Tutorial Sheet 7, EE302 - S2, Control Systems 15th April 2024.

Q-1: (a) Consider $G(s) = \dfrac{1}{s^2+2\zeta s+1}$. Find $\max \zeta \in (0,1)$ such that the gain plot (in log-log scale) has a maxima (at ω_m).

(b) Relate ω_m to ω_d & $\omega_n = 1$
↑ damped frequency. (write inequality $\underline{?} < \underline{?} < \underline{?}$ — $\omega_d, \omega_m, \omega_n$).

Q-2: like $e^{at} = 1 + at + \frac{a^2t^2}{2!} + \cdots$, & $e^{at} = \mathcal{L}^{-1}\left(\frac{1}{s-a}\right)$, $(a \in \mathbb{R})$

define $e^{At} := I + At + \frac{A^2t^2}{2!} + \cdots$ & $\mathcal{L}^{-1}(sI-A)^{-1} = e^{At}$
$(A \in \mathbb{R}^{n\times n})$ ↖ take $\mathcal{L}^{-1}$ entry by entry of $(sI-A)^{-1}$.

Check this for $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ (obtain e^{At}) & verify for the resulting $G(s)$ for $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $C = [-1 \;\; 4]$ (& $D = 0$).

Q-3: For $G(s) = \dfrac{4s+5}{s^2-3s+2}$, obtain two state space realizations (A_1, B_1, C_1, D_1) & (A_2, B_2, C_2, D_2) with A_1 diagonal & $A_2 \equiv$ companion form (see next Question). Verify that each of them give the same $G(s)$. Find matrix $T := \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, $a \neq b$ such that $A_2 T = T A_1$.

- Interpret columns of T as eigenvectors of A_2.
- Why did certain entries of T be chosen as 1?

Q-4: Check that $A = \begin{bmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ * & * & * \cdots & * \end{bmatrix}$ & $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$ & $C = [c_0 \;\; c_1 \cdots c_{n-1}]$

- coefficients of $\det(sI-A)$ →

gives $G(s) = \dfrac{c_0 + c_1 s + \cdots c_{n-1}s^{n-1}}{\det(sI-A)}$: check for any 3rd degree polynomial.

Q-5 Transpose of A, interchange of $B \leftrightarrow C^T$ gives same $G(s)$: Verify for 2×2 A or 3×3 A.

Q-6: Consider $G(s) = \dfrac{s+a}{s^2+3s+2}$, ~~Construct~~ Construct A, B, C as in Q 4, but interchange $B \leftrightarrow C^T$ & $A \to A^T$.
Check that controllability matrix is singular iff we have pole/zero cancellation.

Q-7 (a) Consider A, B as in Q4.
Find $F = [f_1 \;\; f_2]$ such that $(A+BF)$ has characteristic polynomial $= s^2 + 7s + 10$ (& choose A appropriately).

(b) Show that (A, B) is always controllable (for any A).
(A, B) is called controller/controllable canonical form?
? ?

Tutorial Sheet 7 (contd) 15th April 2024 (Source : below) Jobling Chris P. of UK.

Consider further the (A, b) structure of Q-4 (Q-8) $\left(A = \begin{bmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ -a_0 & \cdots & \cdots & -a_{n-1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \right.$ $(=: e_n)$ also called $\boxed{b = B}$

Let $G(s) = \dfrac{1}{s^n + a_{n-1}s^{n-1} + \cdots a_1 s + a_0}$, $\omega = G(s)u$

Below are 2 interpretations/understandings of this (A, b) (in controller canonical form & states x = phase-variables)

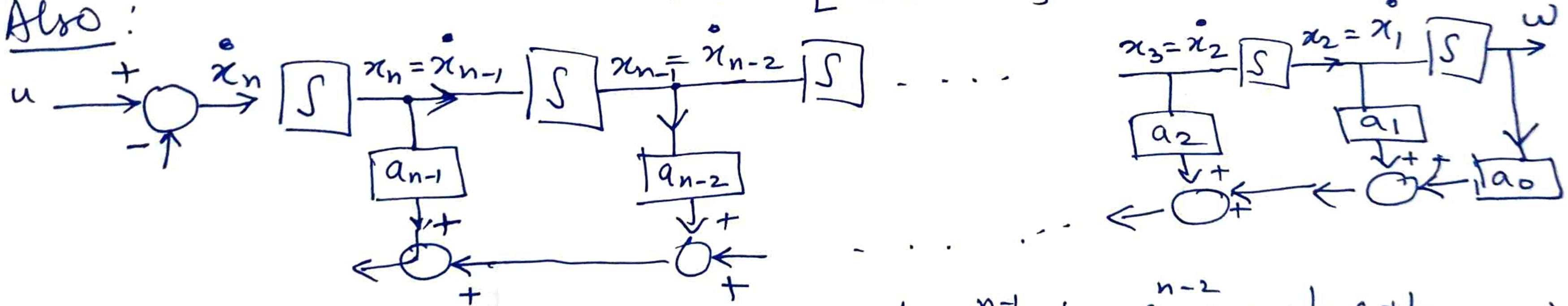
$x_1 = \omega$, $x_2 = \dot{x}_1 = \dot{\omega}$, ... $x_n = \dot{x}_{n-1} = \dfrac{d^{n-1}}{dt^{n-1}}\omega$

$\dot{x}_n = \dfrac{d^n}{dt^n}\omega = -a_0\omega - a_1x_2 - a_2x_3 - a_3x_4 \cdots - a_{n-1}x_n + u$

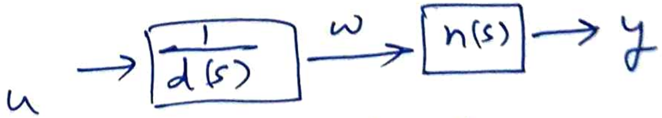
This is same as $\dfrac{d}{dt}x = Ax + bu$ for A, b as defined above.

& $\omega = [1\ 0\ 0 \cdots 0]x$

<u>Also</u>:



For $y = G(s)u$, with $G(s)$ <u>strictly</u> proper $= \dfrac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots\ a_1 s + a_0}$, interpret as $= \dfrac{n(s)}{d(s)}$ (say)



Source github.io of EGLM03 Modern Control Systems

& check that $C = [b_0\ b_1\ \cdots\ b_{n-1}]$ & <u>same</u> A, b works.

See figure in link