EE 302 Tutorial Sheet 3, 13^{th} Feb 2025 Control Systems Cours

Q-1: (a) For $G(s) = \frac{4}{s+8}$, standard negative unity feedback configuration, & step response, plot $e(\infty)$ versus k ($k \equiv$ ~~feedforw~~ forward path gain) (be sure to check FVT validity).

(b) For $G(s) = \frac{4}{s^2+3s+2}$, (c) $\frac{s-3}{s+4}$, (d) $\frac{1}{(s+1)(s+2)(s+3)}$ (poles $-1, -2, -3$)

Q-2: Suppose n & d are monic polynomials with ~~po~~ roots
- non-repeated (individually) and
- disjoint (i.e. n & d are coprime).
- in open LHP, on $-$ve real axis,
- interlaced. (i.e. roots of n & roots of d are interlaced.)

Show: (a) difference in degrees can be atmost 1.

(b) For any +ve & real k, roots of $d(s) + k\,n(s)$ are real & in ~~open~~ $-$ve.

Q-3: (a) Shift the poles/zeros to get symmetry about $j\mathbb{R}$ and then get breakaway / break-in points: for $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$.

(b) Same for $\frac{(s-1)(s+5)}{s(s+1)(s+3)(s+4)}$

Q-4 For what value of $z \in \mathbb{R}$ does the root locus pass through point $-3+2j$? (for $G(s) = \frac{s-z}{s(s+2)}$).

Q-5: Verify (assuming F.V.T. holds).

$e(\infty)$	type 0	type 1	type 2
step input	$\frac{1}{1+K_p}$	0	0
ramp input	∞	$1/K_v$	0
parabolic input	∞	∞	$1/K_a$

Find steady state errors for step, ramp, parabolic inputs for $G(s) = \frac{k(s+0.01)}{s(s+2)(s+3)}$ (as k varies from 0 to $+\infty$, use root locus to check FVT validity)

