EE 113 - Control Systems Module

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Key elements of control

Control Theory

- Regulation & Tracking
- Trajectory Optimization

controller implementation

implement ~ "carefully designed law" → policy equation

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Mathematical aspects get applied in engineering

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Involves analysis (mathematical modelling, and response analysis) and then synthesis (controller design, shaping)

Dynamical systems

- Systems have delays/dynamics/memory
- Generators: when load increases, speed decreases (in some time)
- Heat: effect on temperature
- Vehicle control: steering/brakes: some delay before it has an effect
- Filters: input/output relation is 'dynamic'
- Economic systems: cash-reserve-ratio: effect on markets/liquidity in some time
- Economic systems: interest rates (FD/lending): effect on economy: delays

Overview: read from Polderman & Willems:
Intro to mathematical systems theory: a behavioral approach

Control Theory - Regulation

- Plant: system to be controlled
- Regulation: keep certain to-be-controlled variables at desired values (set-point)

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- Plant: system to be controlled
- Regulation: keep certain to-be-controlled variables at desired values (set-point)
- In spite of external disturbances
- In spite of changes in plant properties
- Examples:
  - Temperature control at home/office
  - Suspension (passive/active) of an automobile (absorbs the irregularities of the road to improve the comfort and safety)
- Regulation: for efficiency, quality control, safety, and reliability.

Control inventions

History

- Christiaan Huygens (1629-1695) invented a flywheel device for speed control of windmills
- Main idea used later: centrifugal fly-ball governor (by James Watt, 1736-1819, the inventor of the steam engine)
- Tuning centrifugal governors that achieved 'fast regulation',

Control inventions

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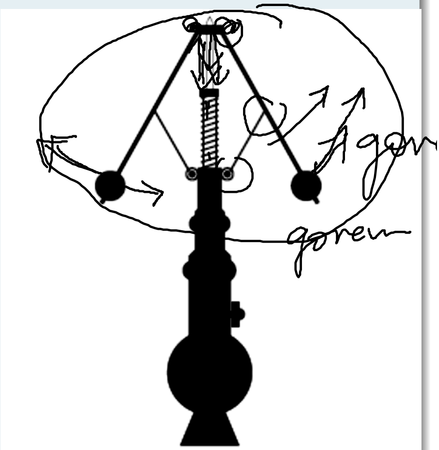
- Tuning centrifugal governors that achieved 'fast regulation', but avoided 'hunting' (James Clerk Maxwell)

Fly ball Governor



(Source: Polderman/Willems 1998 book)

Two key recent control inventions

- About a century ago: two main inventions drove control theory: regulation

1. Proportional-Integral-Differential (PID) controller
2. In 1930s, the 'negative feedback amplifier' by Harold Black (of Bells)

Control Inventions: feedback amplifier

- Had far-reaching applications to telephone technology and other areas of communication.
- Long-distance communication used to be hampered due to the drifting of the gains of the amplifiers used in repeater stations.
- Impressive technological development: it permitted signals to be amplified in a reliable way,

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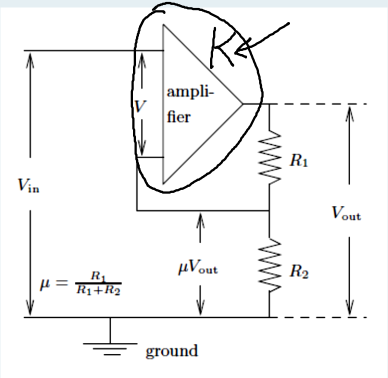
- Long-distance communication used to be hampered due to the drifting of the gains of the amplifiers used in repeater stations.
- Impressive technological development: it permitted signals to be amplified in a reliable way,
- Now: insensitive to the parameter changes inherent in vacuum-tube (and also solid-state) amplifiers.

(Source: Polderman/Willems 1998 book)

Feedback Amplifier



Control Inventions - Feedback Amplifier

- Assume that we have an electronic amplifier that amplifies its input voltage to output voltage with a gain K .

$$V_{out} = KV \tag{1}$$

- Use a voltage divider and 'feed back' μVout to the amplifier input.
- Basic calculations give:

$$V = V_{in} - \mu V_{out} \tag{2}$$

- Combining these two gives a crucial relation

$$V_{out} = \frac{1}{\mu + \frac{1}{K}} V_{in} \tag{3}$$

Feedback amplifier

What's the big deal with this formula?

- Value of K of an electronic amplifier is typically large,

Feedback amplifier

What's the big deal with this formula?

