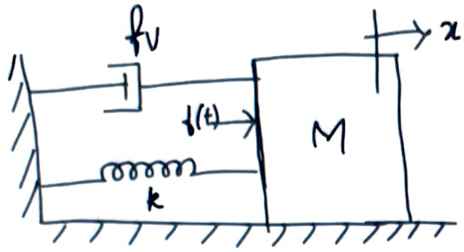
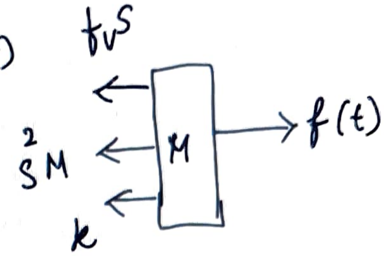
Solution:

Q1)





$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{dx} + k\,x(t) = f(t) \quad \text{—— (1)}$$

$$s^2 M X(s) + s f_v X(s) + k X(s) = F(s)$$

$$\frac{X(S)}{F(s)} = \frac{1}{Ms^2 + f_v s + k}$$

In voltage-current $\rightleftharpoons$ force-velocity

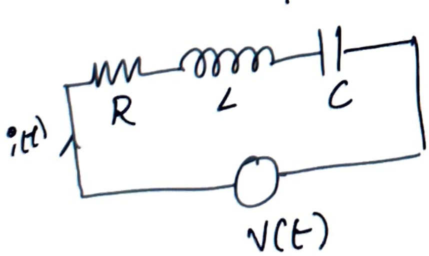
we have force → voltage velocity → current

$f(t) \rightarrow v(t)$ $\dot{x}(t) \rightarrow i(t)$

∴ rewriting Eqn (1).

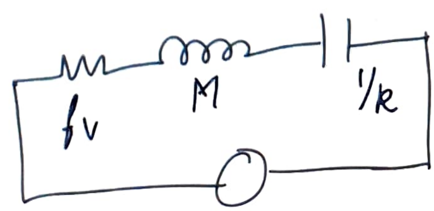
$$M\frac{di(t)}{dt} + f_v i(t) + k\int i(t)\,dt = v(t)$$

↳ is the mesh Eqn for network

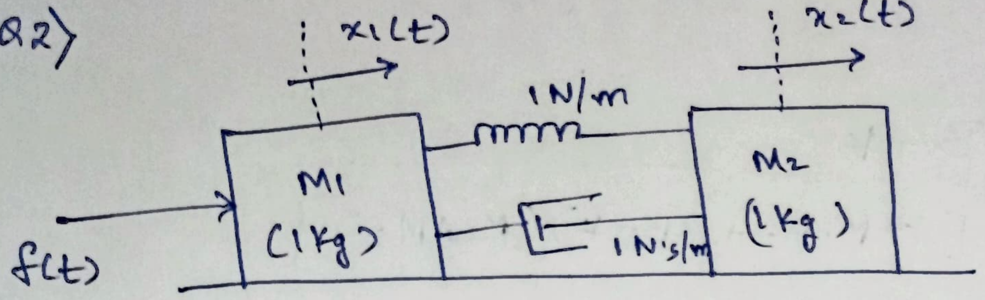


$$V(t) = R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

∴ the Eq. network



Q2)



(frictionless surface)

→ <u>for mass M_1</u>

$$F = M_1 \frac{d^2 x_1}{dt^2} + B\frac{d}{dt}(x_1 - x_2) + K(x_1 - x_2) \longrightarrow (1)$$

→ <u>for mass M_2</u>

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B\frac{d}{dt}(x_2 - x_1) + K.(x_2 - x_1) \longrightarrow (2)$$

Take Laplace transform for eqn (1), and (2) we get

$$\Rightarrow F(s) = [s^2 M_1 + sB + K] X_1 - (sB + K) X_2 \longrightarrow (3)$$

$$\Rightarrow 0 = (s^2 M_2 + sB + K) X_2 - (sB + K) X_1 \longrightarrow (4)$$

from eqn (4) $$X_1 = \frac{(s^2 M_2 + sB + K)}{sB + K} X_2 \longrightarrow (5)$$

substitute (5) in (3). we get.

