Tutorial 25-March 2025 EE302 Control Systems S1

Q-1 (a) Get (approx) gain margin for $G(s)$ (open loop) $= \dfrac{30}{(s+200)(s+2)(s+20)}$

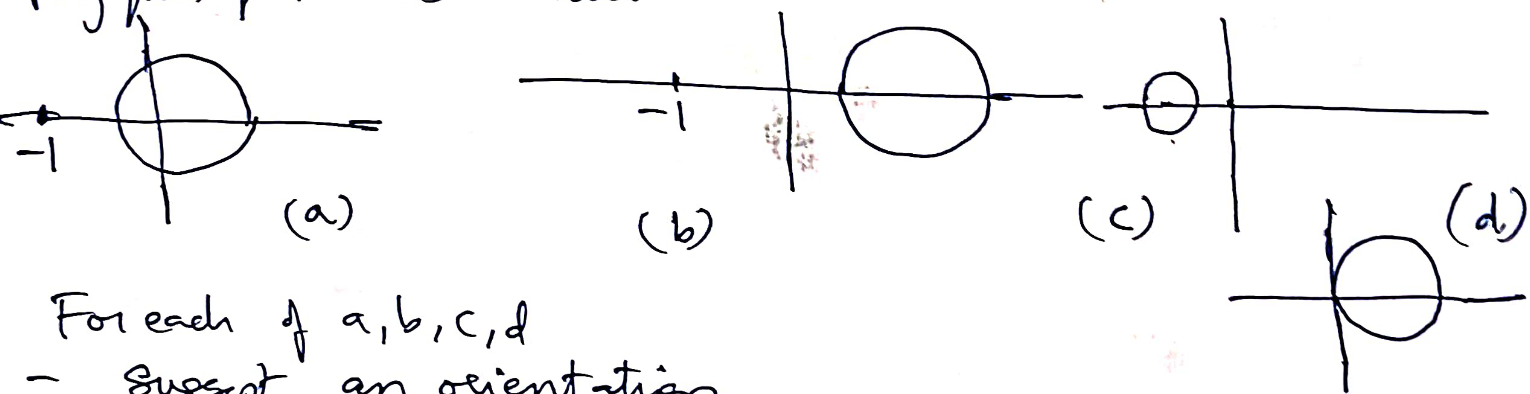
(b) Use Nyquist plot for same $G(s) = \dfrac{30}{(s+2)(s+20)(s+200)}$ exactly.

(c) Verify using Routh-Hurwitz.

Q-2: Consider Bode magnitude plot of $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, $\omega_n, \zeta > 0$ & $\zeta < 1$. (a) Find value of ζ such that a peak occurs (in exact gain plot).

(b) (When a peak occurs) find ω_p at which peak occurs. Relate ($<$, or $=$ or $>$) w.r.t. ω_d (damped frequency) & ω_n (natural frequency)

Q-3: Suppose $G(s)$ is open loop stable & has Nyquist plot as below:



For each of a, b, c, d
- suggest an orientation
- suggest a transfer function. (Pay attention to zeros in LHP/RHP & whether closed loop is stable or unstable.)

(Use Nyquist criteria).

