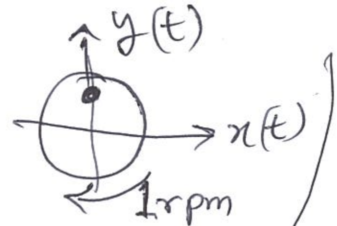
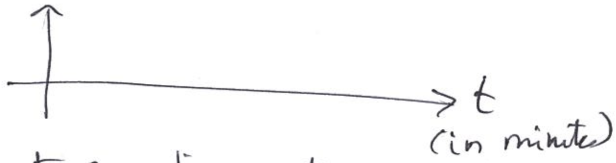
Tutorial ~~7~~, EE 210, Signals & Systems, 8^{th} Nov 2019.

Q-1:



Radius of dot = 1 unit.

(a) Plot $x(t)$ vs time



(b) Mark samples at sampling rates 5 rpm, 4 rpm, 2 rpm, 1.8 rpm, 1 rpm, 0.9 rpm, 0.5 rpm (on the plot in 1a).

Q-2: Consider (just for Q-2) the defn of DFT:

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n]\, e^{-jkn\,2\pi/N}.$$

(a) Write defn of IDFT. (Inverse DFT)

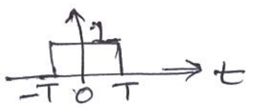
(b) Construct vectors $x \in \mathbb{C}^N$ such that $DFT(x) = IDFT(x)$

(c) Construct real and odd signal x such that $DFT(x) = IDFT(x)$

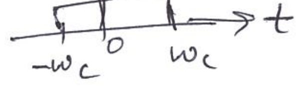
(d) Write a matrix for DFT & IDFT for $N = 2, 3, 4$.

(e) Construct 2 "different" vectors (for $N=4$) such that $DFT(x) = x$.

Q-3: (a) Find Fourier transform of



(b) Find inverse Fourier transform of



(c) Express (a) & (b) in terms of $\text{sinc}(\theta) := \frac{\sin \pi\theta}{\pi\theta}$

Q-4 (a) Give ~~two~~ a function $f(x)$ such that $\mathcal{F}(f) = f$ ($\mathcal{F}$: normalized Fourier transform)

(b) ~~Give~~ Suggest impulse train I (appropriately) such that $\mathcal{F}(I) = I$.

Q-5: Construct difference equation for system with impulse response $h[n] = 2, 3, 0, -5, 6$ for $n = 0, 1, 2, 3, 4$ respectively.

Q-6 Construct difference equation for impulse response

(a) $h[n] = \left(\frac{1}{2}\right)^n u[n]$

(b) $h[n] = 3^n u[n]$

(c) $h[n] = \left(\frac{1}{4}\right)^n u[-n]$

(d) $h[n] = -3$ for $n = 8$
$= 0$, otherwise.

Q-7: Find differential equation for impulse response $h(t)$ as below

(a)

(b) $e^{-3t} u(t)$

(c) $e^{4t} u(t)$

(d) $\left(5e^{-3t} + 6e^{4t}\right) u(t)$

(e) $5\,\delta(t-6)$

Q-8: classify Systems in Q-5,6,7 as
- IIR or FIR
- finite memory or infinite memory.
- classify input & output clearly.

Things to ponder leisurely (not relevant for endsem)

Q-1: For normalized Fourier transform $\mathcal{F}$ (i.e. $\frac{1}{\sqrt{2\pi}}$ constant for scaling) show $\mathcal{F}^4 = I$

Q-2: consider band-limited $x(t)$. Fourier transform $X(j\omega)$ has uncountably infinite values (between $-\omega_M$ to ω_M). Then how can only countably infinite $x[n]$ help perfect reconstruction? (assume appropriate sampling rate).

Q-3 Consider periodic $x(t)$ (with period T). Uncountably infinite values of $x(t)$ (in $t \in [0, T]$) make up $x(t)$. How can only countably infinite Fourier coefficients a_k capture $x(t)$ perfectly?

Q-4: Suggest a discontinuous band-limited $x(t)$. (either periodic or aperiodic).

Q-5 e^{-sT} can be approximated by real-rational to "ample" accuracy, but increasing order (order of diff eqn.) Look up Padé approximation.