EE 210 **Spring 2018-19**

HW 4

Assigned: 12/03/19 Due: 18/03/19

You are encouraged to discuss these problems with others, but you need to write up the actual solutions alone. Your only have to submit your solutions to **the questions marked** [†]. *Drop off your submission in the dropbox labeled EE210 in the EE office by 5.30 pm on the due date.*

1. [†] Prove the following properties of the Fourier transform. Suppose

$$x(t) \xrightarrow{\text{FT}} X(\omega), \quad y(t) \xrightarrow{\text{FT}} Y(\omega).$$

Prove the following properties.

(a) (Linearity) For complex numbers a and b, $ax(t) + by(t) \xrightarrow{\text{FT}} aX(\omega) + bY(\omega)$

(b) (Time shift) For real τ, $x(t-\tau) \xrightarrow{\text{FT}} e^{-j\omega\tau}X(\omega)$

(c) (Frequency shift) For real ω_0, $x(t)e^{j\omega_0 t} \xrightarrow{\text{FT}} X(\omega - \omega_0)$

(d) For real $x(t)$, $X(\omega) = X^*(-\omega)$. For real and even $x(t)$, $X(\omega)$ is real and even. For real and odd $x(t)$, $X(\omega)$ is purely imaginary and odd.

Note: You might want to prove Property (c) using Property (b) and the duality principle.

2. Prove that $\int_{-\infty}^{\infty} \text{sinc}(t)dt = \int_{-\infty}^{\infty} \text{sinc}^2(t)dt = 1$. (Recall that $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.)

3. Obtain the Fourier transform of the signal $x(t)$, where

$$x(t) = \begin{cases} 0 & x \in (\infty, -1.5] \\ (x+1.5) & x \in (-1.5, -0.5] \\ 1 & x \in (-0.5, 0.5] \\ 1.5 - x & x \in (0.5, 1.5] \\ 0 & x > 1.5 \end{cases} .$$

4. [†] Compute the Fourier transform corresponding to the following signals.

(a) $e^{-at}\cos(\omega_0 t)u(t) \quad (a > 0)$

(b) $e^{-3|t|}\sin(2t)$

(c) $x(t) = \begin{cases} 1 - t^2 & t \in (0, 1) \\ 0 & \text{otherwise} \end{cases} .$

(d) $x(t)$ as shown in Figure 1.

5. Compute the inverse Fourier transform corresponding to the following spectra.

(a) $X(\omega) = \frac{2\sin[3(\omega - 2\pi)]}{\omega - 2\pi}$

(b) $X(\omega) = \cos(4\omega + \pi/3)$

(c) $X(\omega)$ as shown in Figure 2(a).

(d) $X(\omega)$ as shown in Figure 2(b).

6. [†] Let $p(t)$ denote the periodic triangular pulse train as shown in Fig. 6.

(a) Compute the Fourier series coefficients p_k corresponding to $p(t)$

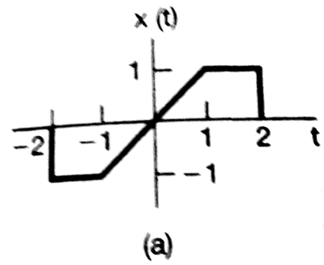

Figure 1: $x(t)$ for Problem 4(d)

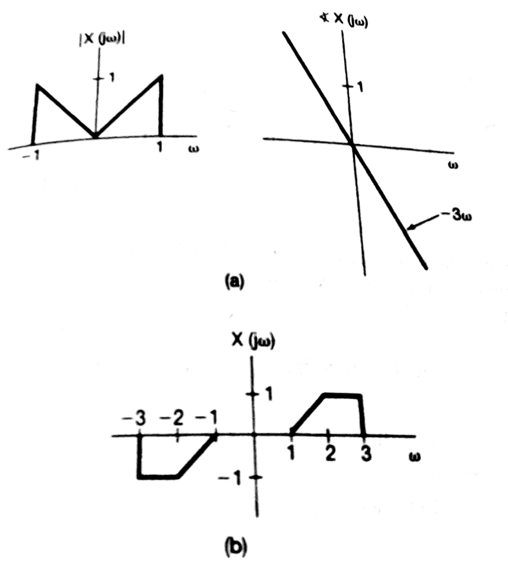

Figure 2: Spectra for Problems 5(c) and 5(d)

(b) Compute and sketch the Fourier transform $P(\omega)$ corresponding to $p(t)$

(c) Let $x(t)$ be an aperiodic signal. Define $y(t) = x(t)p(t)$. Obtain an expression for $Y(\omega)$.

(d) Sketch $Y(\omega)$ when $x(t) = \text{sinc}(t)$.

7. Let $X(\omega)$ be the Fourier transform of the signal $x(t)$ shown in Figure 4. Do the following computations without explicitly evaluating $X(\omega)$.

   (a) $X(0)$

   (b) $\int_{-\infty}^{\infty} X(\omega)d\omega$

   (c) $\int_{-\infty}^{\infty} X(\omega)e^{j\omega}d\omega$

   (d) $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

   (e) Sketch the inverse Fourier transform of $\text{Real}(X(\omega))$. (Here, $\text{Real}(a)$ denotes the real part of a.)

8. [†] Consider an LTI system whose response to the input $x(t) = [e^{-t} + e^{-3t}]u(t)$ is $y(t) = (2e^{-t} - 2e^{-4t})u(t)$.

   (a) Determine the frequency response of this system.

   (b) Determine the impulse response.

   (c) Find the differential equation relating the input to the output.

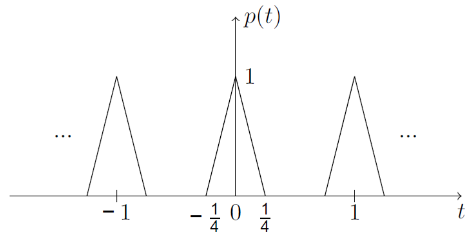

Figure 3: $p(t)$

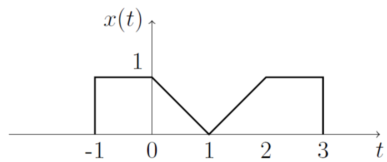

Figure 4: $x(t)$ for Problem 7

9. The goal of this problem is to understand the effect of non-linear phase response. Consider the LTI system with frequency response

$$H(\omega) = \frac{a - j\omega}{a + j\omega}.$$

Here, $a > 0$.

(a) Sketch the magnitude and phase response of the system.

(b) Setting $a = 1$, determine the output of the system to the input

$$\cos(t/\sqrt{3}) + \cos(t) + \cos(\sqrt{3}t).$$

10. [†] Suppose $g(t) = x(t)\cos(t)$ and the Fourier transform of $g(t)$ is

$$G(\omega) = \begin{cases} 1 & |\omega| \leq 2 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Determine $x(t)$

(b) Specify the Fourier transform of $x_1(t)$ such that $g(t) = x_1(t)\cos(2t/3)$

11. The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$y'(t) + 10y(t) = \int_{-\infty}^{\infty} x(s)z(t-s)ds - x(t),$$

where $z(t) = e^{-t}u(t) + 3\delta(t)$.

(a) Find the frequency response of this system.

(b) Find the impulse response of this system.