(a) $w(s) = \frac{4}{s+8}$

$\Rightarrow \lim_{s \to 0} \frac{1}{1+k\left(\frac{4}{s+8}\right)}$

$= \frac{1}{1+k\frac{(4)}{8}}$

$e(\infty) = \frac{1}{1+\frac{k}{2}}$

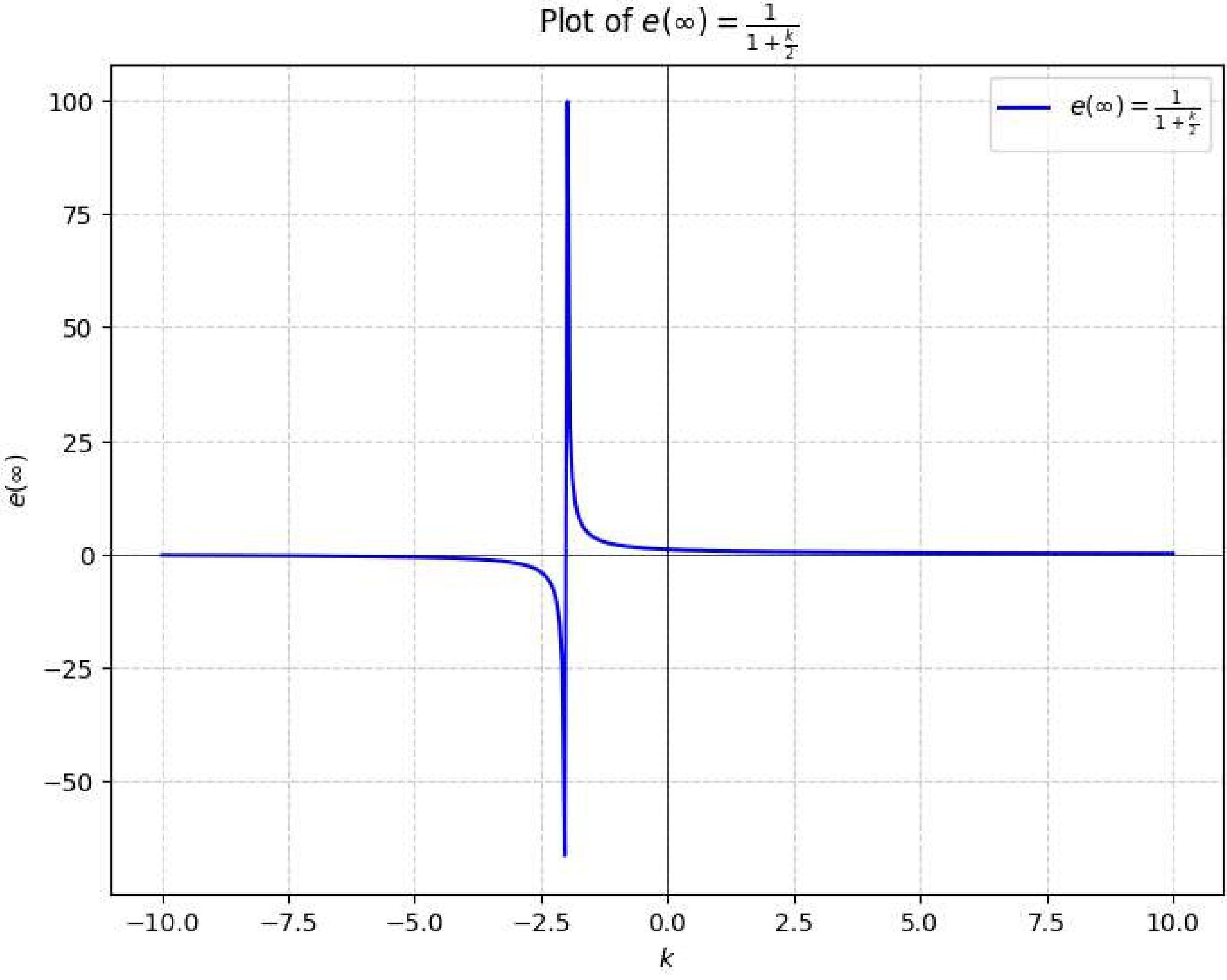
$\therefore$ we are getting a finite value of $e(\infty)$ hence FVT is valid

