Tutorial Sheet 5, EE 302-S1, 11th March 2025.

Q-1: Consider $G(s) = \frac{ds+b}{s+a}$, $d \neq 0$. $\rightarrow \frac{ds+b}{s+a}$.

$u(t) \rightarrow \boxed{} \rightarrow y(t)$ input.

(a) Find the underlying differential equation between u & y.

(b) Rewrite $y(t) = y_1(t) + y_2(t)$ such that the input $u(t)$ would not need to be differentiated. (Hint. Get $G(s) = d + G_1(s)$ — d: constant, $G_1(s)$: strictly proper.)

(c) Conclude that (more generally) one **never** needs to differentiate input, for proper, transfer function, real distinct roots of $d(s)$ with $G(s) = \frac{n(s)}{d(s)}$. (Solve for).

Q-2: Consider $G(s) = \frac{1}{s+3}$ with standard negative unity feedback configuration but with k increasing from $-\infty$ to 0. Draw root locus.

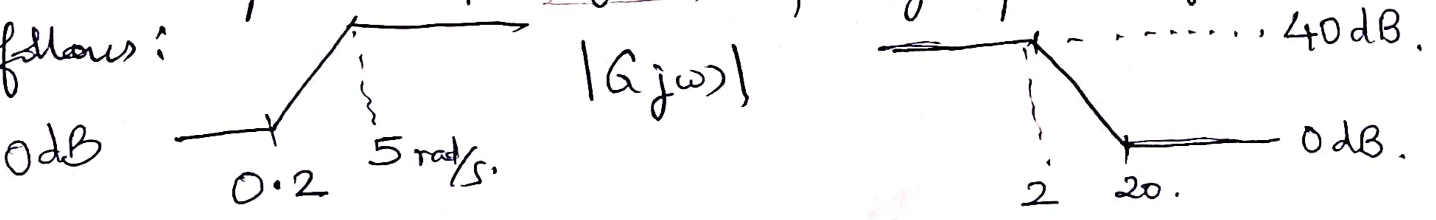
(b) Draw: complete root locus $-\infty \rightarrow 0 \rightarrow +\infty$.

(c) Guess/get rules for $k<0$ (for strictly proper G & biproper G.).

(d) Consider cases: $k<0$, $k>0$, and $-$ve feedback & $+$ve feedback, and $G(s) = \frac{n(s)}{d(s)}$: leading coefficients of n, d: same sign / opposite sign. Explain why we have **not** 2^3 but just 2 cases to study for closed loop pole locus.

Q-3: Draw asymptotic Bode plot for $\frac{1}{s+1}$, $\frac{1}{(s+1)(s+2)}$: Both phase & magnitude.

Q-4: Design low pass filter & high pass filter as follows:



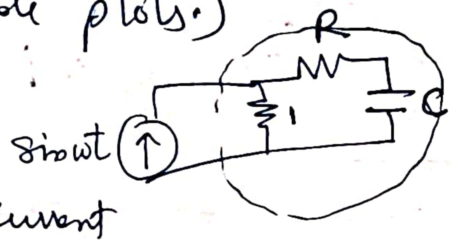
Q-5: (a) Verify that $\sin \omega t$ input gives output $|G(j\omega)| \sin(\omega t + ?)$
(verify using linearity & consider complex valued signals $e^{j\omega t}$.
(b) What about $\cos \omega t$ input?

Q-6: If relative degree of a $G(s) = 0$, can G be low-pass? high pass?

(Q-7) If $G(s)$ is high pass, can $G(s)$ be relative degree $= 1$ or $2, \ldots$? What should relative degree be?

Q-8: If magnitude plot is given, (like in Q-4), how much non-uniqueness is possible in $G(s)$? (Answer for both magnitude plots.)

Q-9: Consider



Current source.

(a) Find impedance $Z(s)$ of this circuit.