$$\Rightarrow F(s) = (s^2 M_1 + sB + K)\frac{(s^2 M_2 + sB + K)}{sB + K} X_2$$

$$- (sB + K) X_2$$

$$\Rightarrow F(s) = \frac{X_2}{(sB+K)}\left[s^4 M_1 M_2 + s^2 (sB+K)(M_1+M_2) + (sB+K)^2 - (sB+K)^2 \right]$$

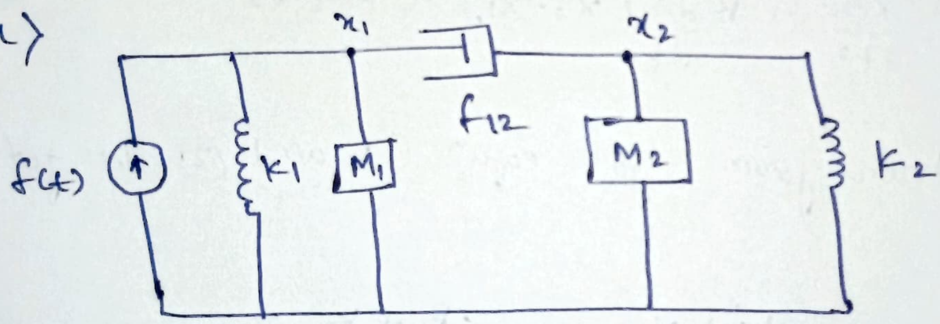
$$\Rightarrow \frac{X_2}{F(s)} = \frac{sB+K}{s^2\left[s^2 M_1 M_2 + (sB+K)(M_1+M_2)\right]}$$

Now, from given Datat, the required T.F is

$$\boxed{\frac{X_2}{F(s)} = \frac{s+1}{s^2\left[s^2+2s+2\right]}}$$

Q3) Mechanical ckt diagram.

a)



at Node x_1.

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + f_{12}\frac{d}{dt}(x_1 - x_2) + K_1 x_1$$

at node x_2 :-

$$0 = M_2 \frac{d^2 x_2}{dt^2} + f_{12}\frac{d}{dt}(x_2 - x_1) + K_2 x_2$$

F-V analogy.

$M \rightarrow L$

$F \rightarrow V$

$f \rightarrow R$

$K \rightarrow 1/C$

$v \rightarrow i$

$x \rightarrow q$

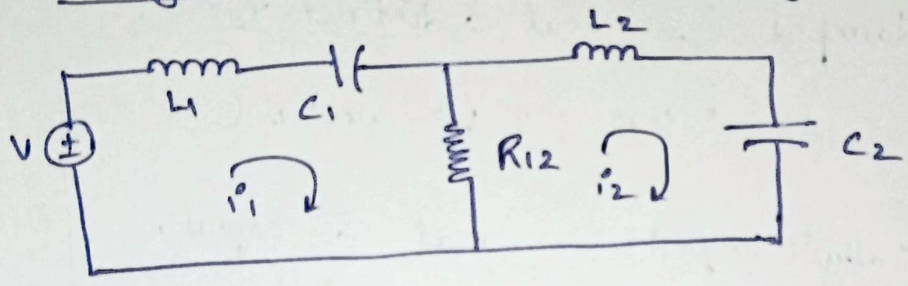
remember.

→ corresponding electrical system (driven by v-source) will have the following system equations.

1st loop → $V = L_1\frac{di_1}{dt} + R_{12}(i_1 - i_2) + \frac{1}{C_1}\int i_1 dt$

2nd loop → $0 = L_2\frac{di_2}{dt} + R_{12}(i_2 - i_1) + \frac{1}{C_2}\int i_2 dt$

Analogous electrical ckt driven by v-source.



b) using F-I analogy.