$\Rightarrow y = \frac{1}{1+\frac{x}{2}}$

$\Rightarrow y(x') = 1$

$\hookrightarrow x' = 1 + \frac{x}{2}$

$\therefore$ It is a rectangular hyperbola



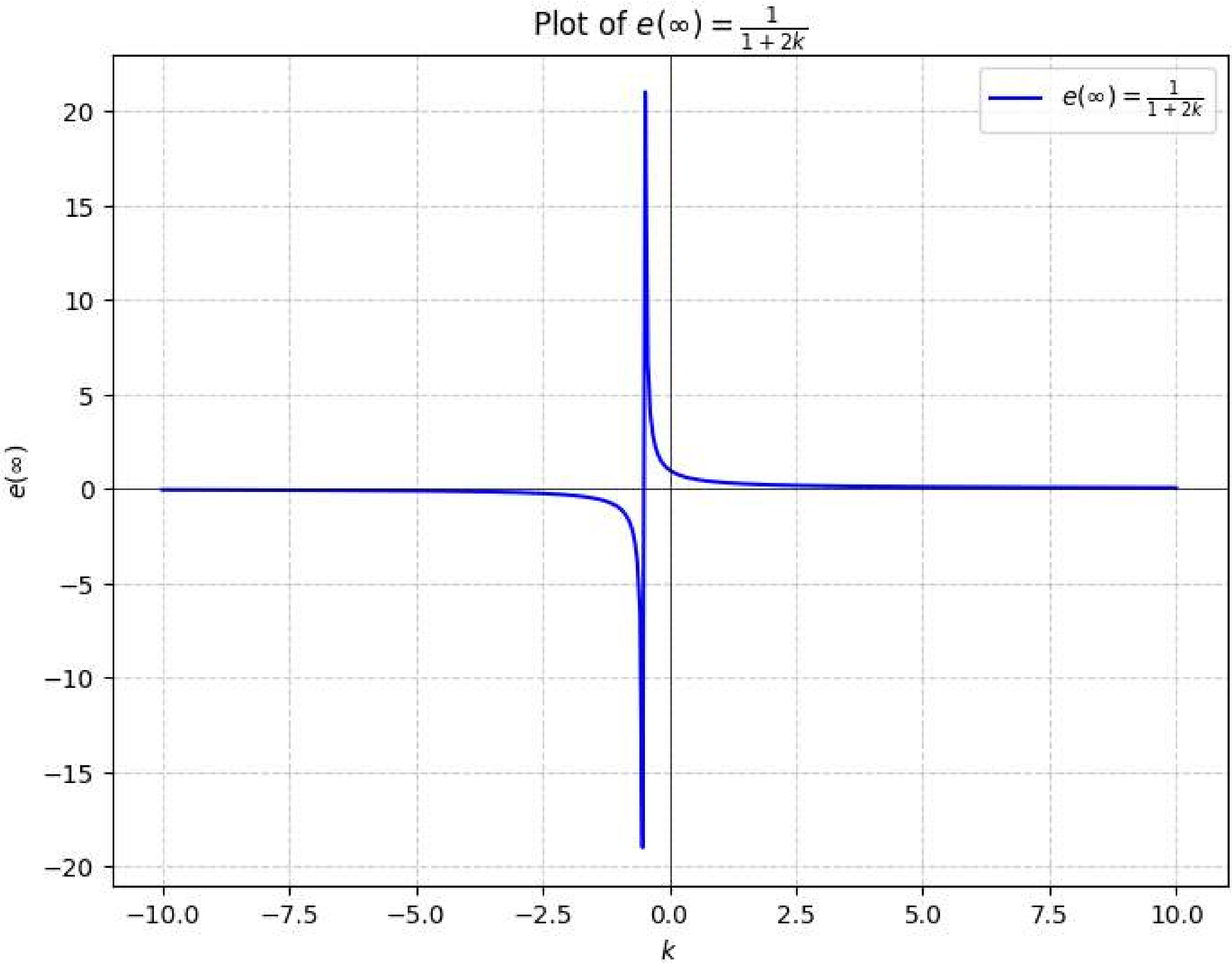
Plot of $e(\infty) = \frac{1}{1+\frac{k}{2}}$
$e(\infty) = \frac{1}{1+\frac{k}{2}}$
100
75
50
25
0
−25
−50
$e(\infty)$
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
k

(b) $G(s) = \dfrac{4}{s^2+3s+2}$

$$\lim_{s\to 0} \frac{1}{1+K\left(\frac{4}{s^2+3s+2}\right)}$$

$$e(\infty) = \frac{1}{1+K\left(\frac{4}{0+0+2}\right)} = \frac{1}{1+2K}$$

∴ we are getting a finite value here too hence FVT is valid



Plot of $e(\infty) = \frac{1}{1+2k}$
$e(\infty) = \frac{1}{1+2k}$
$e(\infty)$
k
20
15
10
5
0
−5
−10
−15
−20
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

