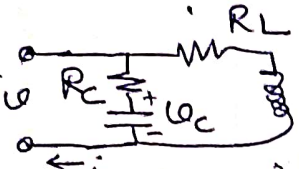
Tutorial Sheet 6, EE302 Control Systems S1

3rd April 2025

Q-1: For $G(s) = \dfrac{3s^2+4s-8}{s^2+3s+2}$, obtain state space realizations (A_1, B_1, C_1, D_1) & (A_2, B_2, C_2, D_2) in which A_1 – diagonal, A_2 – companion form

Why would $D_1 = D_2$? Verify that you get back $G(s)$.

Q-2: For circuit 

write state $x = \begin{bmatrix} v_c \\ i_L \end{bmatrix}$

- Express circuit equations as $\dot{x} = Ax + Bu$, $y = Cx + Du$ $\quad u = v$, $y = i$
- Check if $G(s) = D + C(sI-A)^{-1}B = Y(s)$ ↑ admittance
- Check condition for pole/zero cancellation.
- Check controllability rank condition for controllable (& condition on R_c, C, R_L, L). Assume some of them = 1 (normalize).

Q-3: For $x(k+1) = Ax(k) + Bu(k)$ with $x(0) = a \in \mathbb{R}^n$.

$x(N) - ?$ $a = \begin{bmatrix} ? & ? & \cdots \end{bmatrix} \begin{bmatrix} u(N-1) \\ u(N-2) \\ \vdots \\ u(0) \end{bmatrix}$ (Fill the question marks)

(assume $N > n$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^n$)

Q-4:

Consider controller canonical form of (A, B, C, D) (i.e. $A = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$). Suppose we intend $(A+BF)$ to have characteristic polynomial $= s^2+4s+6$ (for $G(s) = \dfrac{(s-3)(s+4)}{s^2-3s+2}$), find $u = Fx$ law (i.e. find $F = [f_1 \; f_2]$).

Q-5: Given $\dot{x} = Ax + Bu$, $y = Cx + Du$, let $z = Tx$, $T \in \mathbb{R}^{n \times n}$, & $\det T \neq 0$. (change of coordinates) → Find transfer fn from u to y: before/after change