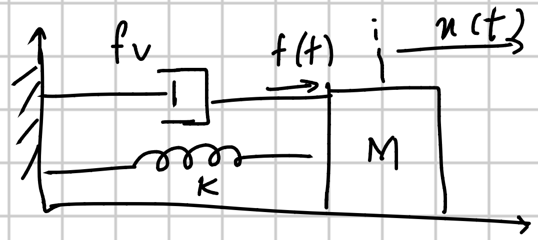
01)



Input = Force
output - Displacement

$$f(t) = f_v \dot{x}(t) + K x(t) + M \ddot{x}(t) \longrightarrow (1)$$

$$F(s) = f_v s X(s) + k X(s) + M s^2 X(s)$$

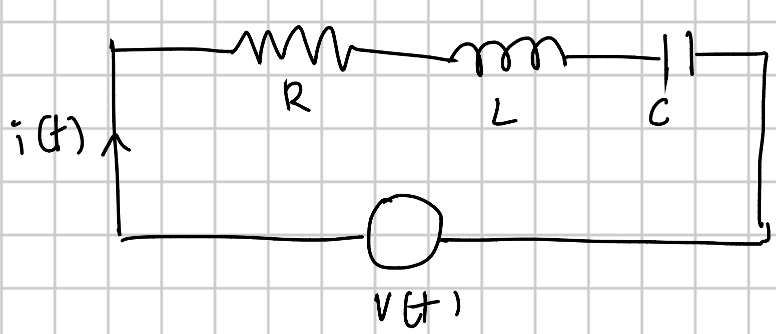
$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

Force - voltage ; current - velocity :
$f(t) - v(t)$ $i(t) - \dot{x}(t)$

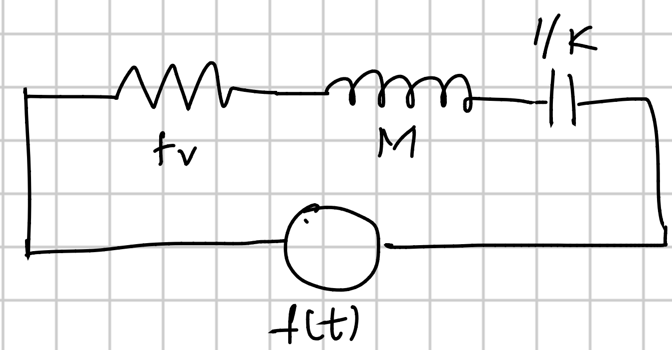
(1) becomes ;

$$v(t) = f_v i(t) + K \int i(t)\,dt + M \frac{di(t)}{dt}$$

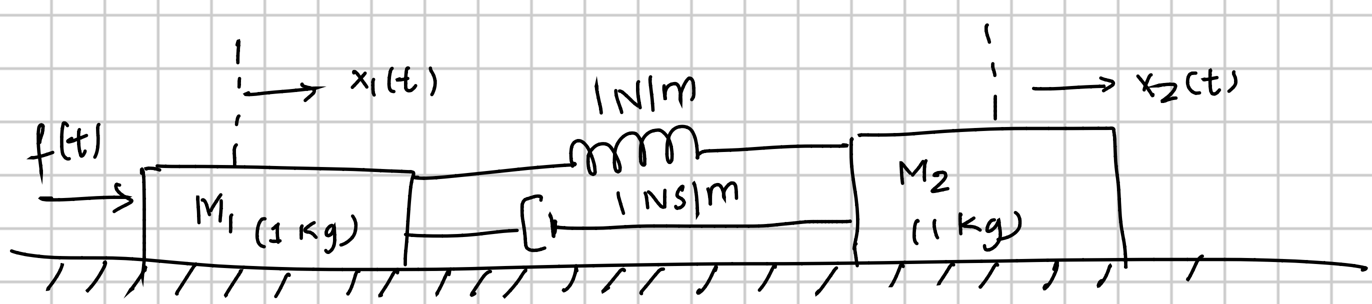
$$v(t) = R i(t) + \frac{1}{C} \int i\,dt + L \frac{di}{dt}$$