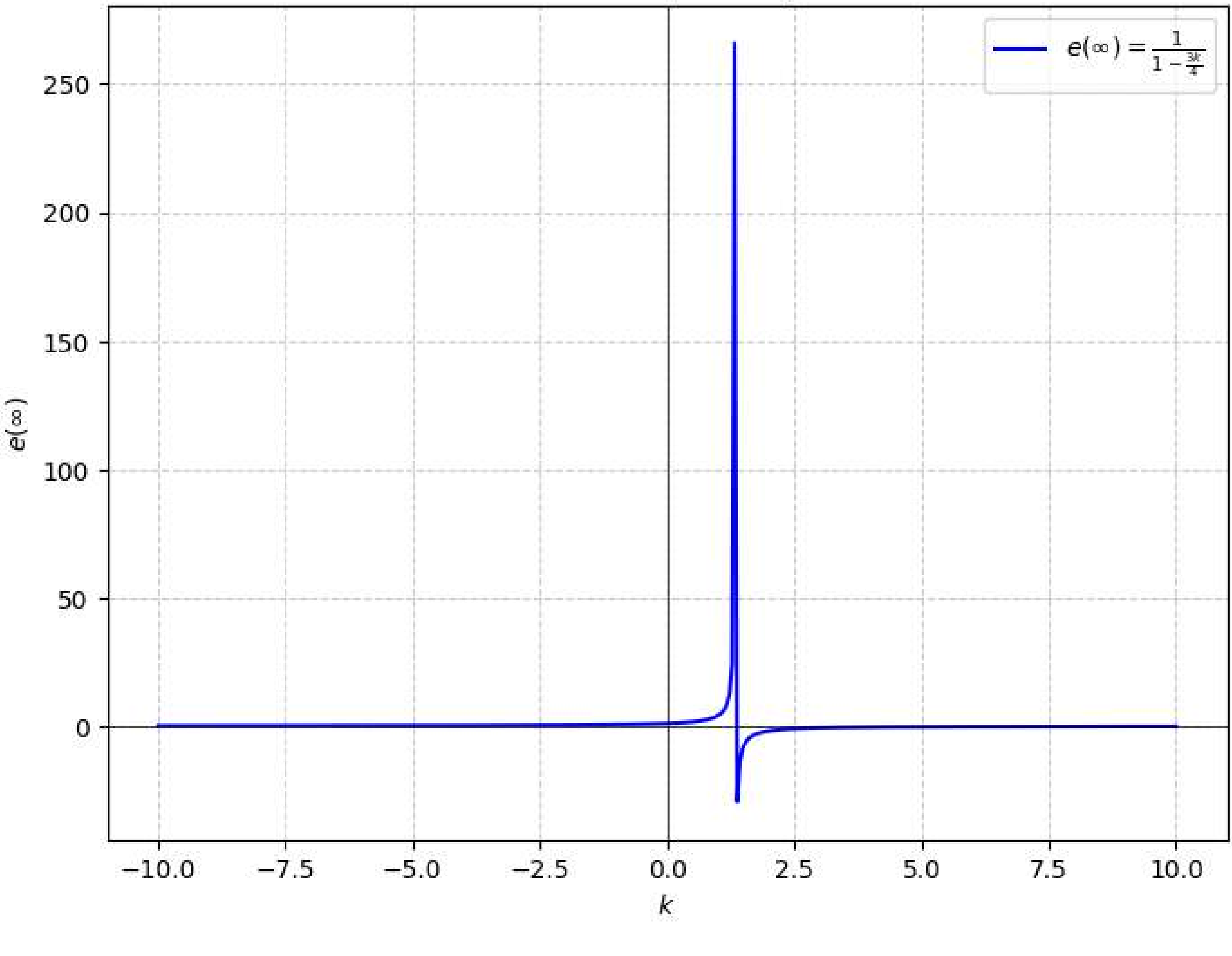
(c) $G(s) = \dfrac{s-3}{s-4}$

$$e(ss) = \lim_{s \to 0} \frac{1}{1+KG(s)}$$

$$= \frac{1}{1+K\left(\frac{0-3}{0+4}\right)}$$

$$e(\infty) = \frac{1}{1-\frac{3K}{4}}$$

Plot of $e(\infty) = \frac{1}{1 - \frac{3k}{4}}$



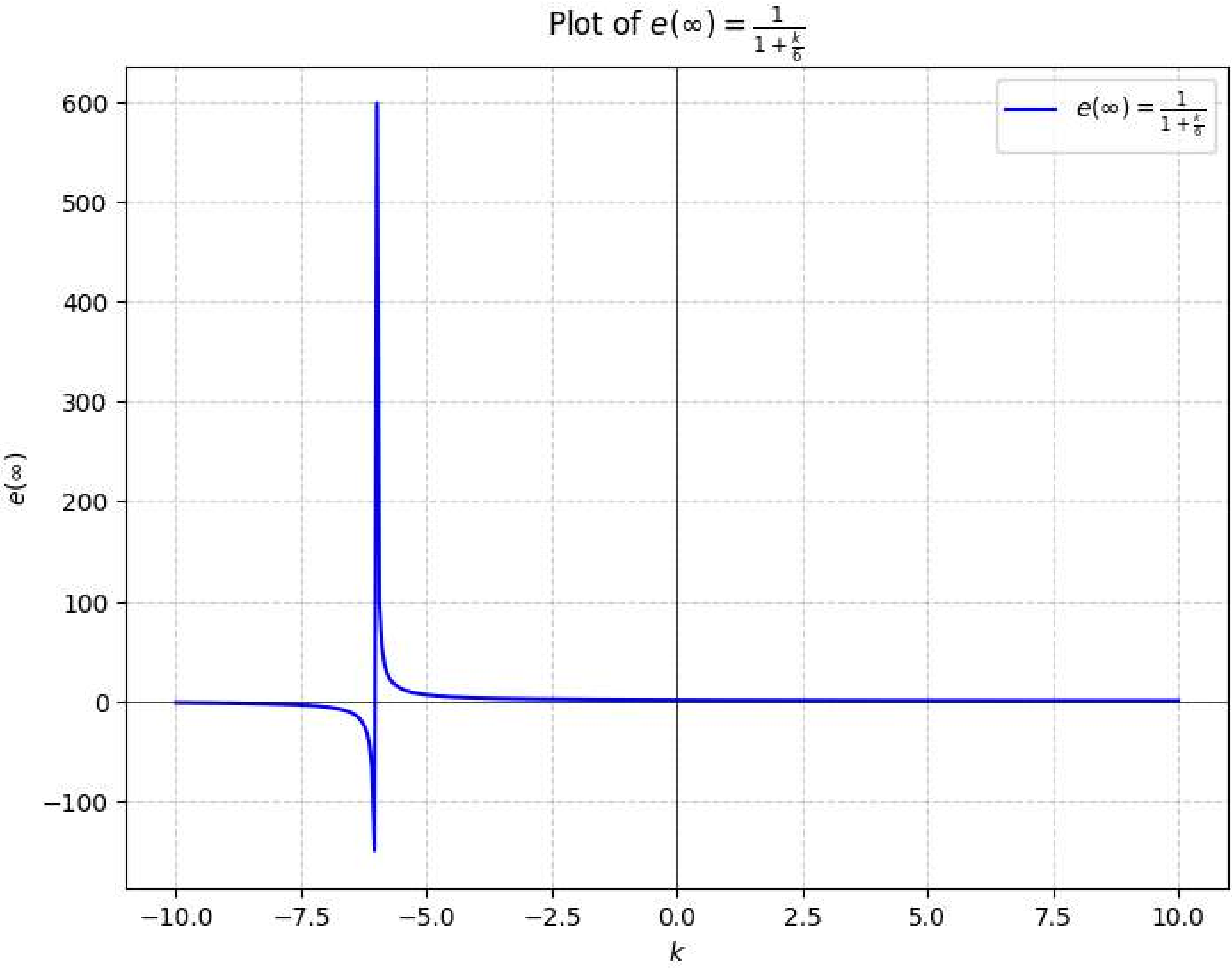
(d) $$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

$$e(\infty) = \underset{s \to 0}{\mathrm{lt}} \frac{1}{1 + K\,G(s)}$$

$$= \underset{s \to 0}{\mathrm{lt}} \frac{1}{1 + K\left(\frac{1}{(s+1)(s+2)(s+3)}\right)}$$

$$= \underset{s \to 0}{\mathrm{lt}} \frac{1}{1 + \frac{K}{6}}$$

$$e(\infty) = \frac{1}{1 + \frac{K}{6}}$$

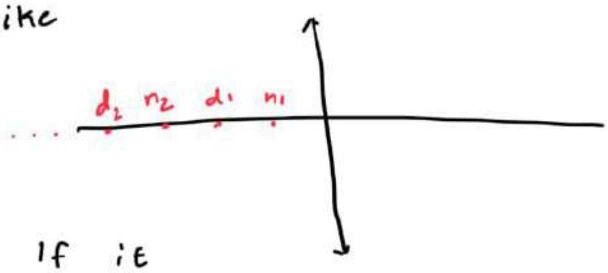


Plot of $e(\infty) = \frac{1}{1+\frac{k}{6}}$
$e(\infty) = \frac{1}{1+\frac{k}{6}}$
600
500
400
300
200
100
0
−100
$e(\infty)$
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
k

Tutorial -3 : Q-02:

→ (a) roots of n and d would look like

- the 'interlaced' property ensures that difference in degrees is atmost 1. If it exceeds one, we will have either n or d having adjacent roots, violating the interlaced property.