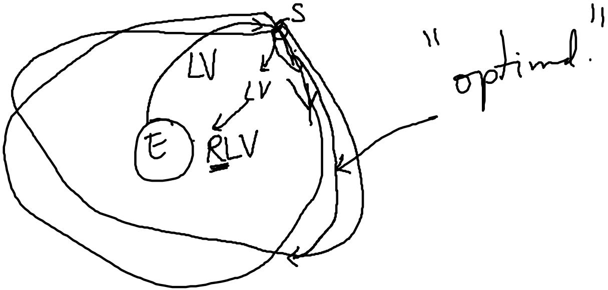
- Value of K of an electronic amplifier is typically large, but also very unstable: due to: sensitivity to aging, temperature, loading
- The voltage divider: can be implemented by means of passive resistors: gives a very stable value for μ.
- Now, for large values of K (although varying values of K):

$$\frac{1}{\mu + \frac{1}{K}} \approx \frac{1}{\mu} \tag{4}$$

and so Black's feedback amplifier gives:
an amplifier with a stable amplification gain $(1/\mu)$
based on an amplifier that has an inherent uncertain gain K.

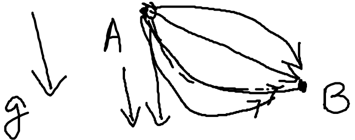
Control theory - trajectory optimization

- Trajectory transfer (for a dynamical system): find a path from a given initial state to a given terminal state.
- Most common example: satellite: go from one periodic orbit to another:



Control theory - trajectory optimization

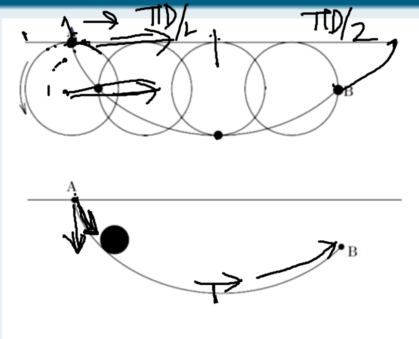
- Trajectory transfer (for a dynamical system): find a path from a given initial state to a given terminal state.
- Most common example: satellite: go from one periodic orbit to another: with least power/energy
- Path with maximum distance from obstacles
- Classic example: Brachystochrone problem (Johann Bernoulli - 1696)
- Find path/curve between two points A and B such that a body falling under its own weight moves in least time.



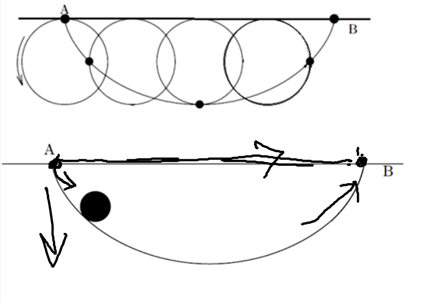
Control theory - trajectory optimization

cycloid

Case 1



Case 2



(Source: Polderman/Willems 1998 book)

Example of Air Conditioning Mechanism

- Air Conditioner with fixed time interval on/off: cheaper implementation
  - For example AC is on for 20 minutes and off for 10 minutes.
  - Timings are preset to save energy consumption.
  - Actual room conditions unconsidered (blind control)
- Air Conditioner with sensor based control: expensive implementation
  - AC with sensor based control takes the actual parameters,
  - analyse them with reference values, adjust the conditioning to the required level.
  - There will be a slight band at reference value, where no control action is taken.
  - Feedback: can make closed loop 'unstable' (even if open-loop was well-behaved)

