graph

$p \xrightarrow{e_1} q$

$L = \begin{bmatrix} 2 & - & -1 \\ & 3 & 0 \\ -1 & & 2 \\ -1 & 0 & 3 \end{bmatrix}$

Tellegen → weak
⟹ strong.

$v_1 = v_p - v_q$



KCL $\quad i = B^T i_{\text{loop-current}}$

through currents.

$$v = A^T v_n$$

voltage drop

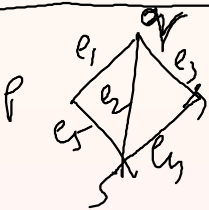
node voltages.

$Ax = b$

$$Ax = b$$

$$\begin{bmatrix} 5 & 6 \\ 7 & 8 \\ 9 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} x_1 + \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix} x_2 = \begin{bmatrix} 11 \\ 0 \\ -3 \end{bmatrix}$$

$$5x_1 + 6x_2 = 11$$



Spanning tree

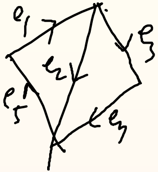
n−1

no loop

$\{e_1, e_2, e_3\}$

$\{e_1, e_2, e_4\}$, $\{e_1, e_3, e_4\}$

weak form.



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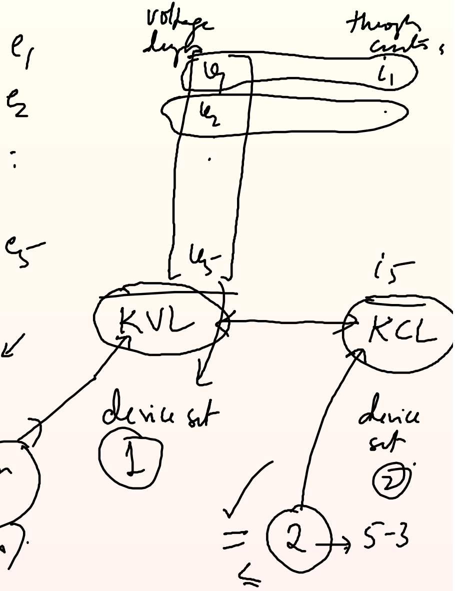
$0 = v_1 i_1 + v_2 i_2 \cdots \quad v_5 i_5$

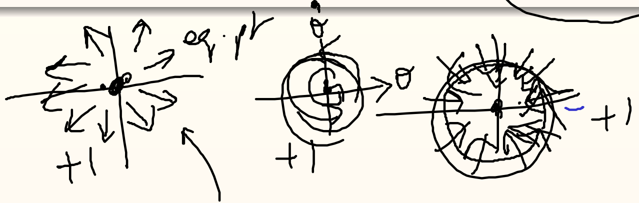
strong form.

$5x_1 + 6x_2 = 0$

$10x_1 + 12x_2 = 0$



Types of equilibrium points (planar), isolated



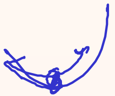
Simple ones: center, node (unstable/stable), focus (unstable/stable), saddle