$\rightleftharpoons$



02)



i/p = $f(t)$
o/p = $x_2(t)$

$B = 1$
$K = 1$

For M_1 :

$$F(t) = M_1 \ddot{x}_1(t) + [\dot{x}_1(t) - \dot{x}_2(t)] + [x_1(t) - x_2(t)]$$

For M_2 :

$$0 = M_2 \ddot{x}_2(t) + (\dot{x}_2(t) - \dot{x}_1(t)) + [x_2(t) - x_1(t)]$$

$$F(s) = M_1 s^2 X_1(s) + sX_1(s) - sX_2(s) + X_1(s) - X_2(s)$$

$$F(s) = [M_1 s^2 + s + 1] X_1(s) - (s+1) X_2(s)$$

$$0 = M_2 s^2 X_2(s) + sX_2(s) - sX_1(s) + X_2(s) - X_1(s)$$

$$0 = [M_2 s^2 + s + 1] X_2(s) - (s+1) X_1(s)$$

$$X_1(s) = \frac{(M_2 s^2 + s + 1)}{s+1} X_2(s)$$

$$F(s) = \left(\frac{(M_1 s^2 + s + 1)(M_2 s^2 + s + 1)}{(s+1)} \right) X_2(s) - (s+1) X_2(s)$$

$$= \frac{X_2(s)}{(s+1)} \left(M_1 M_2 s^4 + M_1 s^3 + M_1 s^2 + M_2 s^3 + s^2 + s + M_2 s^2 + s + 1 - (s+1)^2 \right)$$

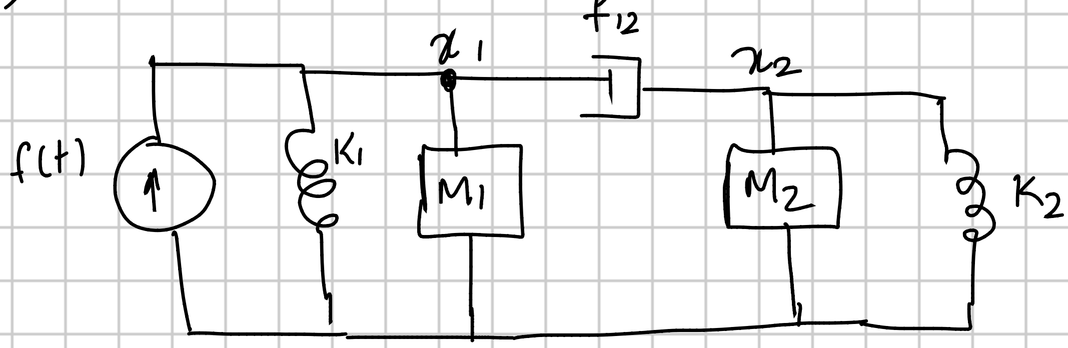
$$= \frac{X_2(s)}{(s+1)} \left[M_1 M_2 s^4 + s^2 (M_1 s + M_1 + M_2 s + M_2) + s^2 + 2s + 1 - (s+1)^2 \right]$$

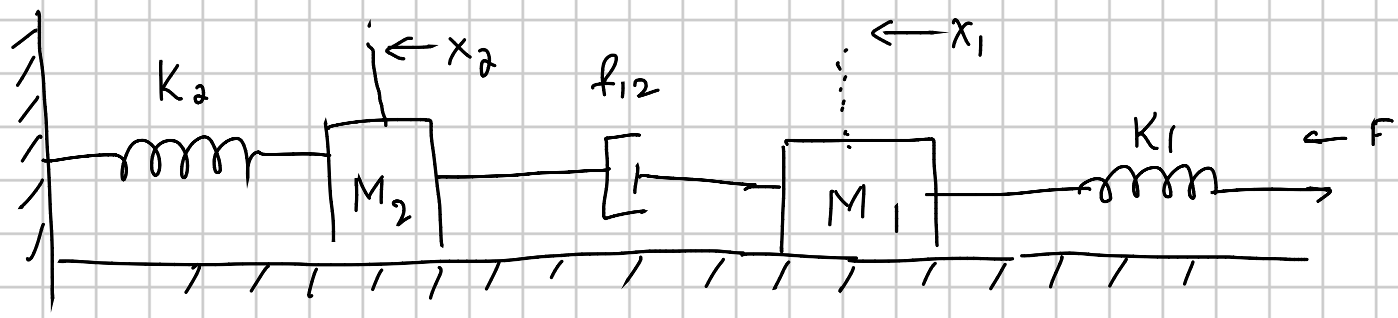
$\boxed{M_1 = 1 \quad M_2 = 1}$

$$= \frac{X_2(s)}{(s+1)} \left[M_1 M_2 s^4 + s^2 (M_1 + M_2)(s+1) + (s+1)^2 - (s+1)^2 \right]$$

