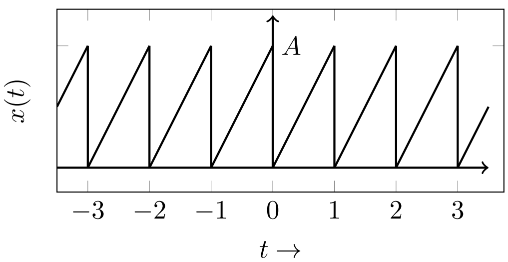
EE 210 **Spring 2018-19**

HW 3

Assigned: 10/02/19 Due: 18/02/19

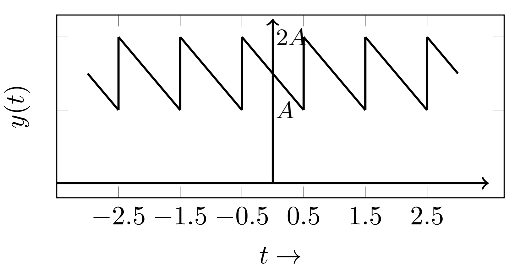
You are encouraged to discuss these problems with others, but you need to write up the actual solutions alone. Start early, and come to office hours (TBA) with any doubts. Your only have to submit your solutions to **the questions marked** [†]. *Drop off your submission in the dropbox labeled EE210 in the EE office by 5.30 pm on the due date.*

1. [†] A periodic signal $x(t)$ is given below.



   (a) Determine its Fourier Series coefficients. Sketch its magnitude and phase spectrum.

   (b) Using the results in part (a) above and without doing elaborate integrations, determine the coefficients of the Fourier series of the periodic signal $y(t)$ shown below. Sketch the magnitude and phase spectrum.



2. A 2π periodic signal $x(t)$ is specified over one period as

$$x(t) = \begin{cases} \dfrac{t}{A} & 0 \leq t < A \\ 1 & A \leq t < \pi \\ 0 & \pi \leq t < 2\pi \end{cases}$$

   Represent the function as a Fourier series.

3. The Fourier series coefficients of a periodic signal $x(t)$ is given by

$$d_k = \begin{cases} jk, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$$

   The fundamental period of the signal is $T_0 = 4$. Determine the signal $x(t)$.

4. [†] Suppose we are given the following information about signal $x(t)$:

   i) $x(t)$ is a real signal

   ii) $x(t)$ is periodic with period T=6 and has Fourier coefficients a_k

iii) a_k=0 for $k = 0$ and $k > 2$
iv) $x(t) = -x(t-3)$
v) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{1}{2}$
vi) a_1 is a positive real number

Show that $x(t) = A\cos(Bt + C)$ and determine the value of constants A, B and C.

5. [†] Consider the following three discrete-time signals with fundamental period of 6:

$$x[n] = 1 + \cos\left(\frac{2\pi n}{6}\right), \quad y[n] = \sin\left(\frac{2\pi n}{6} + \frac{\pi}{4}\right), \quad z[n] = x[n]y[n]$$

   (a) Find the Fourier series coefficients of $x[n]$
   (b) Find the Fourier series coefficients of $y[n]$
   (c) Find the Fourier series coefficients of $z[n]$

6. [†] Consider a causal continuous-time LTI system whose input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d}{dt}y(t) + 4y(t) = x(t)$$

   Find the Fourier series representation of the output $y(t)$ for each of the following inputs.

   (a) $x(t) = \cos(2\pi t)$
   (b) $x(t) = \sin(4\pi t) + \cos(6\pi t + \pi/4)$

7. [†] Consider the periodic impulse train in continuous time $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$. Plot the approximations obtained by truncating the Fourier series of $x(\cdot)$. Specifically, plot $\sum_{k=-N}^{N} a_k e^{j(\frac{2\pi}{T})kt}$ for $N = 5$, 10, 100, where $\{a_k\}$ denotes the Fourier series of x. Interpret your plots.

8. [†] Consider a continuous-time LTI system $\mathcal{S}$ whose frequency response is

$$H(\omega) = \begin{cases} 1 & |\omega| \geq 250 \\ 0 & \text{otherwise} \end{cases}.$$

   The input to this system is a signal $x(t)$ with fundamental period $T = \pi/7$, and Fourier series coefficients a_k. If the output signal $y(t) = x(t)$, for which values of k is a_k guaranteed to be zero?

9. [†] Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ -1 & -2 \leq n \leq -1 \\ 0 & \text{otherwise} \end{cases}.$$

   Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k],$$

   determine the Fourier series coefficients of the output $y[n]$.

10. Consider two discrete-time signals $x[n]$ and $y[n]$ that are both periodic with period N. Suppose that

$$x[n] \xrightarrow{\text{DTFS}} a_k, \quad y[n] \xrightarrow{\text{DTFS}} b_k.$$

   Exploit the duality principle to show that the property

$$x[n]y[n] \xrightarrow{\text{DTFS}} \sum_{l \in <N>} a_l b_{k-l}$$

   implies the property

$$\frac{1}{N} \sum_{l \in <N>} x[l]y[n-l] \xrightarrow{\text{DTFS}} a_k b_k.$$