EE302-S1, Control Systems, Endsem (25th April 2025)

Notes & instructions: read these very carefully and then read all questions before starting to attempt:

- Attempt all 8 questions: each question carries 10 marks. Unnecessarily long, convolved, redundant or irrelevant text could attract marks-reduction. Show intermediate steps briefly for all calculations.
- Unless otherwise explicitly specified, k is real and positive, and feedback configuration is always the standard negative unity feedback configuration, with the constant gain k, compensator $C(s)$ (if any), and the open loop transfer function $G(s)$ all in the forward path.
- For Bode plot, mark corner frequencies clearly in both gain and phase plots and mark 0 dB line, DC-gain value line, high-frequency roll-off rates (or high-frequency gain, in dB). For Nyquist plot, mark $\omega = 0$, $\omega = \infty$ and show orientation clearly. Mark real-axis intersection values.
- If a state space realization of a certain kind is to be 'provided', then please provide the required kind: no need to derive or elaborate or prove.
- Some questions might not have the sought answer (by intent). In such a case, give reasons why the sought answer is not possible.
- If you feel a question has ambiguity and/or needs clarification, then assume yourself appropriately, state and justify your assumption and then solve the problem with that assumption. Do not call any TA nor instructor for your query.

Ques 1: Consider the discrete time system $x(k+1) = Ax(k) + Bu(k)$ with $B \in \mathbb{R}^n$ and $x(0) = 0$.

(a) What matrix ought to have full rank for the system to allow one to steer x to an arbitrary final state $x(N)$, at some appropriate time instant N using a suitable input sequence $u(\cdot)$?

(b) Prove this matrix's full rank condition is a necessary and sufficient condition for the ability to steer from initial condition 0 to arbitrary final condition using some input.

Ques 2: Consider the open loop transfer function $G(s) = \dfrac{s-12}{s^2+3s+2}$. Consider the gain $k_c > 0$ that causes closed loop to have imaginary axis poles and the corresponding frequency ω_c (i.e. imaginary axis closed loop pole) at that value of k_c.

(a) Use Routh table to calculate k_c and use Routh table to calculate ω_c.

(b) Use Bode plot of $G(s)$ (both magnitude and phase plots) and use these plots to calculate both ω_c and k_c.

(c) Plot the Nyquist plot of $G(s)$ and use this plot to calculate ω_c and k_c.

Ques 3: (a) Define e^{At} using a series representation for a matrix $A \in \mathbb{R}^{n\times n}$, and time-variable $t \in \mathbb{R}$.

(b) With brief reasons, provide the range of t and the class of matrices A for which this series converges.

(c) For $A = \begin{bmatrix} 5 & 6 \\ -3 & -4 \end{bmatrix}$, obtain e^{At} using the fact that $\begin{bmatrix} 5 & 6 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}^{-1}$

(d) For this A, obtain e^{At} using inverse Laplace transform of a 2×2 matrix.

Ques 4: (a) For a state space system $\frac{d}{dt}x = Ax + Bu$, $y = Cx + Du$, of the SISO system with input u and output y, define when the system is called state-controllable.

(b) Formulate the pole-placement problem.

(c) List a necessary and sufficient condition for the state space system to be state-controllable.

Ques 5: Consider the system $\frac{d}{dt}y - 2y = u$ with input u and output y and initial condition $y(0) = 3$.

(a) Obtain a function $u(t)$ of time that should be given to the system to get output $y(t) = 3e^{-4t}$.

(b) Obtain a feedback law $u = fy$ (with f a constant, real number) such that we have $y(t) = 3e^{-4t}$.

(c) Describe briefly in what way the method in (b) is better, w.r.t. perturbations in the initial condition $y(0)$.

(d) Same question as (c), but w.r.t. perturbations in the system pole 2.

Ques 6: Consider the transfer function $G(s) = \dfrac{6s^2 + 22s + 12}{2s^2 + 7s + 3}$.

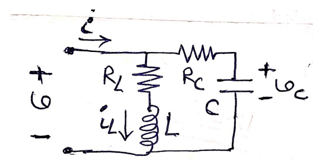
(a) Provide a state space realization with $A \in \mathbb{R}^{2\times 2}$, and (A, B) controllable.

(b) Provide a state space realization with $A \in \mathbb{R}^{2\times 2}$, and (A, C) observable.

(c) Provide a state space realization with $A \in \mathbb{R}^{2\times 2}$, with both controllability and observability.

(d) Same as (6a), but for $G(s) = \dfrac{6s^2 + 22s + 12}{7s + 3}$.

Ques 7: Consider the RLC circuit shown with $R_C = 1$ and $R_L = 1$, while $C > 0$ and $L > 0$ are arbitrary.



(a) Find the impedance transfer function $Z(s)$ from input source $i(t)$ to output voltage $v(t)$ (using any method).

(b) For $Z(s)$, obtain a state space realization with the states as capacitor voltage v_C and inductor current i_L. Show intermediate calculations.

(c) What are the poles of $Z(s)$ and what is the relation with the eigenvalues of the matrix A?

(d) With suitable reasons, use the state space realization to get a condition between L and C for a pole-zero cancellation in the impedance $Z(s)$.

Ques 8: Consider the transfer function $G(s) = \dfrac{1}{s^2 - 3s + 2}$.

(a) Obtain a state space realization with A diagonal.

(b) Obtain a state space realization with (A, B) in controller canonical form.

(c) For each of the two state space realizations, obtain a feedback law $u = Fx$ such that the closed loop has two poles (repeated) at -1.