(b) $d(s) + Kn(s)$ for $K > 0$

both $d(s)$ and $n(s)$ have roots in OLHP.

K is just scaling the $n(s)$.

so roots of $d(s) + Kn(s)$ should remain in OHLP

$\forall\ K > 0$.

TUT3 Q3

(a) $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$

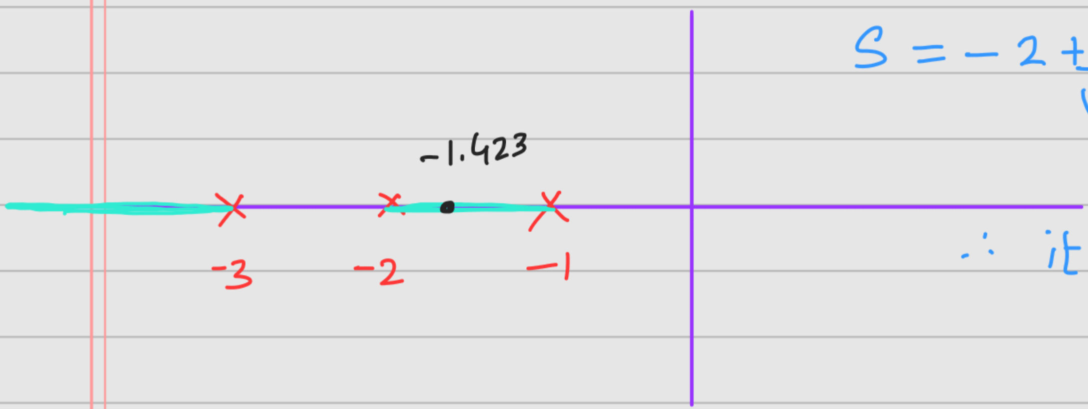
$s = s' - 2 \Rightarrow G(s') = \dfrac{1}{(s'-1)\, s\, (s'+1)}$

$1 + K\,G(s)H(s) = 0 \Rightarrow K = -\dfrac{1}{G(s)}$

$\therefore K = -(s'-1)\, s'(s'+1) = -(s'^3 - s')$

$\dfrac{dK}{ds'} = -3s'^2 + 1 \qquad \dfrac{dK}{ds'} = 0 \Rightarrow s' = \pm\dfrac{1}{\sqrt{3}}$

$S = s' - 2 \Rightarrow S = -2 + \dfrac{1}{\sqrt{3}}\ ;\ -2 - \dfrac{1}{\sqrt{3}}$



$S = -2 + \dfrac{1}{\sqrt{3}}$ lies on root locus

$\therefore$ it is valid

(b) $G(s) = \dfrac{(s-1)(s+5)}{s(s+1)(s+3)(s+4)}$

$s = s' - 2 \Rightarrow G(s') = \dfrac{(s'-3)(s'+3)}{(s'-2)(s'-1)(s'+1)(s'+2)}$

$K = -\dfrac{1}{G(s)} = -\dfrac{(s'-2)(s'-1)(s'+1)(s'+2)}{(s'-3)(s'+3)}$

$$K = -\frac{(s'^2-1)(s'^2-4)}{(s'^2-9)} = -\frac{(s'^4-5s'^2+4)}{(s'^2-9)}$$

$$\frac{dK}{ds} = -\left[\frac{(s'^2-9)(4s'^3-10s') - 2s'(s'^4-5s'^2+4)}{(s'^2-9)^2}\right]$$

$$\frac{dK}{ds} = 0 \Rightarrow 4s'^5 - \underline{36s'^3} - \cancel{10s'^3} + 90s' - 2s'^5 + \cancel{10s'^3} - 8s' = 0$$