(b) For one period, find active power absorbed by the circuit. (get $\int_0^T v(\tau)\, i(\tau)\, d\tau$ for $T = \frac{2\pi}{\omega}$.)

(c) Relate active power w.r.t. $\mathrm{Re}(Z(j\omega))$

Q-10: Plot complete root locus ($k: -\infty \to +\infty$)
for $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$, $\dfrac{1}{(s+1)(s+2)}$, $\dfrac{s}{s^2+\omega^2}$

Tutorial - 4

Q1: (a) $G(s) = \dfrac{ds+b}{s+a}$, $d \neq 0$

$u(t) \longrightarrow \boxed{} \longrightarrow y(t)$

$$\frac{y(s)}{u(s)} = \frac{ds+b}{s+a}$$

$\Rightarrow sy(s) + ay(s) = dsu(s) + bu(s)$

$\Rightarrow$ Taking inverse laplace:

$$\frac{dy}{dt} + ay = d\frac{du}{dt} + bu$$

(b) $G(s) = \dfrac{ds+b}{s+a} = \dfrac{ds+da-da+b}{s+a}$

$$= \frac{d(s+a)}{(s+a)} + \frac{b-da}{s+a}$$

$$= d + \frac{b-da}{s+a}$$

$$\Rightarrow \frac{y(s)}{u(s)} = d + \underbrace{\frac{b-da}{s+a}}_{G_1(s)}$$

Taking inverse laplace after rearranging

$$y(s) = \underbrace{du(s)}_{y_1(s)} + \underbrace{\left(\frac{b-da}{s+a}\right)u(s)}_{y_2(s)}$$

$y_1(s) = du(s) \xrightarrow{\text{I.L.T.}} y_1(t) = du(t)$

$$y_2(s) = \left(\frac{b-da}{s+a}\right) u(s)$$

$$\Rightarrow y_2(s)(s+a) = (b-da)u(s)$$

$$\text{I.L.T.} \Rightarrow \frac{dy_2(t)}{dt} + ay(t) = (b-da)u(t)$$

(c) Suppose we have $G(s) = \frac{N(s)}{D(s)}$, where degree $(N(s)) \leq$ degree $(D(s))$ then we can perform partial fraction expansion (decomposition) as:

$$G(s) = \frac{N(s)}{D(s)} = k + \frac{c_1}{s+p_1} + \cdots \frac{c_n}{s+p_n}$$

$$\Rightarrow \frac{y(s)}{u(s)} = k + \frac{c_1}{s+p_1} + \cdots + \frac{c_n}{s+p_n}$$

and each term will correspond to its respectively.

$$y_i(s)(s+p_i) = c_i u(s)$$

$$\text{I.L.T.} \Rightarrow \frac{dy_i(t)}{dt} + p_i y_i(t) = c_i u(t)$$

and $$y(t) = k u(t) + \sum_{i=1}^{n} y_i$$

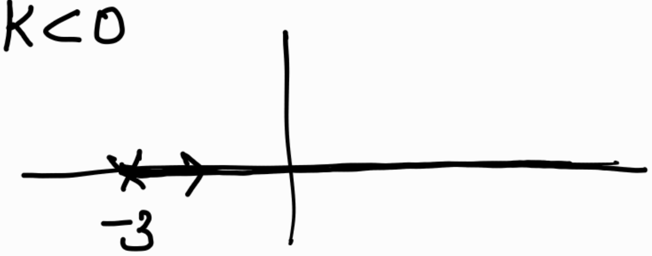
None of this analysis involved derivatives of $u(t)$