$F \to I$ $\quad K \to 1/L$

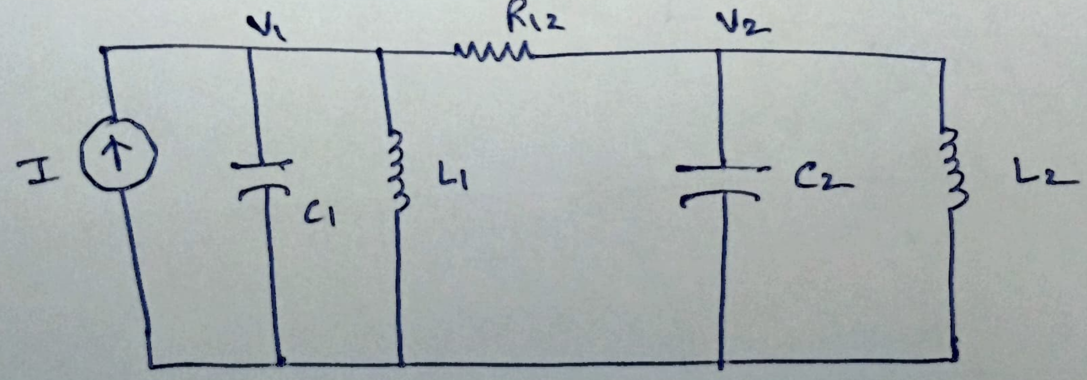
$M \to C$ $\quad v \to V$

$f \to 1/R$ $\quad x \to \phi$

1st Node eqn $\quad I = C_1\frac{dV_1}{dt} + \frac{V_1 - V_2}{R_{12}} + \frac{1}{L_1}\int V_1 dt$

2nd Noc eqn. $\quad 0 = C_2\frac{dV_2}{dt} + \frac{V_1 - V_2}{R_{12}} + \frac{1}{L_2}\int V_2 dt$

Analogous ele. ckt driven by I-source.



Q 4) Based on the given characteristic eqn and nature of roots we can judge nature of response.

Note:

→ underdamped → complex roots

$D < 0$ i.e $[(b^2 - 4ac) < 0]$

→ Over damped → real & distinct roots.

$D > 0$, $[b^2 - 4ac > 0]$

→ Critically Damped → real & equal.

$D = 0$

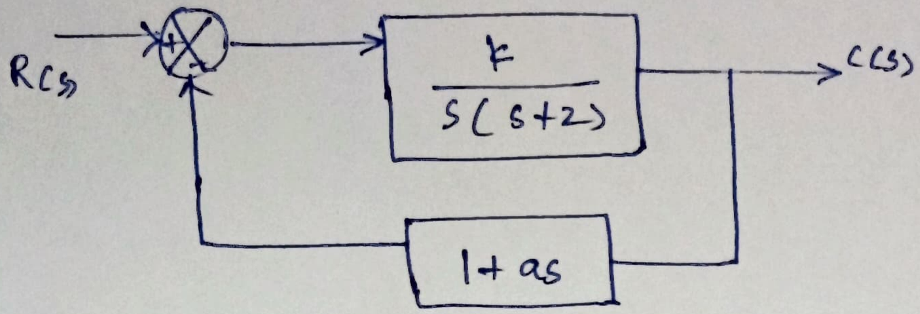
for: (i) $s^2 + 5s + 6$, $D = 1$ i.e > 0

(ii) $s^2 + 100 \Rightarrow s = \pm j10$

(iii) $s^2 + 2s + 2 \Rightarrow D = -4 < 0$

$$\boxed{\begin{array}{l} (i) \longrightarrow C \\ (ii) \longrightarrow A \\ (iii) \longrightarrow B \end{array}}$$

Q5>



denominator of CLTF ⇒ char. eqⁿ ie $1+G(s)H(s)$

$$1+\frac{K}{s(s+2)}\times(1+as) \Rightarrow$$

$$\Rightarrow \boxed{s^2+(2+Ka)s+K=0} \rightarrow \text{characteristic eq}^n.$$

comparing it with standard 2nd order

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

on comparing.

