EE 210 **Spring 2017-18**

Mid-semester examination

Max marks: 100 Time: 2 hours

You will not require a calculator. Begin the answer to each question on a fresh page. Throughout, $\star$ denotes the convolution operation, u denotes the unit step signal.

The second sheet of this handout is a mid-term feedback form. You are requested to detach the sheet, fill it up, and hand it over after you are finished with the exam. While your answer booklet will be collected after 2 hours, you can stay back for 10 minutes and fill in the feedback sheet after submitting your answer booklet. The feedback will remain anonymous, and will help me adapt the course delivery in the second half of the semester.

1. [**10** marks] Check if the following system is memoryless, linear, time-invariant, causal, and stable. (x denotes the input signal, and y the output signal.)
$$y(t) = x(\sin(t))$$
Justify your answers.

2. [**20** marks] Define the signal $x(t)$ as follows.
$$x(t) = \begin{cases} 0 & t < 0 \\ 1 & t \in [0,1] \\ 2-t & t \in (1,2] \\ 0 & t > 2. \end{cases}.$$
Sketch carefully the following signals.
   (a) $x(-2t-1)$
   (b) $x(t) \star u(t)$
   (c) $x'(t) \star u(t)$

3. [**15** marks] Consider the LTI system defined by the following input-output relationship:
$$y[n] = x[n] - x[n-1].$$
   (a) Sketch the impulse response $h[n]$ of this system.
   (b) Sketch the step response $s[n]$ (the output corresponding to the a unit step input).
   (c) Obtain the frequency response $H(\omega)$ corresponding to this system. Sketch the amplitude response $|H(\omega)|$ over $\omega \in [-\pi, \pi]$.

4. [**20** marks] Consider the continuous time periodic signals $x(t)$ and $y(t)$ with period 1, defined as:
$$x(t) = t^2 \quad (t \in [0,1)),$$
$$y(t) = t \quad (t \in [0,1)).$$
   (a) Compute the Fourier series coefficients a_k corresponding to $x(t)$.
   (b) Compute the Fourier series coefficients b_k corresponding to $y(t)$.
   (c) Compute $\sum_{k=-\infty}^{\infty} |a_k|^2$.

5. [**15** marks] $x[n]$ is a discrete-time periodic signal with period 4 as shown below.

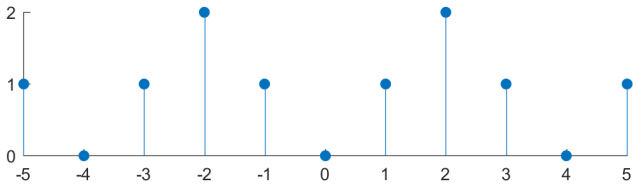

Figure 1: $x[n]$

(a) Compute the discrete-time Fourier series coefficients a_k of $x[n]$.

(b) Sketch the spectrum a_k.

(c) The signal $x[n]$ is passed through an LSI system with frequency response

$$H(\omega) = \begin{cases} 0 & -\pi \leq \omega < -2\pi/3 \\ 1 & -2\pi/3 \leq \omega \leq 2\pi/3 \\ 0 & 2\pi/3 < \omega \leq \pi \end{cases}.$$

Obtain the output signal $y[n]$.

6. [**20** marks] Consider a signal $x(t)$ that is periodic with period T, satisfying

$$\int_0^T |x(t)|^2 dt < \infty.$$

The goal of this problem is to derive a Fourier series representation of $x(\cdot)$ of the form

$$x(t) = B_0 + \sum_{k \geq 1} B_k \cos\left(k\frac{2\pi}{T}t\right) + \sum_{k \geq 1} C_k \sin\left(k\frac{2\pi}{T}t\right).$$

(a) Derive the formulas for B_k and C_k (i.e., the analysis equations for this transform).

(b) Compute the coefficients B_k and C_k for the following signal, periodic with period 6.

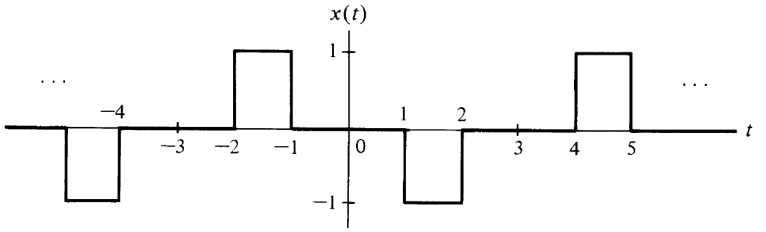

Figure 2: $x(t)$

Mid-Term Feedback

1. **Teaching feedback:** Please enter your feedback on the classroom teaching so far. Do mention specifically any aspects of the teaching style have / have not worked for you.

2. **Feedback on course material:** Comment on the coverage of different topics in the course so far. For example, let me know which of the topics covered have been most and least clear to you.

3. On a scale of 1-10, rate the difficulty of this mid-sem exam.