Q2) $G(s) = \frac{1}{s+3}$

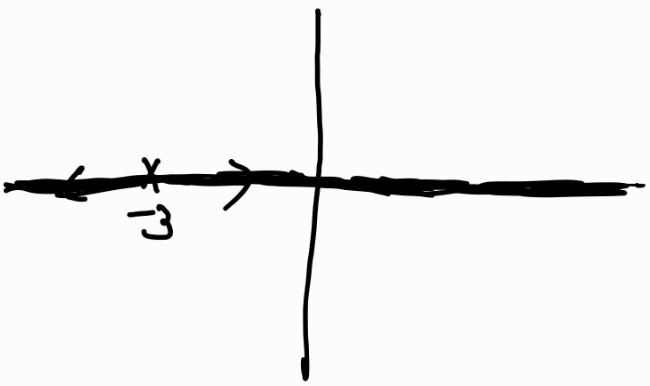
$1 + KGH$

$1 + \frac{K}{s+3} = 0 \qquad s+3+k=0 \Rightarrow s = -(3+k)$

(a) $K<0$



b) $K \in R$



c) $K<0$

i) A point on the real axis lies on the root locus if the total number of poles & zeroes to the right of it is even.

ii) No. of branches of the RL equals no. of open poles.

iii) RL starts at $K=0$ & ends at open-loop zeroes $(K=-\infty)$

iv) Angle of asymptotes $= \frac{2n\pi}{\text{Poles}-\text{zeroes}}$

d) $1+GH=0$ (-ve unity feedback)

$k>0 \rightarrow$ Direct Root Locus

$k<0 \rightarrow$ Inverse RL

+ve unity feedback — $k>0$ IRL

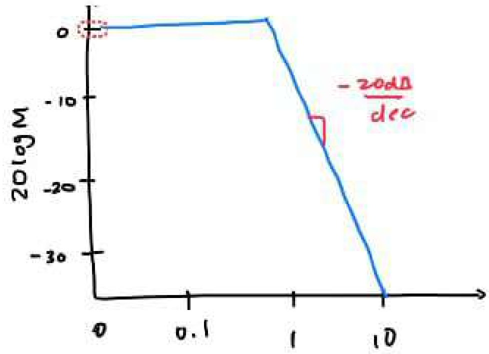
+ve unity feedback — $k<0$ DRL

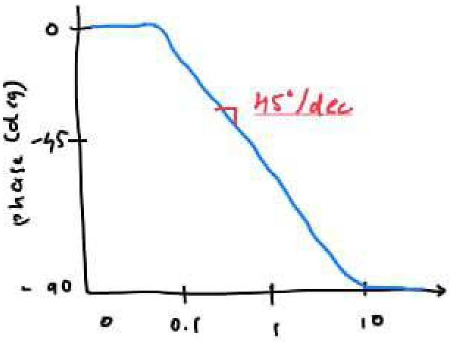
So, to study closed loop pole locus, Only 2 cases are enough.

$\left.\begin{array}{l}\text{-ve unity, } k>0 \\ \text{+ve unity, } k<0\end{array}\right\}$ DRL

$\left.\begin{array}{l}\text{+ve unity, } k>0 \\ \text{-ve unity, } k<0\end{array}\right\}$ IRL

