Tutorial - 1 EE 302 Control System

1] Find the Laplace transform of the following

(a) $f(t) = 5e^{3t}u(t)$

(b) $f(t) = 5t\,e^{3t}$

(c) $f(t) = 8\cos(3t)$

(d) $f(t) = 4t\sin(8t)$

(e) $f(t) = 5e^{-2t} + t$

($u(t) \Rightarrow$ unit step function)

2] Find inverse Laplace transform of the following

(a) $F(s) = \dfrac{1}{s+4}$

(b) $F(s) = \dfrac{1}{(s+8)^2}$

(c) $F(s) = \dfrac{5s}{s^2+9}$

(d) $F(s) = \dfrac{s+2}{(s+2)^2+4}$

(e) $F(s) = \dfrac{9}{(s+1)^2+9}$

(f) $F(s) = \dfrac{s^2+2s+1}{s^2+5s+6}$

3] Perform partial fraction expansion & find inverse Laplace transform

(a) $F(s) = \dfrac{3}{s^2+3s+2}$

(b) $F(s) = \dfrac{3}{s(s^2+s-2)}$

(c) $F(s) = \dfrac{2}{(s+1)(s+4)^2}$

(d) $F(s) = \dfrac{8}{s(s^2+2s+2)}$

(e) $F(s) = \dfrac{4s}{s^2+11s+30}$

4] Check the validity of initial value theorem for the following functions. Find the solution, if applicable for the same.

(a) $f(t) = (3e^{-2t} + 9)\,u(t)$

(b) $f(t) = (7e^{-8t} + 2e^{-5t})\,u(t)$

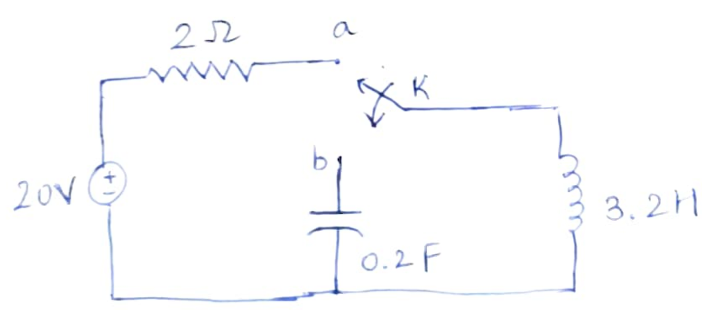
($u(t) \Rightarrow$ unit step function)

5] Check the validity of the final value theorem for the following functions. Find the solution, if applicable for the same.

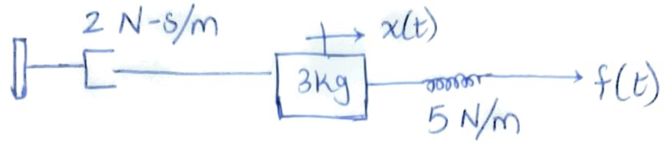
(a) $f(t) = 5\sin(3t) + 2e^{-2t} + e^{-t}$

(b) $f(t) = 7e^{-10t} + 6e^{-2t} + 3e^{-t}$

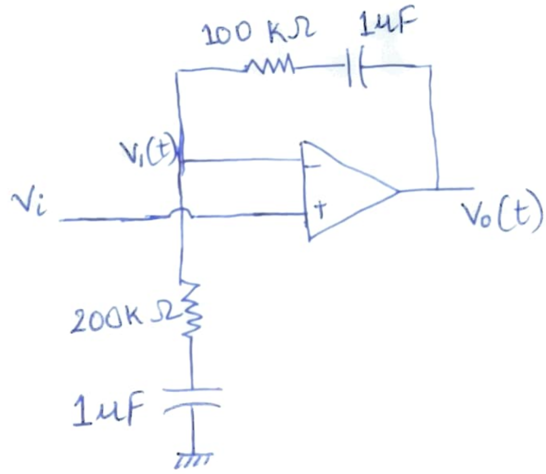
6] In the circuit shown, switch K is moved from position 'a' to position 'b' at $t=0$. (Steady state at 'a' prior to $t=0$) Solve for $i(t)$ using Laplace transform



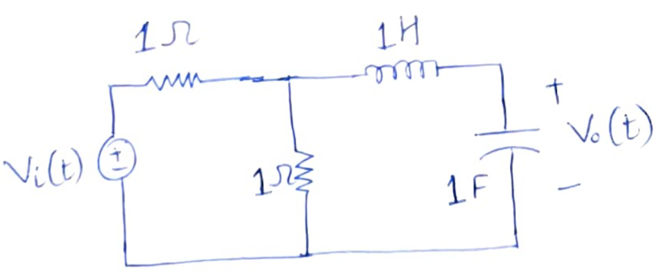
7] Find the Transfer function $G(s) = \frac{X(s)}{F(s)}$, for translational mechanical system as shown below



8] Find the transfer function $G(s) = \frac{V_o(s)}{V_i(s)}$ for the op-amp circuit.



9] Find the transfer function $G(s) = \frac{V_o(s)}{V_i(s)}$ for the below network.



Q-10 Check that $e^{3t} * \sin 4t$ obtained through convolution definition & using their Laplace transforms are same. (Signals are zero for negative time.)

Q-11: Check that $f * g = g * f$ and the convolution integral reduces from $-\infty$ to ∞ when both f, g are zero for $t < 0$.

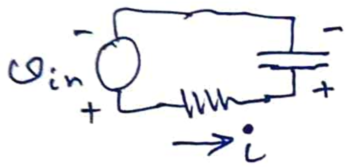
Q-12: Check that $f' * g = g' * f = \frac{d}{dt}(f * g)$ and thus that the following 3 commute

$$f \rightarrow \boxed{s} \rightarrow \boxed{G(s)} \rightarrow \quad = \quad g \rightarrow \boxed{F} \rightarrow \boxed{s} \rightarrow$$

what about initial conditions? $(F(s) = \mathcal{L}(f(t)))$.

(In Q-12, assume f, g are zero for $t < 0$).

Q-13: Consider the RC circuit



$v_c(t)$ and the transfer fn from v_{in} to current i.

Suppose initial voltage across capacitor $= 5V$., capacitance $= 0.2F$, $R = 30\Omega$.

Check that the effects of initial voltage across capacitor & input V_{in}: both effects on current i, check that these 2 effects are additive.

thus transfer fn from $v_c(0) \rightarrow I(s)$ & $v_{in} \rightarrow I(s)$ can be found individually & be added (to get $I(s)$).

Q-14: Consider diff eqn

$$4\frac{d}{dt}y - \frac{6d^2}{dt^2}y + 3y = \frac{d}{dt}u - 6u.$$

Find transfer fn $G(s)$ and check that for input $u(t) = 4e^{3t}$, (for $t \in (-\infty, \infty)$)

the output $y(t) = 4\, G(s)\big|_{s=3} e^{3t}$ solves the diff eqn.

For $u(t)$ being zero for $t<0$, Find p in eqn below:

$$y(t) = p e^{3t} + \text{natural response part}$$