+1

Index ±1

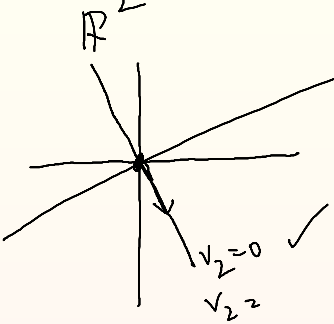
−1

Non-simple ones: index-2

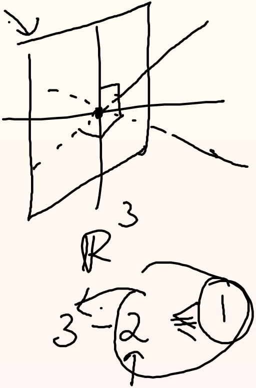


−1

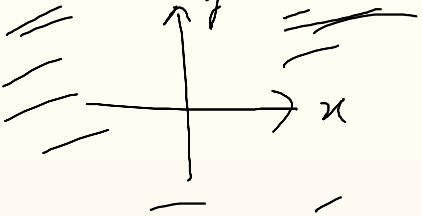
optimization



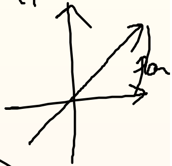
$v_1 \perp$ set S



plane

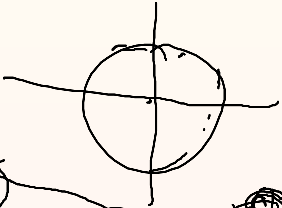


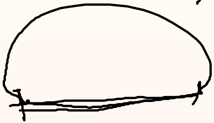
line



circle

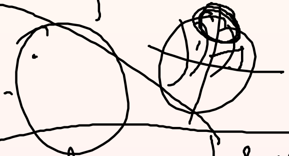
θ_1





homeomorphic

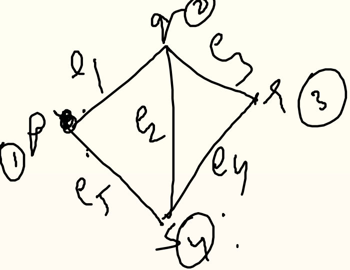
θ_2



plane $\times$ line

(x, y) , (z)

$\boxed{\text{Circle 1} \times \text{circle 2} = \text{torus}}$ cross-product



4-nodes.

$\in \mathbb{R}^{4\times 4}$

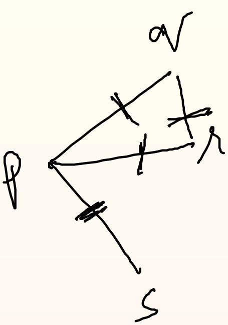
D - diagonal matrix $\rightarrow$ degree

$$D = \begin{array}{c} \\ p \\ q \\ r \\ s \end{array} \begin{array}{c} \begin{array}{cccc} p & q & r & s \end{array} \\ \begin{bmatrix} 2 & & & 0 \\ & 3 & & \\ & & 2 & \\ 0 & & & 3 \end{bmatrix} \end{array}$$

Adjacency

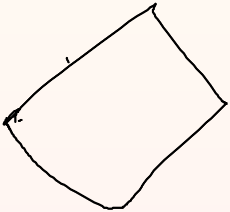
$$A_{adj} = \begin{array}{c} \begin{array}{cccc} p & q & r & s \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ p \\ q \\ r \\ s \end{array}$$

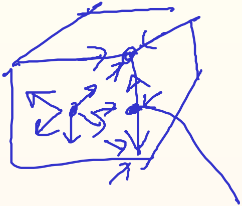
$$L = D - A_{adjacency}$$



Complete g

Cycle





edge = saddle — -1

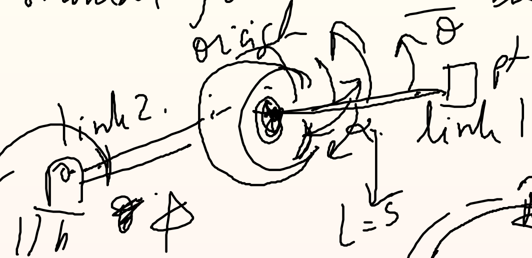
Face, vert = node — $+1$

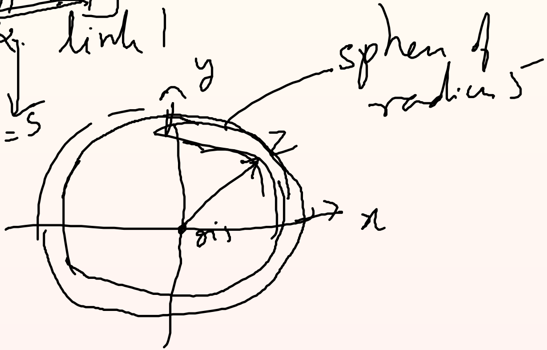
$$F - E + V = 2$$

Hairy ball theorem

Tutorial on Thursday: will ~~send problems~~ today

shoulder joint = ball & socket jt





C × S