EE 210 **Spring 2018-19**

HW 1

Assigned: 14/01/19 Due: 21/01/19

You are encouraged to discuss these problems with others, but you need to write up the actual solutions alone. Start early, and come to office hours (TBA) with any doubts. Your only have to submit your solutions to **the questions marked** [†]. *Drop off your submission in the dropbox labeled EE210 in the EE office by 5.30 pm on the due date.*

Preliminaries

A signal $x(t)$ is said to be *even* if $x(t) = x(-t)$ for all t, and *odd* if $x(t) = -x(-t)$ for all t. An arbitrary signal $x(t)$ can be decomposed as a sum of an even and an odd signal as follows:

$$x(t) = x_e(t) + x_o(t),$$

where

$$x_e(t) = \frac{x(t) + x(-t)}{2},$$
$$x_o(t) = \frac{x(t) - x(-t)}{2}.$$

x_e is referred to as the *even part* of x, and x_o is referred to as its *odd part*.

Note: Is the above decomposition of a signal as a sum of an even and an odd signal unique?

Note: While the above definitions are given for continuous-time signals, it is easy to write out the corresponding versions for discrete-time signals.

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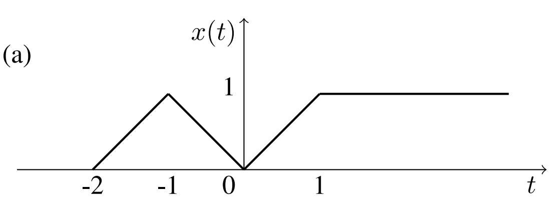
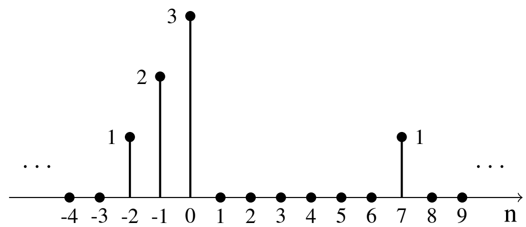
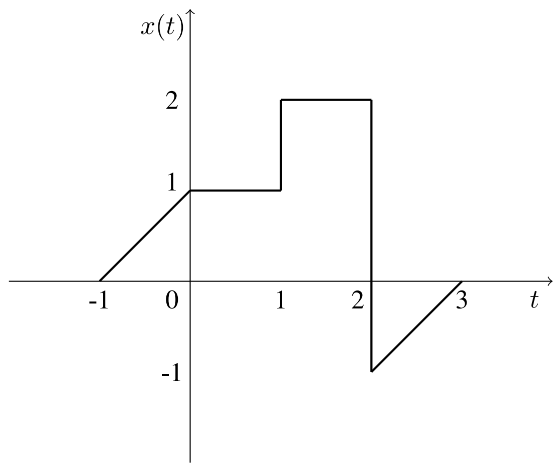
1. [†] Carefully sketch the following signals. Mark all the critical points.
   (a) $g(t) = tu(-t-1) - u(-t-1)$
   (b) $h(t) = e^{-tu(t)}, \quad -1 \leq t \leq 1$
   (c) $m(t) = \left(\frac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+4}\right)\delta(1-t)$
   (d) $p[n] = (\frac{1}{2})^n u[n-3]$
2. Determine and sketch the even and odd parts of the signals depicted in figures below. Label your sketches carefully.

(a)

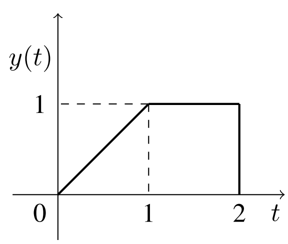


(b) 

3. For the continuous time signal $x(t)$ shown below, sketch and label carefully each of the following signals: (a) $x(t-1)$, (b) $x(2-t)$, (c) $x(2t+1)$, (d) $[x(t)+x(-t)]u(t)$.



4. [†] Consider the signal $y(t) = (1/5)x(-2t-3)$ shown below. Determine and carefully sketch the original signal $x(t)$.



5. [†] A system is said to be *invertible* if distinct input signals always result in distinct output signals. In other words, for an invertible system, the input signal corresponding to any given output signal can be uniquely determined.

   Identify whether each of the following systems is invertible. (x denotes the input signal, and y the output signal.) Justify your answers.

   (a) $y[n] = \sum_{k=-\infty}^{n} x[k]$
   (b) $y[n] = x[n]x[n-2]$
   (c) $y[n] = nx[n]$
   (d) $y[n] = x[2n]$

6. [†] Check if the following systems are linear, time-invariant and causal. (x denotes the input signal, and y the output signal.) Justify your answers.

   (a) $y(t) = \sin(x(t) - x(0))$
   (b) $y(t) = x(\sin(t))$
   (c) $y(t) = \max_{s\in[t-1,t]}\{x(s)\}$

(d) $y(t) = x(t/3)$

7. [†] Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$. Compute and plot each of the following convolutions.

   (a) $y_1[n] = x[n] \star h[n]$

   (b) $y_2[n] = x[n+2] \star h[n]$

   (c) $y_3[n] = x[n] \star h[n+2]$

8. [†] Let

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{elsewhere} \end{cases}$$

and

$$h[n] = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & \text{elsewhere} \end{cases}$$

where $N \leq 9$ is an integer. Determine the value of N, given that $y[n] = x[n] \star h[n]$, $y[4] = 5$ and $y[14] = 0$.