EE 210 — Spring 2018-19

HW 5

Assigned: 8/4/19 — Due: 15/4/19

You are encouraged to discuss these problems with others, but you need to write up the actual solutions alone. Drop off your submission in the dropbox labeled EE210 in the EE office by 5.30 pm on the due date.

1. Consider a signal $x(t)$ that is bandlimited in $[-\pi, \pi]$. Your pointy-haired boss samples this signal at $x(n/2)$, $n \in \mathbb{Z}$. He claims that $\phi(t) = sinc(2t)$ is the *unique interpolation* signal such that

$$x(t) = \sum_{n \in \mathcal{Z}} x(n/2)\phi(t - n/2).$$

Is your boss correct? If not, provide another choice for $\phi(\cdot)$.

Note: Problem designed by Prof. Animesh Kumar.

2. [†] Consider a class of signals $\mathcal{S}$ whose spectrum has finite support, such that for $x(t) \in \mathcal{S}$,

$$X(\omega) \neq 0 \text{ for } |\omega| \in (\pi, 2\pi)$$

and $X(\omega) = 0$ otherwise.

What is the minimal sampling rate required for perfect reconstruction of this class of signals? Come up with a corresponding reconstruction approach.

Note: Problem designed by Prof. Animesh Kumar.

3. [†] Compute the discrete time Fourier transform for each of the following signals:

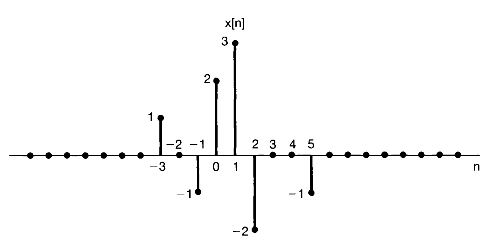
   (a) $\sin(n)$

   (b) $\cos(\pi n/3 + \pi/4)$

   (c) $2^{-|n|}$

   (d) $\delta[n-1] + \delta[n+1]$

4. [†] Consider the signal depicted below. Let the Fourier transform of this signal be written in rectangular form as $X(\omega) = A(\omega) + jB(\omega)$. Sketch the time domain signal whose transform is $Y(\omega) = B(\omega) + A(\omega)e^{j\omega}$.



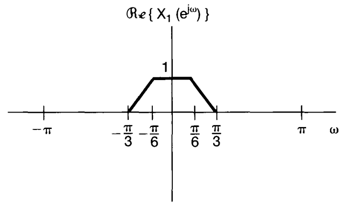
5. [†] Consider the LSI system with impulse response $h[n] = 2^{-n}\cos(\pi n/2)u[n]$. Determine the frequency response of this system.

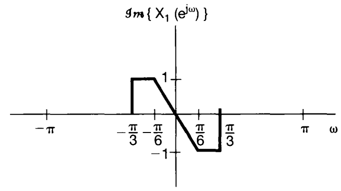
Now, consider the input signal $x[n] = \cos(\pi n/2)$. Detemine the corresponding output signal.

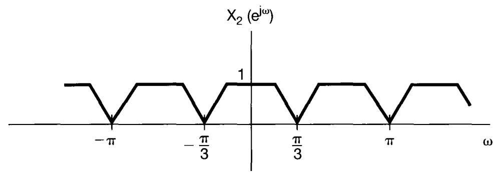
6. [†] The DTFT of the signal $x_1[n]$ is shown below.

   (a) Consider the signal $x_2[n]$ whose spectrum is shown below. Express $x_2[n]$ in terms of $x_1[n]$.

   (b) Consider the signal $x_3[n]$ with spectrum as shown below. Express $x_3[n]$ in terms of $x_1[n]$.



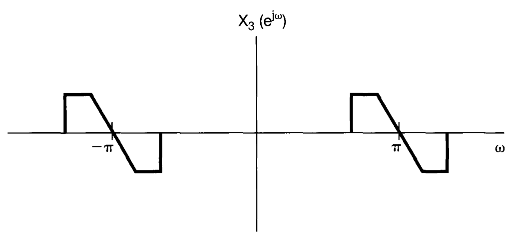




(c) Compute

$$\alpha := \frac{\sum_{-\infty}^{\infty} n x_1[n]}{\sum_{-\infty}^{\infty} x_1[n]}$$

without inverting the spectrum of $x_1[n]$.


$X_3(e^{j\omega})$
$-\pi$
π
ω