EE302 Tutorial Sheet 2, Section S2, Tutorial 1st Feb 2024.

Q-1: Plot in the same figure the step responses of following transfer fns-

$$\frac{s-1}{2s+3}, \quad \frac{1}{s+2}, \quad \frac{1}{s+3}, \quad \frac{1}{1+0.3s}, \quad \frac{1}{1+0.2s}, \quad \frac{s+2}{4s+5}$$

Q-2: Suppose $G(s)$ is proper and 1st order.
then check if $G(0) = \lim_{t\to\infty}$ step response: under what assumptions?

Q-3(a) Suppose $G(s)$ is proper and 1st order and stable (i.e. poles in OLHCP) $\equiv$ OLHP
If $y(t)$ is the step response, then $y(0^+) \neq 0 \Leftrightarrow G(s)$ has a zero.

(b) Show that $y(0^+) \cdot y(\infty) < 0 \Leftrightarrow G(s)$ has a non-minimum phase zero.

Q-4: Use Initial value theorem for finding initial value (i.e. $t \to 0^+$ not at $t=0$) of impulse response and step response of $\frac{10}{3s+61}$, $\frac{10s-9}{3s+61}$, $\frac{10}{s-3}$

Q-5: For proper $G(s)$, relative degree of $G(s) := \deg d - \deg n$ $\left(G(s) = \frac{n(s)}{d(s)}\right)$ $(d \neq 0)$. $n, d \equiv$ polynomials.
Consider step response $y(t)$ of $G(s)$.
Suppose relative degree of $G(s) = 7$.
then what can be told about $y(0)$, $\dot{y}(0^+)$, $\ddot{y}(0^+)$, ... $\left.\frac{d^6}{dt^6}y\right|_{0^+}$, $\left.\frac{d^7}{dt^7}y\right|_{0^+}$, $\left.\frac{d^8}{dt^8}y\right|_{0^+}$

Q-6: Suppose $G(s)$ is biproper and has pole, zero in OLHP, G is 1st order.
Suppose $G(0) > 0$. (DC gain $\equiv G(0)$ for stable $G(s)$).
depending on step response $y(t)$'s initial value $y(0^+)$ & $\dot{y}(0^+)$ (+ve or −ve), what can be told about whether pole or zero is closer to $j\mathbb{R}$?

Q-7:
Suppose impulse response of $G(s)$ (proper, stable) is $a\delta + be^{-t/T}$ (for $t \geq 0$). $T > 0$.
Find relative condition between $a, b, T \in \mathbb{R}$ for
— zero to be closer than pole (and both in OLHP) from $j\mathbb{R}$.
— pole to be closer than zero from $j\mathbb{R}$ (& both in OLHP)
— zero at origin $s=0$.

Q 8 onwards: see following pages.

Q8) Find steady state value for the following

(a) $\frac{100}{(s+6)^2+64}$ for unit step input

(b) $\frac{1}{s^2+3s+2}$ for unit impulse input

Q9) Find the step response for $\frac{-1}{s+10}$. Also find the step response when an zero at $s=10$ is added.

[(ie) step response of the non-minimum phase system $\frac{-(s-10)}{s+10}$]

Q10) For unit step input find initial rise rate for the following $\frac{5}{s+5}$, $\frac{20}{s+20}$. Also find rise time & settling time.

Q-11 :- For the following transfer functions, write the general form of step response

a] $G(s) = \frac{400}{s^2+12s+400}$,

b] $G(s) = \frac{900}{s^2+90s+900}$

c] $G(s) = \frac{225}{s^2+30s+225}$,

d] $G(s) = \frac{625}{s^2+625}$

Q. 12 :- For the given transfer functions, find ζ & ω_n

a] $G(s) = \frac{36}{s^2+4.2s+36}$,

b] $\frac{20}{s^2+6s+44}$

c] $G(s) = \frac{s+2}{s^2+9}$,

d] $\frac{5}{(s+3)(s+6)}$

Also state the nature of each response (overdamped, underdamped, & so on).

Q. 13 :- For each of the second order systems given below find ζ, ω_n, T_s, T_p, T_r & % overshoot.

a] $T(s) = \frac{16}{s^2+3s+16}$

b] $= \frac{0.04}{s^2+0.02s+0.04}$

c] $T(s) = \frac{1.05\times10^7}{s^2+1.6\times10^3 s+1.05\times10^7}$

Q. 14 :- Find the transfer function of a second-order system that yields a 12.3% overshoot and a settling time of 1 second.

Q15 Consider a first order system of the form

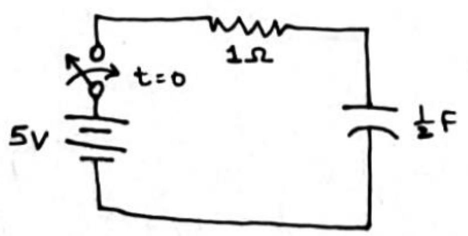
$$\tau \frac{dx}{dt} = -x + u \quad , \quad y = x$$

we say that the parameter τ is the time constant for the system, since when the input is zero, the system approaches the origin as $e^{-t/\tau}x(0)$. For this model, show the following:-

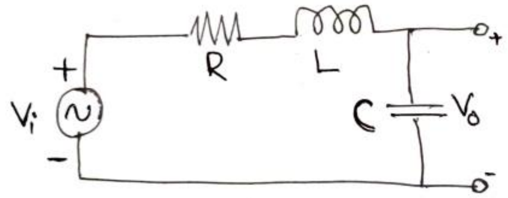
(i) Show that the rise time for a step response of the system is approximately 2τ.

(ii) Show that the 1%, 2% and 5% settling times approximately equal to 4.6τ, 4τ and 3τ respectively.

Q16 Find the capacitor voltage in the network shown below, if the switch closes at $t=0$. Assume zero initial conditions. Also find the time constant, rise time, and settling time for the capacitor voltage.



Q. 17 Consider a series RLC circuit:
For $R = 1\Omega$, $L = 1H$ and $C = 1F$, the time constant of circuit (in sec) is



Simple example to notice how feedback is more robust than ~~oaper~~ open loop controller.

This problem is for concluding the robustness of feedback controller w.r.t. changes in system pole and/or initial condition $x(0)$.

Q.18 Consider model $\dot{x} = 3x + u$ & $x(0) = 4$.

- Check input $u(t)$ that gives $x(t) = x(0)e^{-2t}$ (after control action)

($u = -5x$ would have sufficed: but obtain $u(t)$ explicitly :)

- Now apply $u(t)$ from above explicitly to
$\dot{x} = 3x + u$ & $\mathcal{L}(\quad) = \mathcal{L}(\quad)$ to get

$$sX(s) - 3X(s) - x(0) = U(s).$$

thus $X(s) = \frac{1}{(s-3)}\left[\quad\right]$ & check that the $u(t)$ indeed gives $x(0)e^{-2t}$.

Now use same $u(t)$ for actual system $\dot{x} = 3.1x + u$ and/or actual initial condition $x(0) = 3.9$

check if $x(t)$ is still $x(0)e^{-2t}$ or how different $x(t)$ could be due to the mismatch in system equation/initial condition.