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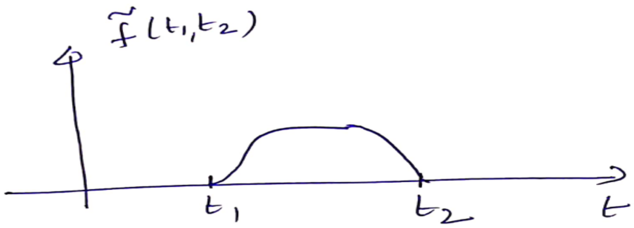
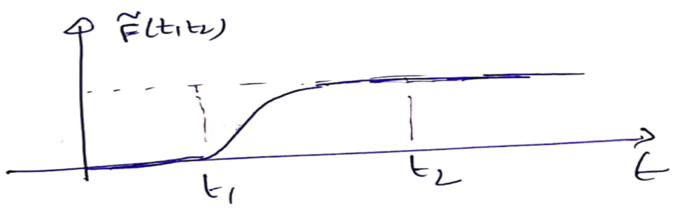
EE210: Signals and Systems, Tutorial-3, August 2019

Problem 1.
$$f(t) = e^{(-1/t)}; t > 0$$
$$= 0 \quad : t < 0$$

1. Show that $\lim_{t\to 0^+} \frac{d^n f(t)}{dt^n} = 0$; for all n . (Hint: Use $\lim_{a\to\infty} \frac{f(x)}{e^{ax}} = 0$; p(x) is any polynomial function.)
2. By shifting ,scaling and reflection of f(t) & $\tilde{f}(t1,t2)$; Show that





3. Show that, using $\tilde{F}(t1,t2)$, one can smoothly switch from any $g_1 : (-\infty, t_1] \longrightarrow \mathbb{R}$ to any $g_2 : (t_1, \infty] \longrightarrow \mathbb{R}$; provided a finite interval $[t_1, t_2]$ for switching.

Problem 2. There are many C^∞ function with compact sypport. (C^∞ stands for functions that are differentiable any number of times and the derivative is still continuous. These are verrrry smooth functions and can be differentiated any number of times.) Other than the sinusoids, cosinusoids, polynomials, exponentials, the example in the Problem 1 is also an example of C^∞ function.

Let D denote the set of C^∞ functions that have compact support. This problem is to see that there are plenty of functions within D.

Check that for any $g \in D$ and $h \in C^\infty$, we have $g \cdot h \in D$.

Show that the set D is closed under

1. Addition
2. Scaling
3. Conventional multiplication

4. Convolution
5. $\frac{d}{dt}$, $\frac{d^2}{dt^2}$, $a_1\frac{d}{dt} + a_2\frac{d^2}{dt^2}$ and in fact any 'polynomial' in $\frac{d}{dt}$

Also note that this is NOT closed (in general) under Fourier transform, nor under integration.

Problem 3. From the time domain defination of convolution, Show that

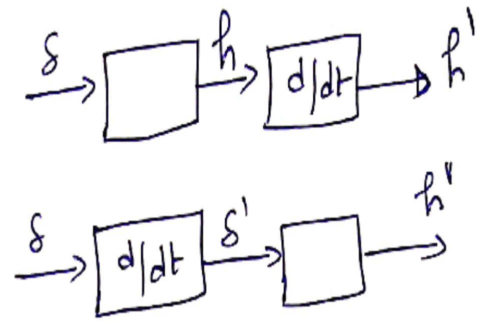
1. Convolution distributes over addition
2. Convolution is associative
3. Conventional is commutative

And from these properties, Show that convolution is just like multiplication.

Problem 4. If the fourier transform of f(t) is given by F(jw), Find the fourier tansform of $\frac{d}{dt}f(t)$ in terms of F(jw)

Problem 5. Using the property given in ques no. 4 ., Show that $h^{'} * x = h * x^{'}$ where $'$ implies $\dfrac{d}{dt}$. Also show the same by using the properties mentioned in ques no 3 .

{ Hint:



These two operations are Equivalent }