$\Rightarrow \omega_n^2 = K$

given, $\omega_n = 4$ rad/sec

$\boxed{\therefore K = 4^2}$

$2\xi\omega_n = 2+aK$

$\Rightarrow 2\times0.7\times4 = 2+16a$

$\Rightarrow \boxed{a = 0.225}$

$$\boxed{\begin{matrix} K=16 \\ a=0.225 \end{matrix}}$$

Q 6)

→ no zeros

→ DC gain = 10

→ % overshoot = 8%.

→ $t_s = 10$ sec [2%]

$$\boxed{\% \text{ overshoot} = e^{\left[\frac{-\pi \xi}{\sqrt{1-\xi^2}}\right]} \times 100}$$

$$\Rightarrow 0.08 = e^{\left(-\pi\xi/\sqrt{1-\xi^2}\right)}$$

Apply natural log and simplify

$$\Rightarrow 2.52 = \frac{\pi \xi}{\sqrt{1-\xi^2}}$$

solve for ξ, we get $\boxed{\xi = 0.62}$

$$\Rightarrow \because T_s = \frac{4}{\xi \omega_n} \qquad \Rightarrow 10 = \frac{4}{0.627\omega_n}$$

$$\Rightarrow \boxed{\omega_n = 0.637 \text{ rad/sec}}$$

$$\rightarrow \text{DC gain} = \frac{K}{\omega_n^2} \qquad \Rightarrow 10 = \frac{K}{(0.637)^2}$$

$$\Rightarrow \boxed{K = 4.05}$$

The transfer function $= \dfrac{4.05}{s^2 + 0.798s + 0.405}$

6) contd.

$T_p = 2$ sec [check if possible]

$$\boxed{T_p = \frac{\pi}{\omega_d}}$$

$$\omega_d = \left(\sqrt{1-\xi^2}\right)\omega_n$$

$$\Rightarrow \boxed{\omega_d = 0.496}$$ substitute values

Now,

$$T_p = \frac{\pi}{0.496} \simeq 6.34 \text{ sec}$$

$$T_p = 2 \text{ sec given} \neq 6.34 \text{ sec}$$

$\therefore$ $T_p = 2$ sec is not possible

Q7) Consider $G(S) = \dfrac{2P}{(S+2)(S+P)}$

Partial fraction for step response

$$Y(S) = \frac{2P}{S(S+2)(S+P)} = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S+P}$$

Residues: $2P = A(S+2)(S+P) + BS(S+P) + CS(S+2)$

$$\Rightarrow A = 1, \quad B = \frac{-P}{P-2}, \quad \boxed{C = \frac{2}{P-2}}$$

as $P \longrightarrow \infty$ $C \longrightarrow 0$ indicating that contribution of term $\frac{C}{S+P}$ is lower as $P \longrightarrow \infty$.

Q8. The DE given is

$$4\frac{d^2c(t)}{dt^2} + 8\frac{dc(t)}{dt} + 2c(t) = 16\,r(t)$$

Take Laplace transform.

$$\Rightarrow \left[4s^2 + 8s + 2\right]C(s) = 16\,R(s)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{16}{4s^2 + 8s + 2}$$

$$= \frac{4}{s^2 + 2s + \left(\frac{1}{2}\right)}$$

→ comparing with standard eqn i.e $\frac{K\omega_n^2}{s^2 + 2\xi\omega_n \cdot s + \omega_n^2}$; K - gain