$\rightarrow \quad \dfrac{1}{s+1}$

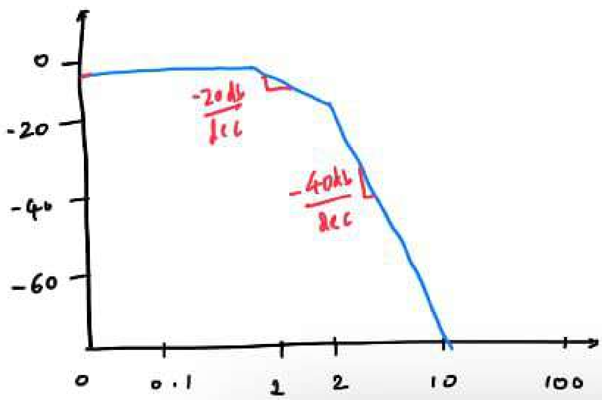


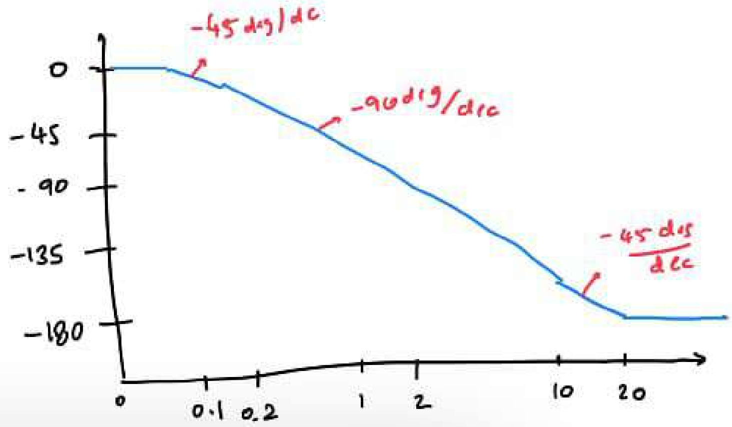


$\rightarrow \quad \dfrac{1}{(s+1)(s+2)}$

	Freq.	
	1	2
Pole at -1	-20	-20
pole at -2	0	-20
	-20	-40

	Freq			
	0.1	0.2	10	20
Pole at -1	-45	-45	0	0
pole at -2	0	-45	-45	0
	-45	-90	-45	0





Tut 4 Q5

a) $\sin \omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j}$

Let $u(t) = \sin \omega t$ and $g(t)$ be transfer function

$\therefore y(t) = u(t) * g(t)$

$$u(t) \longrightarrow \boxed{g(t)} \longrightarrow y(t)$$

$= \sin \omega t * g(t)$

$= \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j} * g(t)$

By using linearity property of convolution.

$$y(t) = \frac{1}{2j}\left(e^{j\omega t} * g(t) - e^{-j\omega t} * g(t)\right)$$

But we know that when input is exponential signal, the o/p follows

$y(t) = e^{j\omega t} * g(t)$; $y(t) = |G(j\omega)|\, e^{j(\omega t + \angle\phi)}$ where $\phi = \angle G(j\omega)\big|_{\omega}$

$\therefore$ Using this property, we can write.

$$y(t) = \frac{1}{2j}\left(|G(j\omega)|\, e^{j(\omega t + \angle\phi)} - |G(j\omega)|\, e^{-j(\omega t + \angle\phi)}\right)$$

$$\therefore y(t) = |G(j\omega)| \left(\frac{e^{j(\omega t + \angle\phi)} - e^{-j(\omega t + \angle\phi)}}{2j}\right)$$

$y(t) = |G(j\omega)| \sin(\omega t + \angle\phi)$ where $\phi = \angle G(j\omega)$

Consider $\cos \omega t = \dfrac{e^{j\omega t} + e^{-j\omega t}}{2}$ and do same analysis

Q6) If relative degree is 0, by Bode magnitude plots, we can say that the magnitude at higher frequencies doesn't decay. So, Transfer function cannot certainly be a LPF.

$$G(s) = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}$$

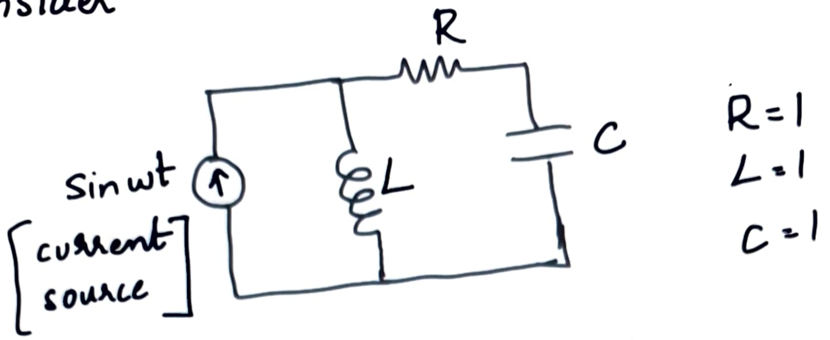
As $\omega \to 0$, magnitude of $G(s) \to a_n/b_n$ & as $\omega \to \infty$, mag $\to \frac{a_0}{b_0}$. It can be a Band pass / stop filter.