$$\boxed{\frac{X_2(s)}{F(s)} = \frac{s+1}{s^4 + 2s^3 + 2s^2}}$$

03)





→ For M_1 :

$$F(t) = M_1 \ddot{x}_1(t) + f_{12}(\dot{x}_1(t) - \dot{x}_2(t)) + k_1 x_1$$

For M_2 :

$$0 = M_2 \ddot{x}_2(t) + f_{12}(\dot{x}_2(t) - \dot{x}_1(t)) + k_2 x_2$$

(a) Voltage as a source

Voltage = Force.

Both equations become

$$v = V_R + V_1 + V_C$$

$$I = \frac{dq}{dt}$$

$$v = IR + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt$$

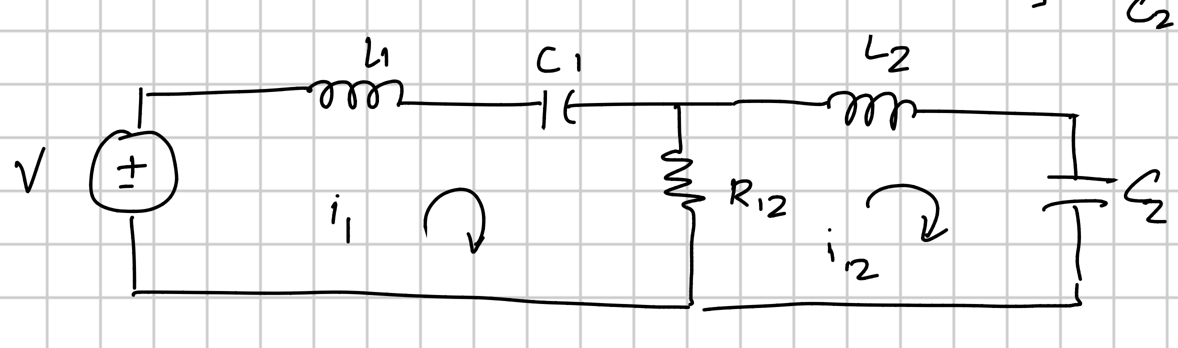
$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$V(s) = sRQ + s^2LQ + \frac{Q}{C}$$

$M \to L$; $R \to B$; $K \to 1/C$

$$V(t) = L_1 \dot{i}_1(t) + R_{12}\left[i_1(t) - i_2(t)\right] + \frac{1}{C_1}\int i_1\,dt$$

$$0 = L_2 \dot{i}_2(t) + R_{12}\left[I_2(t) - i_1(t)\right] + \frac{1}{C_2}\int i_2\,dt$$



(b) current as source

$$I = I_R + I_L + I_C$$

$$V = \frac{d\phi}{dt}$$

$$I = \frac{V}{R} + \frac{1}{L}\int v\,dt + C\frac{dV}{dt}$$

$$I = \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi + C\frac{d^2\phi}{dt}$$

$$I(s) = \frac{s}{R}\phi(s) + \frac{\phi}{L} + s^2 C\phi$$

$$I = Cs^2\phi + \frac{1}{R}s\phi + \frac{1}{L}\phi$$

$$M \rightarrow C; \quad B \rightarrow \frac{1}{R}; \quad K \rightarrow \frac{1}{L}$$

$$I = C_1\frac{dv_1}{dt} + \frac{v_1 - v_2}{R_{12}} + \frac{1}{L_1}\int v_1\,dt$$

$$0 = C_2\frac{dV_2}{dt} + \frac{V_1 - V_2}{R_{12}} + \frac{1}{L_2}\int v_2\,dt$$