$$\Rightarrow 2s'^5 - 36s'^3 + 82s' = 0$$

$$\Rightarrow 2s'(s'^4 - 18s'^2 + 41) = 0$$

On solving we get,

$s' = 0$ and $s' = \pm\sqrt{9+2\sqrt{10}}$ and $s' = \pm\sqrt{9-2\sqrt{10}}$

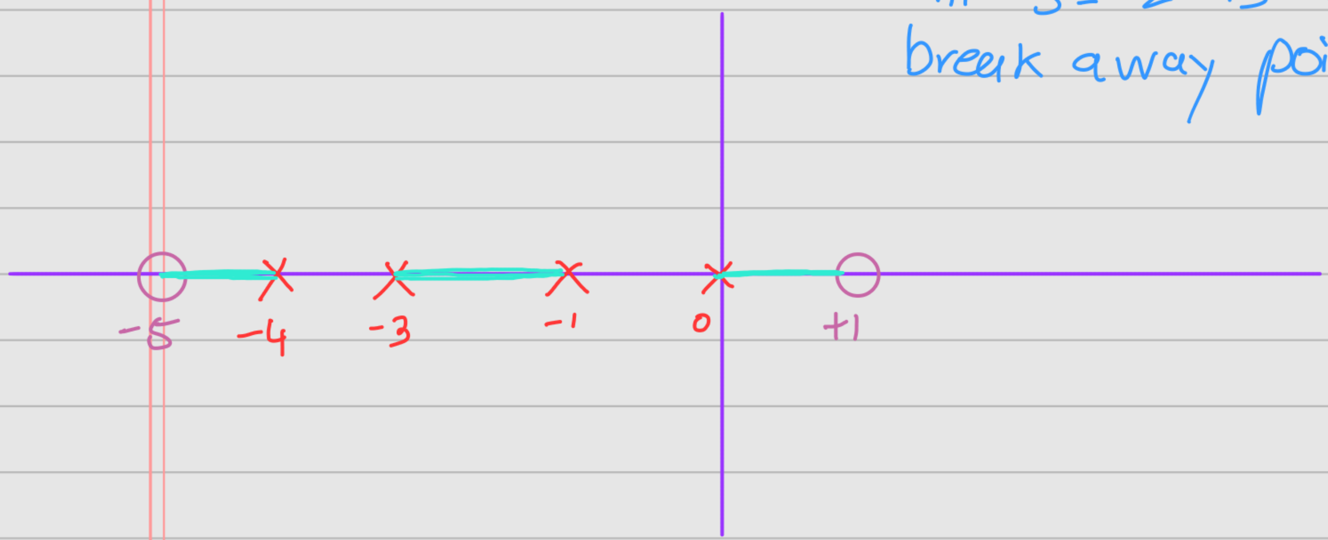
$\therefore s = s' - 2$

$\therefore s = -2$

$s = -2 \pm \sqrt{9+2\sqrt{10}} = 1.914, -5.914$

$s = -2 \pm \sqrt{9-2\sqrt{10}} = -0.364, -3.635$

Only $s = -2$ is valid break away point.



Q4: For what values of $z \in \mathbb{R}$ does the root locus pass through the point $-3+2j$ for

$$G(s) = \frac{s-z}{s(s+2)}$$

1) for $-3+2j$ to be a closed loop pole we have

$$\angle K G(s) H(s) \Big|_{s=-3+2j} = 180^\circ$$

$$\Rightarrow \angle \frac{K(-3-z+2j)}{(-3+2j)(-1+2j)} = 180^\circ$$

$$\Rightarrow -\tan^{-1}\left(\frac{2}{3+z}\right) - \tan^{-1}\left(\frac{8}{1}\right) = 180^\circ$$ simplifying we have

$$\Rightarrow \left(\frac{2}{3+z} + 8\right) \Big/ \left(1 - \frac{16}{3+z}\right) = 0$$

$$\Rightarrow z = -3 - \frac{1}{4} = -13/4$$

2) use magnitude condition to find K at $z = \frac{-13}{4}$, $s = -3+2j$:

$$|K G(s) H(s)|_{s=-3+2j} = 1 \Rightarrow \left| \frac{(-3+2j-z)K}{-1-8j} \right| = 1$$

$$\Rightarrow K = \frac{\sqrt{65}}{\sqrt{\left(-3+\frac{13}{4}\right)^2 + 4}} = \sqrt{\frac{65}{65/16}} = 4$$

Hence at $Z = \frac{-13}{4}$ and $K = 4$, $s = -3+2j$ lies on the root locus.