$\omega_n^2 = \frac{1}{2}$; $\boxed{\omega_n = \frac{1}{\sqrt{2}}}$

and $2\xi\omega_n = 2 \Rightarrow \xi\omega_n = 1$ $\boxed{\xi = \sqrt{2}}$

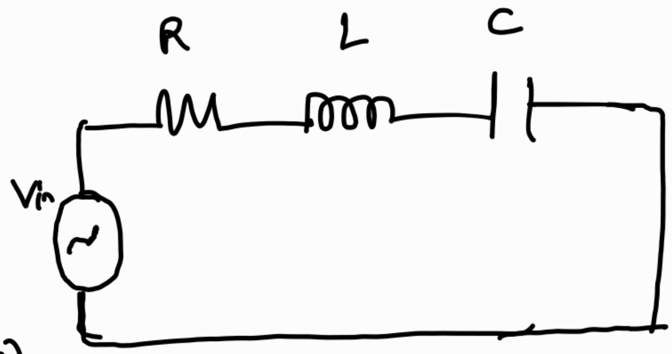
Damping ratio $(\xi) = \sqrt{2}$

Natural freq. $(\omega_n) = \frac{1}{\sqrt{2}}$ rad/sec

Q9) $V = 10\sin\omega t$

$R = 6\Omega$

$L = 3H,\ C = 3F$



i) Using KVL,

$$V_{in}(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s)$$

$$V_{out}(s) = \frac{1}{sC}I(s)$$ [$\because$ asked across capacitor]

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{s^2LC + sRC + 1}$$

ii) Damping ratio is derived from the characteristic equation,

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Comparing with $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$.

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad 2\zeta\omega_n = \frac{R}{L} \Rightarrow \boxed{\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}} \quad \therefore \zeta \propto R$$

iii) $\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{3}$ rad/s

Damping ratio, $\zeta = \frac{6}{2}\sqrt{\frac{3}{3}} = 3$

iv) $\zeta < 1 \Rightarrow \frac{R}{2}\sqrt{\frac{C}{L}} < 1 \Rightarrow 0 < R < 2\Omega$

Question 10

[A]

(a) $\frac{1}{s+1}$ DC Gain = 1

Time constant = 1

(a) → 3

(b) $\frac{2}{s+0.5}$ DC - Gain = 4

(b) → 1

(c) $\frac{4}{s+0.1}$ DC Gain = 40

(d) $\frac{3}{s+3}$ DC Gain = 1

TC = 0.33

(d) → 2

(e) $\frac{1}{s+2}$ DC Gain = 0.5

TC = 0.5

(e) → 4

(B)

(a) → 3

$T_p = 9.7s$

$T_s = 5.8$

(b) → 2

$T_p = 8.43\ s$

$T_s = 4.12s$

(d) → 4

$T_p = 3.97s$

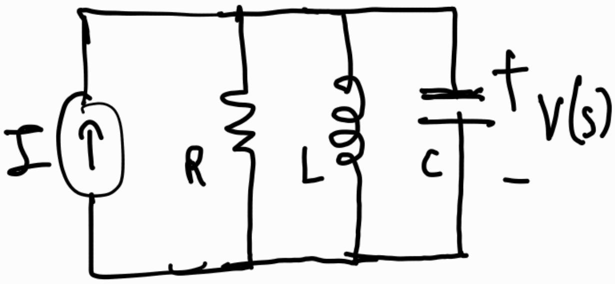
$T_s = 1.9s$

(e) → 1

$T_p = 1.75$

Q11) $I = 3\sin\omega t,\ R = 18\,\Omega,\ L = 9\,H,\ C = 1F$

i) From KCL,



$$I_{in}(s) = \frac{V(s)}{R} + \frac{V(s)}{SL} + SCV(s)$$

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{1}{\frac{1}{R} + \frac{1}{SL} + SC}$$

$$H(s) = \frac{1}{s^2LC + s\frac{L}{R} + 1}$$

ii) Damping ratio is derived from characteristic equation, $s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad 2\zeta\omega_n = \frac{1}{RC}$$

$$\boxed{\zeta = \frac{1}{2R}\sqrt{\frac{L}{C}}} \qquad \therefore \zeta \propto \frac{1}{R}$$

iii) $\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{3}$ rad/s

$$\delta = \frac{1}{2 \times 18}\sqrt{\frac{9}{1}} = \frac{1}{12}$$

iv) $$\frac{1}{2R}\sqrt{\frac{L}{C}} < 1$$

$$R > \frac{1}{2}\sqrt{\frac{L}{C}}$$

$$R > \frac{1}{2}\sqrt{\frac{9}{1}}$$

$$R > 1.5\,\Omega$$