Solutions for Tutorial - 5

(1) $I(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$ where $\omega_0 = \frac{2\pi}{T}$

(2) (a) $\frac{Y(z)}{X(z)} = \frac{z - 5/3}{z(z-2)}$ poles : $z = 0, 2$
zeros : $z - 5/3$

$y(n) = \frac{5}{6} \delta(n-1) + \frac{1}{6} 2^n u(n)$

(b) Causal

(3) (a) $\frac{Y(z)}{X(z)} = H_1(z) = \frac{2z^{-1}+1}{1+z^{-1}}$

$h_1(n) = \delta(n) + (-1)^{n-1} u(n-1)$

(b) $\frac{X(z)}{Y(z)} = H_2(z) = \frac{1+z^{-1}}{1+2z^{-1}} \Rightarrow h_2(n) = \delta(n) - (-2)^{n-1} u(n-1)$

(c) $h(n) = h_1(n) \circledast h_2(n)$

$H(z) = H_1(z) H_2(z) = 1 \xrightarrow{I.Z.T} \boxed{h(n) = \delta(n)}$

$\boxed{h_1(n) \circledast h_2(n) = \delta(n)}$

(4) (a) $y_1(n) = 2^n u(n)$ (b) $y_2(n) = -2^n u(n-1)$

(c) $y_2(n)$ has DTFT because ROC includes unit circle.

(d) $y_2(n)$ is bounded because ROC includes unit circle

or $\sum_{n=-\infty}^{\infty} |y_2(n)| < \infty$

(5) (a) $X[k] = [0, 4-2i, -4, 4+2i]$

(b) $X[k] = [0, 4.045 - 0.138i, -1.54508 - 4.028i, -1.54508 + 4.028i, 4.045 + 0.138i]$

(6) (a) $X(s) = \frac{1}{s-4} + \frac{1}{s+6} - \frac{1}{s}$ ROC : $\sigma > 4$ or $Re\{s\} > 4$

(b) $X(s) = -\frac{1}{s-4} - \frac{1}{s+6} + \frac{1}{s}$ ROC ; $\sigma < -6$ or $Re\{s\} < -6$

(8) (a) $x(t) = e^{-t} \longrightarrow y(t) = \frac{1}{2} e^{-t}$

$x(t) = e^{-t}u(t) \longrightarrow y(t) = \frac{1}{2}\left[e^{-t} - e^{-3t}\right]u(t)$

$x(t) = e^{-3t} \longrightarrow y(t) = \infty$

$x(t) = te^{-3t}u(t) \longrightarrow y(t) = te^{-3t}u(t)$

(9) (a)

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1-j}{\sqrt{2}} & -j & \frac{-1-j}{\sqrt{2}} & -1 & \frac{-1+j}{\sqrt{2}} & j & \frac{1+j}{\sqrt{2}} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \frac{-(1+j)}{\sqrt{2}} & j & \frac{1-j}{\sqrt{2}} & -1 & \frac{1+j}{\sqrt{2}} & -j & \frac{-1+j}{\sqrt{2}} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{-1+j}{\sqrt{2}} & -j & \frac{1+j}{\sqrt{2}} & -1 & \frac{1-j}{\sqrt{2}} & j & \frac{-1-j}{\sqrt{2}} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{1+j}{\sqrt{2}} & j & \frac{-1+j}{\sqrt{2}} & -1 & \frac{-1-j}{\sqrt{2}} & -j & \frac{1-j}{\sqrt{2}} \end{bmatrix}$$

$$\tilde{F} = \frac{1}{N} F^* = \frac{1}{8} \times \left[\begin{array}{cccc:cccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1+j}{\sqrt{2}} & j & \frac{-1+j}{\sqrt{2}} & -1 & \frac{-1-j}{\sqrt{2}} & -j & \frac{1-j}{\sqrt{2}} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{-1+j}{\sqrt{2}} & -j & \frac{1+j}{\sqrt{2}} & -1 & \frac{1-j}{\sqrt{2}} & j & \frac{-1-j}{\sqrt{2}} \\ \hdashline 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{-1-j}{\sqrt{2}} & j & \frac{1-j}{\sqrt{2}} & -1 & \frac{1+j}{\sqrt{2}} & -j & \frac{-1+j}{\sqrt{2}} \\ 1 & -j & -1 & j & 1 & -j & -1 & +j \\ 1 & \frac{1-j}{\sqrt{2}} & -j & \frac{-1-j}{\sqrt{2}} & -1 & \frac{-1+j}{\sqrt{2}} & j & \frac{1+j}{\sqrt{2}} \end{array}\right]$$

→ Blocks are different.