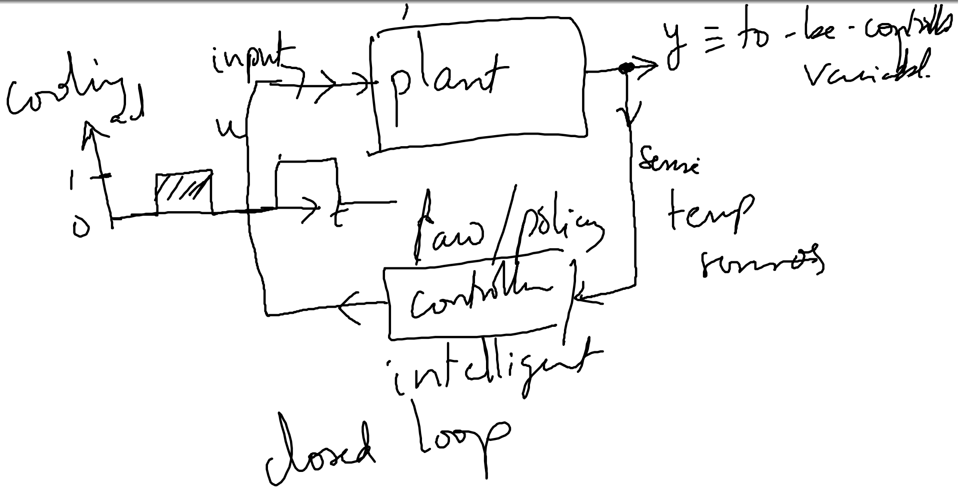
Open Loop Control System

- Advantages of Open Loop Control System
  - Simple in construction and design
  - Economical (no need of sensors)
  - Easy to maintain
  - Generally stable
  - Convenient to use (since output could be difficult to access/measure)
- Disadvantages of Open Loop Control System
  - They could be inaccurate (especially if situation changes)
  - Any change in output cannot be corrected automatically

Closed Loop Control System

- Advantages of Closed Loop Control System
  - Closed loop control systems are usually more accurate
  - Error is corrected due to presence of feedback signal
  - Wide bandwidth (range of frequencies for which system responds desirably)
  - Facilitates automation
  - The sensitivity of system may be made small to make system more stable
  - This system is less affected by noise/modelling/system-uncertainties: robust

- Disadvantages of Closed Loop Control System
  - They are costlier
  - They are complicated to design/implement
  - They are less reliable (complex design means more scope for breakdowns/degradation)
  - Require more maintenance
  - Feedback could lead to oscillatory/unstable response (unless we all learn control-theory well)



cooling
input
u
1
0
t
plant
y ≡ to-be-controlled variable
sense
temp sensors
law/policy
controller
intelligent
closed loop

Exercise:

- Find example from our daily experience that involves
  - open loop control
  - closed loop control (involves feedback)

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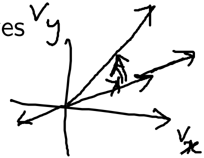


- Consider: $A = \begin{bmatrix} 7 & -10 \\ 5 & -8 \end{bmatrix}$

  $v \xrightarrow{Av} Av$

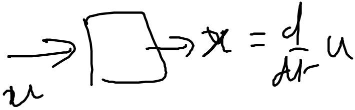
- In general, when A acts on a vector v (i.e. Av), the vector v gets scaled (lengthened/shortened/flipped) and also rotated.
- Vectors: $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
- Check that vector v_1 gets purely scaled (when A acts on v_1): what is the scaling?

  $v \neq 0$.

- Check that vector v_2 also gets purely ....
- Suggest a square real 2×2 matrix in which no real vector gets purely scaled.

Also find what is discrete-time impulse δ_d and continuous-time impulse δ_c

Positive/negative feedback



- $\frac{d}{dt}x = u$
- Suppose $u = +5x$ (positive feedback)
- Suppose $u = -5x$ (negative feedback)
- Solve both differential equations (exponentially growing/decaying)

LTI systems and exponential signals

- Linear Time Invariant (LTI) systems:
  systems governed by linear,

LTI systems and exponential signals

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LTI systems and exponential signals

- Linear Time Invariant (LTI) systems:
  systems governed by linear, constant coefficient,
  ordinary differential equations
- Signals: exponential functions e^{at}: function of time t
- a need not be real: can consider complex-valued functions of real-variable time t

Exponential signals are ‘eigenfunctions’

- For (square) matrices, we speak of eigenvectors
- Consider: $A = \begin{bmatrix} 7 & -10 \\ 5 & -8 \end{bmatrix}$
- Vectors: $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
- For some real square matrices, need to allow eigenvectors to be complex

LTI systems and exponential signals

- LTI systems have all exponential signals as eigen-'functions'
- Scaling: depends on the system. 'Transfer function'
- Transfer function also rings a bell (resonates?) with 'resonance'!
- Poles and zeros for input to output map
- Convolution is how input gets mapped to output

LTI systems and sinusoids signals

- For system with transfer function $G(s)$
- Sinusoids are also exponentials: real part zero
- Transfer function: complex (when evaluated at $j\omega$): magnitude and angle depends on ω: 'phasor' analysis
- If input is $\sin(\omega t)$, then output is $|G(j\omega)| \sin(\omega t + \angle G(j\omega))$
- Bode plot: plot $|G(j\omega)|$ versus ω (in log-log scale)
- and also: $\angle G(j\omega))$ versus ω (in normal-log scale)
- Low-pass filters/high-pass filters

Particular solution and homogenous solution

- Consider $\frac{d}{dt}u - u = \frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y$
- Let $u(t) \equiv 0$, then any $y(t) = c_1 e^{-2t} + c_2 e^{-t}$ solves
- When $u(t) = e^{at}$, then ?
- If homogenous solution goes to zero, then particular solution worth studying!
- Exponents in homogenous solution: 'poles' of the system
- 'Zeros' of the system: those exponents where gain $G(s)$ is zero
- Stability: all poles are in the LHP

What is Laplace transform?