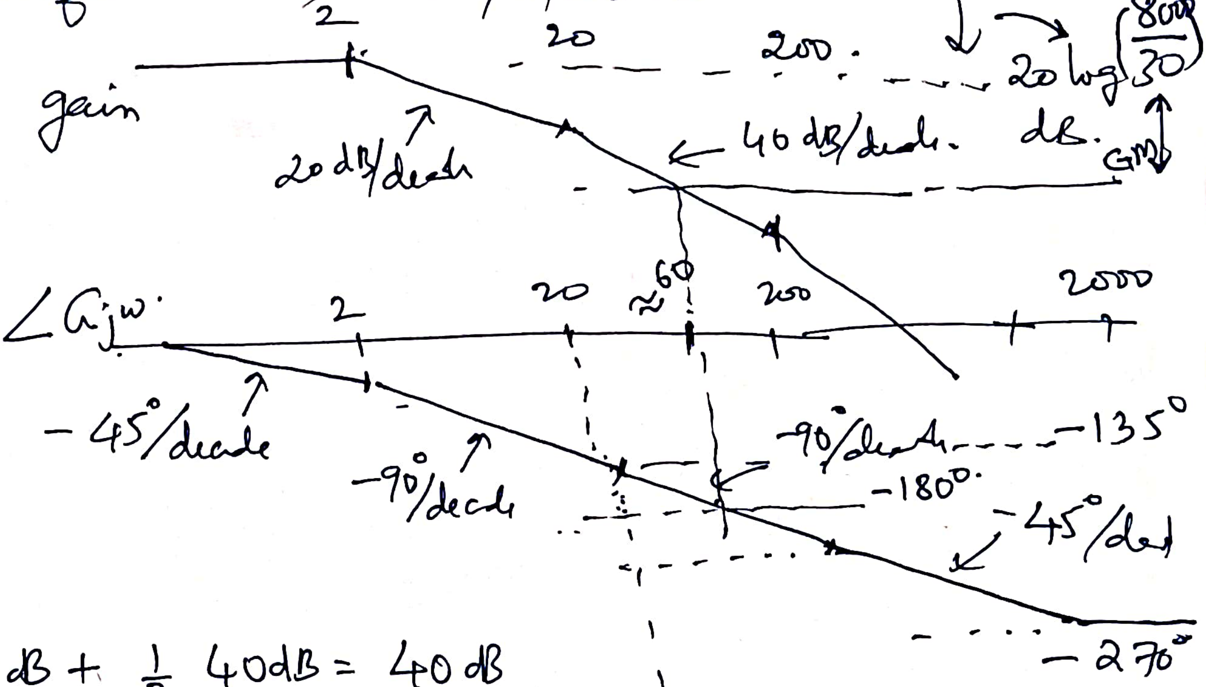
Q-4: Draw Bode & Nyquist plots of

$G(s) = \dfrac{1}{(s-1)(s-2)}$, $\dfrac{(s+1)(s+3)}{(s-1)(s-4)}$.

$\dfrac{s-3}{s+3}$ $\quad \dfrac{s+5}{s-5}$ $\quad \dfrac{s+6}{s+2}$ $\quad \dfrac{-s-6}{s+2}$

Tut 5 (Brief solutions) 25/3/2025

Q-1a Gain



$GM \approx 20dB + \frac{1}{2} 40dB = 40dB$

≈ 100.

(Below the $-20 \log \frac{8000}{30}$ line).

So total $GM \approx 100 \times \frac{8000}{30} \approx \frac{800{,}000}{30} \approx \frac{80000}{3}$

Q-1c: Routh table for

$s^3 + 222s^2 + 4440s + 8000 + 30k$.

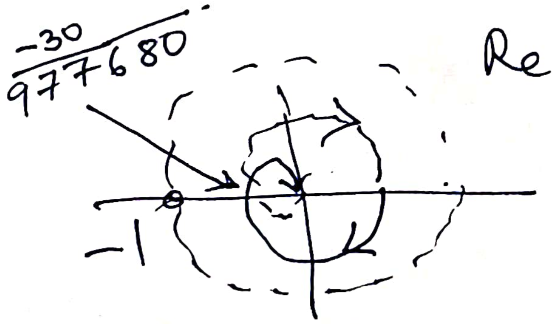
1	4440
222	8000+30k.
985680 − 8000 − 30k	*
8000 + 30k	

Stable for $k < \frac{977680}{30} \approx \frac{97768}{3}$

Q-1b: Im $(-jw^3 - 222w^2 + 4440jw + 8000) = 0$

(Ignore factor 30 now)

$w(w^2 - 4440) = 0 \Rightarrow w = 0$ or $w = \pm\sqrt{4440}$



$$\mathrm{Re}\, G(jw)\Big|_{w=\sqrt{4440}} = \frac{30}{8000 - 222 \times 4440} = \frac{-30}{977680}$$

Q-2:

We get some peak $\Leftrightarrow$ $0 < \zeta < \frac{1}{\sqrt{2}}$.

When we do have a peak (at ω_p) then

$(\omega_d = \omega_n\sqrt{1-\zeta^2})$ $\qquad \omega_p = \omega_n\sqrt{1-2\zeta^2}$

Thus $\qquad$ Recall that no peak if "too much" damping (peak $\approx$ resonance).

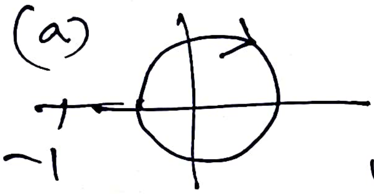
$\omega_p < \omega_d < \omega_n$.

No peak $\equiv$ $\omega_p = 0$ rad/s. $\quad (0 < \zeta < 1)$

Q-3: open loop stable $\equiv$ $P=0$ in $N = P - Z$.

So encirclement if any has to be ~~ac~~ clockwise (so that $N = -1$) (Z cannot ~~be~~ be $-$ve)

(a)



unstable for large gain: take $\frac{s-1}{s+2}$ or $\frac{-s+1}{2s+1}$

(b) orientation ~~can be anything~~ will be clockwise (~~depending on~~ can be $\frac{s+1}{s+2}$ or $\frac{s+2}{s+1}$.)

(c) —||— clockwise. (can be $-\left(\frac{s+1}{s+2}\right)$ or $\frac{-(s+2)}{s+1}$.

(d) —||— can be $\frac{1}{s+1}$ or $\frac{s}{s+1}$).

Q-4 - Pay attention to

angle of approach to origin as $\omega \to \infty$,

- orientation (since $N = P - Z$ has to hold).
- all-pass: magnitude is constant.