Q5) $e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$ $R(s) \to$ input.

a) Step input

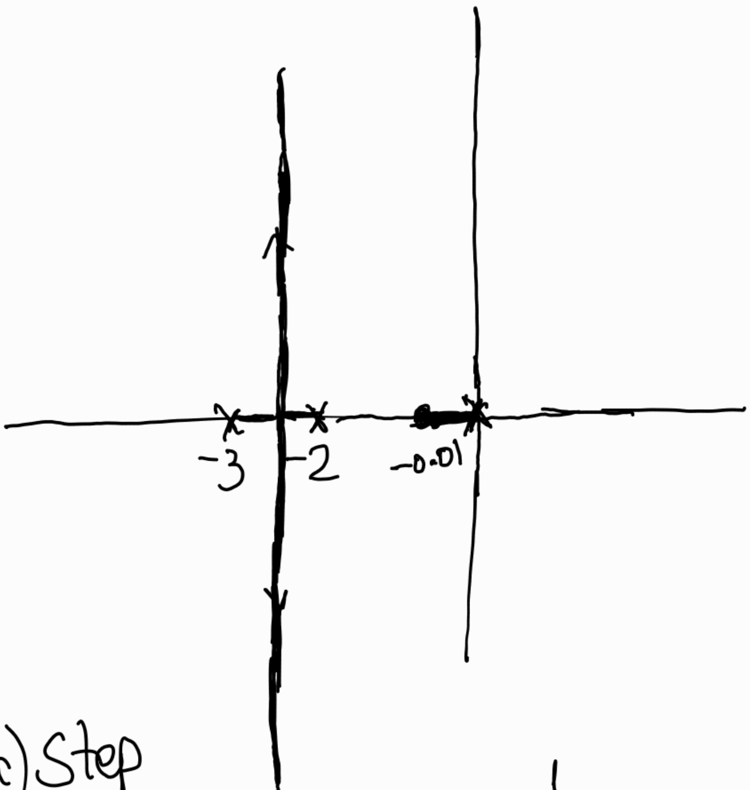
$$e(\infty) = \lim_{s\to 0} \frac{s\frac{1}{s}}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)} \sim \frac{1}{1+K_p}$$

b) Ramp input

$$e(\infty) = \lim_{s\to 0} \frac{s\frac{1}{s^2}}{1+G(s)} = \lim_{s\to 0} \frac{1}{s(1+G(s))} = \frac{1}{\lim_{s\to 0} sG(s)} = \frac{1}{K_v}$$

c) Parabolic input

$$e(\infty) = \lim_{s\to 0} \frac{s\frac{2}{s^3}}{1+G(s)} = \lim_{s\to 0} \frac{2}{s^2(1+G(s))} = \frac{1}{K_a}$$



Locus doesn't cross through imaginary axis, so system is stable. Steady state error exist $\forall K > 0$.

a) Step

$$e(\infty) = \lim_{s\to 0} \frac{s\frac{1}{s}}{1+\frac{s+0.01}{s(s+2)(s+3)}} = \lim_{s\to 0} \frac{s}{s+\frac{s+0.01}{(s+2)(s+3)}} = 0.$$

b) Ramp

$$e(\infty) = \lim_{s\to 0} \frac{s\frac{1}{s^2}}{1+\frac{s+0.01}{s(s+2)(s+3)}} = \lim_{s\to 0} \frac{1}{s+\frac{s+0.01}{(s+2)(s+3)}} = \frac{6}{0.01}$$

$$= 600$$

c) Parabolic

$$e(\infty) = \lim_{s \to 0} \frac{s \frac{2}{s^3}}{1 + \frac{s+0.01}{s(s+2)(s+3)}} = \lim_{s \to 0} \frac{2}{s^2 + \frac{s(s+0.01)}{(s+2)(s+3)}} \rightarrow \infty$$