$\Rightarrow$ Zeroes at $s=0$ force the gain to rise at low frequencies, creating HPF characteristics.

Eg. $\frac{s}{s+a}$, $\frac{s^2}{s^2+\ldots}$

Q7) If $G(s)$ is a HPF, it should attenuate the low frequencies and pass the high f. If the relative degree is 1 or 2, then by Bode Magnitude plots at high f there is a decay in magnitude response. So, if $G(s)$ is HPF, its relative degree cannot be 1 or 2, it should be equal to zero.

Q9 : Consider



a) Find impedance $Z(s)$ of the circuit.

$$Z(s) = \frac{Ls\left[R + \frac{1}{Cs}\right]}{Ls + \left(R + \frac{1}{Cs}\right)} = \frac{s(s+1)}{s^2 + s + 1}$$

b) For one period find active power absorbed by circuit.

$$\int_0^T v(\tau)\, i(\tau)\, d\tau =$$

$$Z(j\omega) = \frac{j\omega\,[j\omega + 1]}{(j\omega)^2 + j\omega + 1} = \frac{-\omega^2 + j\omega}{(1-\omega^2) + j\omega} = \frac{\omega^4 + \omega j}{1 - \omega^2 + \omega^4}$$

$$Z = Z_0 \angle \phi$$

$$V = IZ = Z_0 \sin(\omega t - \phi)$$

$$P_{act} = \int_0^T Z_0 \sin(\omega t - \phi) \sin \omega t \, dt \quad , \quad T = \frac{2\pi}{\omega}$$

expanding $\sin(\omega t - \phi) = \sin \omega t \cos\phi - \cos \omega t \sin\phi$

$$P_{act} = \int_0^T Z_0 \sin^2\omega t \cos\phi \, dt - \int_0^T Z_0 \sin\omega t \cos\omega t \sin\phi \, dt$$

$$\Rightarrow \frac{Z_0 \cos\phi}{2} \int_0^T (1 - \cos 2\omega t) dt - \frac{Z_0 \sin\phi}{2} \int_0^T \sin 2\omega t \, dt$$

$$\Rightarrow \frac{Z_0 \cos\phi}{2} \left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T - \frac{Z_0 \sin\phi}{2} \left[-\frac{\cos 2\omega t}{2\omega} \right]_0^T$$

$$\Rightarrow \frac{Z_0 \cos\phi}{2} [T] - \frac{Z_0 \sin\phi}{2} \left[\frac{-1}{2\omega} (1-1) \right]$$

$$\Rightarrow \frac{Z_0 \cos\phi}{2}$$

c) Relate active power w.r.t $Re[Z(j\omega)]$

$$P_{act} = \frac{Z_0 \cos\phi}{2} = \frac{Re[Z(j\omega)]}{2}$$

Date :

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

angle of asymptote

$$= \frac{180(2q+1)}{P-Z}$$

$\left(\begin{array}{l} P = \text{no of poles} \\ Z = \text{no of zeros} \end{array} \right)$

$$= \frac{180(2q+1)}{3-0}$$

$$= 60(2q+1)$$

$, q = 0, 1, 2, \ldots |P|-|Z|-1$

$= 0, 1, 2$

$$= 60, 180, 300$$

centroid $= \dfrac{\text{sum of poles} - \text{sum of zeros}}{P-Z}$

$$= \frac{(-1-2-3-0)}{3}$$

$$= -2$$

Break-away and Break-in points:

$k + (s+1)(s+2)(s+3) = 0$

→ get the closed-loop transfer function and equate the denominator to 0.

