Tutorial Sheet 1, EE302, S1 - BTech 16th Jan 2025

- Attempt all questions before the tutorial.
- Unless otherwise stated explicitly, all functions $f(t)$ are zero for $t < 0$.)

Q-1:

(a) Find laplace transform for $5e^{3t}\sin 2t$ and $(6t+1)e^{-2t}$

(b) Find inverse Laplace transform of

$$\frac{1}{(s+3)^2}, \quad \frac{2s+3}{s^2+4s+13}, \quad 2s, \quad \frac{3s+2}{s-5}$$

Q-2: Use Initial Value theorem (IVT) & Final value theorem: if they are applicable to find step responses' following values $y(0^+)$, $\dot{y}(0^+)$, $\ddot{y}(0^+)$, $y(\infty)$ for system $G(s) = \frac{as+b}{s+c}$, $\frac{2s}{s^2+4}$, $\frac{3}{s^2+4}$

$$\frac{10}{s+5}, \quad \frac{10}{s-5}$$

Q-3: Use IVT to match the Laplace transform with the given functions: $3e^{-t}$, $4e^{5t}$, $4\sin\omega t$, $38\sin\omega t$, $\cos\omega t$,

3, $\frac{4\omega}{s^2+\omega^2}$, $\frac{4}{s-5}$, $\frac{\omega}{s^2+\omega^2}$, $\frac{s}{s^2+\omega^2}$, $\frac{3}{s+1}$ (give reasons)

Q-4: Consider step response $y(t)$ of system $G(s) = \frac{as+b}{s+c}$, $a \neq 0$, $c > 0$, $b \neq 0$.

Show that $y(0^+) \cdot y(\infty) \neq 0$ and

$$y(0^+) \cdot y(\infty) < 0 \iff \frac{-b}{a} > 0$$

(i.e. zero of $G(s)$ is in RHP).

Q-5: Assume transfer function $G(s)$ is proper and let $y(t)$ be step response.

Show that G is biproper (i.e. relative degree $= 0$) if and only if $y(0^+) \neq 0$.

Q-6: Obtain convolution of e^{-3t} & e^{5t} (both are zero in -ve time)

by (a) integrating in time domain

(b) Using Laplace transform (and partial fraction expansion)

(c) ~~by~~ interpreting $G(s)$ as gain for input e^{-3t}

(d). —"— for input e^{5t}

See Q-7 below.

(e) partial fraction expansion (for above simple case) can be found by substituting $s=5$ & $s=-3$ in $1 = a(s+3) + b(s-5)$.

Relate this to c, d above.

Q-7: Consider particular solution $y(t)$ in solution to differential equation $\left(\frac{d^2}{dt^2} + 3\frac{d}{dt} + 2\right)y = \left(\frac{d}{dt} - 5\right)u$ for $u(t) = e^{7t}$.

Check/verify that $y(t) = G(s)\Big|_{s=7} e^{7t}$ is the particular soln (for $G(s) = \frac{s-5}{s^2+3s+2}$)

Q-8: Convolution of f, g (both $=0$ for $t<0$) satisfies

$\frac{d}{dt}(f * g) = \dot{f} * g = f * \dot{g}$ & convolution is "commutative".

- Relate this to interchange of sequence of the following blocks

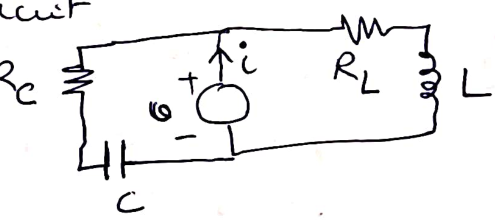
$f \to \boxed{?} \to \boxed{s} \to \;\equiv\; g \to \boxed{s} \to \boxed{??} \to \;\equiv\; \delta \to \boxed{?} \to \boxed{?} \to \boxed{?} \to$

- What should be assumed about initial conditions of f & g?

Q-9: Suppose $f, g : \mathbb{R} \to \mathbb{R}$ & $f * g\,(t) := \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,d\tau$.

Check that for $f, g = 0$ for $t<0$, integral is just $\int_0^t f(t-\tau)g(\tau)\,d\tau$.

Q-10: Consider RLC circuit



- let initial conditions $=0$.

- find impedance $Z(s) = \frac{V(s)}{I(s)}$

- find admittance $G(s) = \frac{I(s)}{V(s)}$.

Use that inductor is impedance sL & capacitor $\frac{1}{sC}$ & use modelling in s-domain

- poles of either $G(s)$ or $Z(s)$ cannot be complex: why?

- Comment about proper/improper (of Z, G) using open/short behavior of C, L at high frequencies.