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EE210: Signals and Systems, Tutorial-1, August 2019

Problem 1. Consider convolution of two sequences $u(n)$ & $h(n)$ (both with finite support) to give $y(n)$. Suppose $u(n), h(n) = 0$ for $n < 0$.

(a) Show that one can construct a 'Toeplitz Matrix T' using entries of h such that

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \end{bmatrix} = T \begin{bmatrix} y(0) \\ y(1) \\ \vdots \end{bmatrix}$$

with entries of T coming from h.

(b) Relate support of h, u with y.

(c) Construct polynomial $H(z), U(z)$ and relate it to $Y(z)$. Suppose the support is fixed, but h and u are changed for this fixed support. Can *support* of y change for different h, u, or does the support of y depend only on the supports of h and u?

Problem 2. Consider the above problem, but with u and h sequences that are nonzero for integer values in a more general range as follows. Using all 3 methods (convolution definition, using Toeplitz matrix, and using z-transform) obtain the support of output y. Suppose support of a sequence is from s (start) to e (end), and denote support of impulse response sequence $h(k)$ as s_h to e_h, and support of input sequence u(k) as s_u to e_u. Relate the supports of h, u and y in terms of s_h, e_h, s_u and e_u.

Problem 3. For continuous time and discrete time,

(a) Check that convolution with $h(t)$ or $h(k)$ is a linear map from u to y.

(b) Show that zero input gives zero output.

(c) Show that this map is time-invariant also.

Problem 4. Find the impulse response for the difference equation:

$$y(k) = y(k-1) + u(k) \qquad \text{(causal system)}$$

Does the impulse response satisfy $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$?

Problem 5. (a) Describe two (different) sequences of functions f_n that converges to the δ_c distribution

(b) Show that $\int_{\infty}^{\infty} f_n(t)\phi(t)dt \to \phi(0)$. ($\phi$ is any function)

(c) Are the functions f_i in $\mathcal{L}^2$?

Problem 6. Show that if $f(t)$ is real-valued and an even function, then $\hat{f}(jw)$ is a real and even function of w. Check if for $f(x)$ an even and real function of x, whether Fourier transform and inverse transform gives same outcome.

Problem 7. Consider two discrete time systems

$$y_1(n) = u(n) + u(n-1)$$
$$y_2(n) = u(n) - u(n-1)$$

Which do you call low pass filter, which one is high pass filter?

Problem 8. (a) Check if the δ_c is in $\mathcal{L}^1$?

(b) Is δ_c a function?

(c) Consider the sequences of function f_n that converge to the δ_c. What do their Fourier transform sequence $(\hat{f}_n)$ converge to?

Problem 9. Consider following sets of continuous time functions:
a) F_c (compact support), b) F_+ Support in $\mathbb{R} = [0\ \infty)$,
c) F_- Support in $\mathbb{R} = (-\infty\ 0]$, d) F support in $\mathbb{R}$.
One can choose two functions in how many different ways? (At most 4×4 ways.)

(a) Which of these choices give convolution that is always well defined?

(b) Show that $f_1 * f_2 = f_2 * f_1$. ($* \equiv$ convolution)

Problem 10. Show that $\mathcal{F}(f_1 * f_2) = \hat{f}_1 \cdot \hat{f}_2$, whenever convolution and Fourier transform are defined.