$$\frac{dk}{ds} = 0$$

→ such s values will give you the break-away and break-in points

$\cancel{s^2 + 3s + 2}$

$$(s+1)(s+2)(s+3) + k = 0$$
$$(s^2 + 3s + 2)(s+3) + k = 0$$
$$s^3 + 6s^2 + 11s + 6 + k = 0$$

Imaginary axis crossing

R-H Table:

s^3	1	11
s^2	6	$6+k$
s^1	$\frac{60-k}{6}$	
s^0		

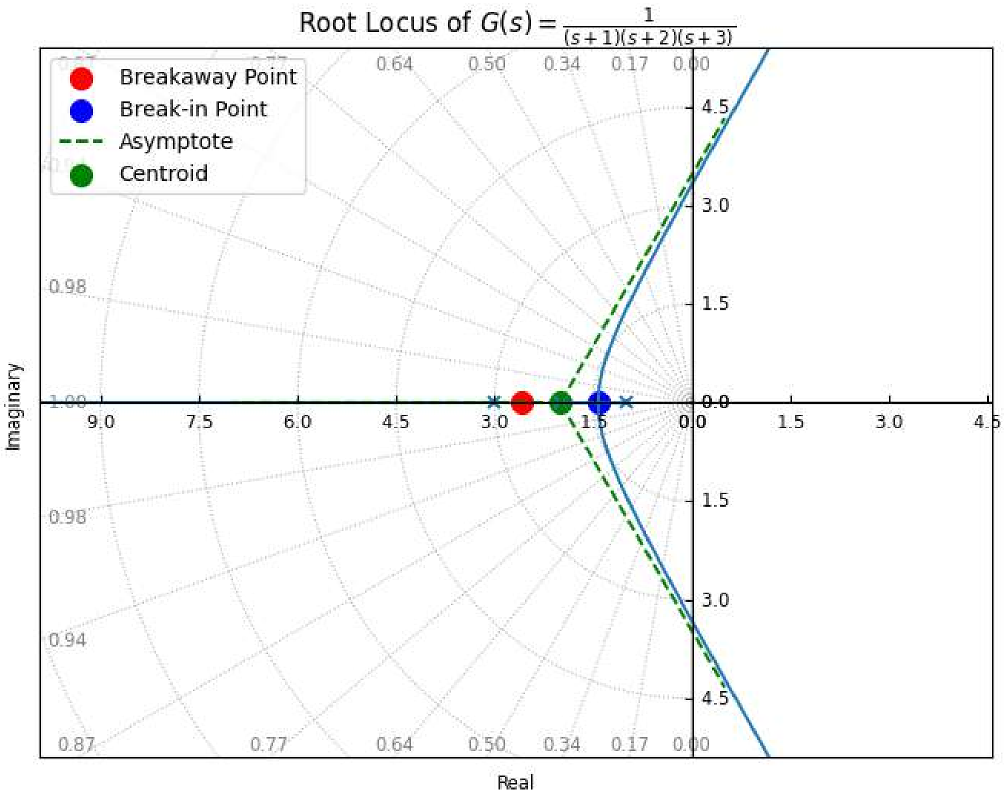
$\Rightarrow 60 - k = 0$

$k = 60$

Auxiliary eq: $6s^2 + 6 + k = 0$

$$6s^2 + 66 = 0$$
$$s^2 = -11$$
$$s = \pm j\sqrt{11}$$

Root locus plot for sys[16]