- Some similarities with Fourier transform, but not as much 'duality' between time/frequency domains
- Laplace transform of $f(t)$ gives $F(s)$: this is also called 'frequency' domain
- $s = j\omega$: many Fourier transform properties get recovered
- $F(s) := \int_0^{+\infty} f(t)e^{-st}dt$
- View (for $s = +5$ (say)) e^{-5t} as decaying-weighting before area under $f(t)$ is calculated.
- If s is kept a (complex) variable, then $F(s)$ has 'full' info about $f(t)$:

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- Technical detail: integral from 0 to ∞: integral might not exist!
  $f(t)$ ought not 'grow too quickly': don't worry now. All practical signals have Laplace transforms!

Laplace transform, definition and properties

Function $f, f_1, f_2, g : [0, \infty) \to \mathbb{R}$: piecewise continuous

$$F(s) = \mathfrak{L}(f)(s), \text{ with } F(s) := \int_{0^-}^{\infty} f(t) e^{-st} dt$$

with $\text{real}(s) > \sigma_0$ large-enough, and inverse[1] defined using σ_0

[1] $$f(t) = \mathfrak{L}^{-1}(F)(t), \text{ with } f(t) := \frac{1}{2\pi j} \lim_{\omega_0 \to \infty} \int_{\sigma_0 - j\omega_0}^{\sigma_0 + j\omega_0} F(s) e^{st} dt$$

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with $\text{real}(s) > \sigma_0$ large-enough, and inverse[1] defined using σ_0

- Linearity: $\mathfrak{L}(\alpha_1 f_1 + \alpha_2 f_2) = \alpha_2 F_1(s) + \alpha_2 F_2(s)$ for any real/complex constants α_1 and α_2
- Delayed f: $\mathfrak{L}(\sigma_T(f)) = e^{-sT} F(s)$ (with $T \geqslant 0$ and f-'zeroed'). $(\sigma_T(f)(t) := f(t - T))$.
- Derivative of f: $\mathfrak{L}(\frac{d}{dt} f) = sF(s) - f(0^-)$ and
- Integral of f: $\mathfrak{L}(\int_0^t f(\tau) d\tau) = \frac{F(s)}{s}$

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- Convolution and product:
  $(f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$, $\mathfrak{L}(f * g) = F(s)G(s)$
- Dirac delta: $\delta * f = f$ and $\mathfrak{L}(\delta) = 1$
- IVT: $f(0^+) = \lim_{t \downarrow 0} f(t) = \lim_{s \to \infty} sF(s)$
  (provided LHS exists, i.e. no impulses/their derivatives at $t = 0$.)
- FVT: $f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$
  (provided LHS exists, i.e. f neither diverges, nor oscillates)
- Time multiplication $\mathfrak{L}(tf(t)) = -\frac{d}{ds}F(s)$
- Complex shift: $\mathfrak{L}(e^{at}f(t)) = F(s-a)$
- Time scaling: $\mathfrak{L}(f(\frac{t}{a})) = aF(as)$ (for $a > 0$)

Polynomials/exponentials/sinusoids

- $\mathfrak{L}(1) = \frac{1}{s}$ (note: functions are only on $[0, \infty)$)
- $\mathfrak{L}(t) = \frac{1}{s^2}$
- $\mathfrak{L}(e^{at}) = \dfrac{1}{s-a}$
- $\mathfrak{L}(\sin(\omega t)) = \dfrac{\omega}{s^2+\omega^2}$ and $L(\cos(\omega t)) = \dfrac{s}{s^2+\omega^2}$
  (Use IVT to be sure of which is of which.)
- $\mathfrak{L}(e^{at}\sin(\omega t)) = \dfrac{\omega}{(s-a)^2+\omega^2}$

Positive/negative feedback

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- Suppose $u = +5x$ (positive feedback)
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