$$G(s) = \frac{1}{(s+1)(s+2)}$$

Angle of asymptote

$$= \frac{180(2q+1)}{P-Z}$$

$$= \frac{180(2q+1)}{2}$$

$$= 90(2q+1)$$

$q = 0, 1, \ldots, P-Z-1$

$P-Z-1 = 1$

$\Rightarrow q \in \{0, 1\}$

$\Rightarrow 180(1), 180(3)$

$180, 540$

$$\text{centroid} = \frac{\text{sum of poles} - \text{sum of zeros}}{P-Z}$$

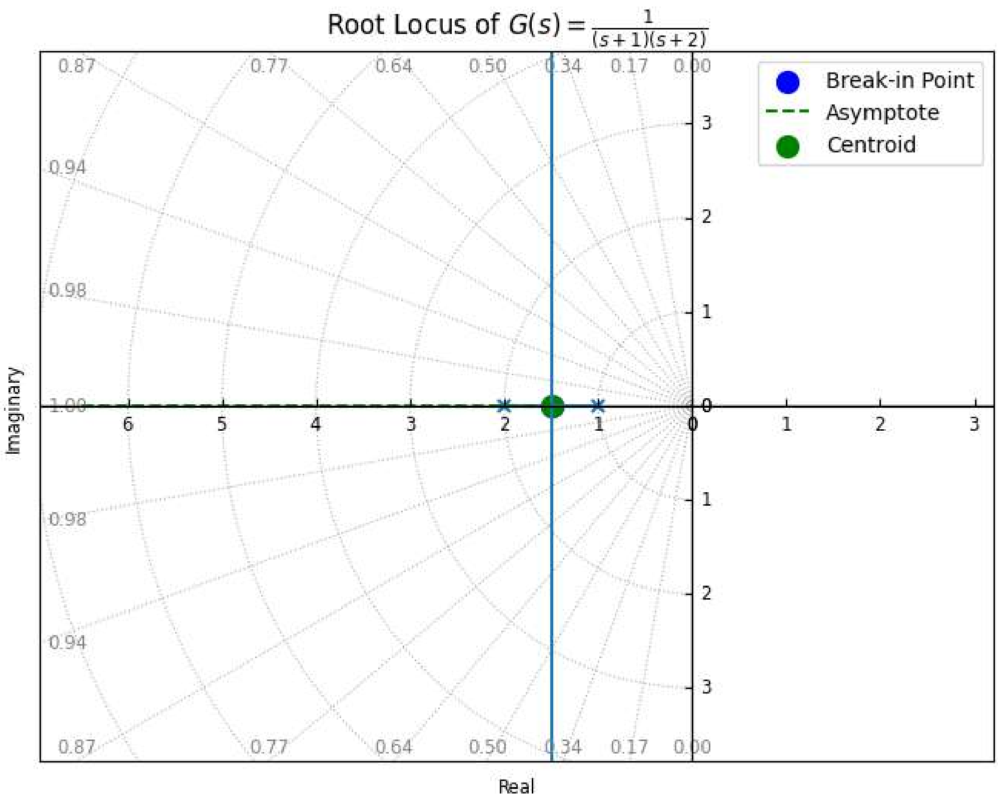
$$= \frac{-3}{2}$$

Break away / Break-in points:

$K + (s+1)(s+2) = 0$

$$\frac{dK}{ds} = 0$$

Root locus plot for sys[18]



$$G(s) = \frac{s}{s^2+\omega^2}$$

angle of asymptotes:

$$= \frac{180(2q+1)}{|P|-|Z|}$$

$$= \frac{180}{1}(2q+1) \quad q \in (0,1,\ldots |P|-|Z|-1)$$

$$\Rightarrow 180^\circ$$

centroid: $\frac{0 + j\omega - j\omega - 0}{1}$

$$= 0$$

Break-away / Break-in points:

$$\frac{KG(s)}{1+KG(s)} = 0$$

$$\frac{\frac{Ks}{s^2+\omega^2}}{1+\frac{Ks}{s^2+\omega^2}} = 0 \Rightarrow \frac{Ks}{Ks+s^2+\omega^2} = 0$$

s

~~solve~~ $Ks + s^2 + \omega^2 = 0$ → denominator of CLTF

solve for $\frac{dK}{ds} = 0$

Img. axis Crossing:

RH Table:

s^2	1	ω^2
s^1	K	0

Auxiliary eq:

$$s^2 + \omega^2 = 0$$

$$s = \pm j\omega$$

Root locus plot for sys[21]