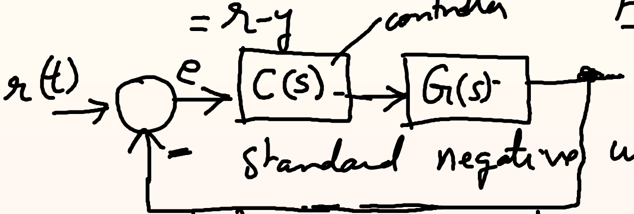
Steady state error analysis

(assuming steady state "exists", i.e. stability & F.V.T. is applicable)



standard negative unity feedback configuration

$r(t) =$ unit step (position) $\frac{1}{s}$ or unit ramp $\frac{1}{s^2}$ or unit parabolic input $\frac{1}{2}t^2$ $\frac{1}{s^3}$

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)}$$

$r \longrightarrow e$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + C(s)\,G(s)}$$

$e(\infty)$ for $r(t) =$ unit step $\longrightarrow R(s) = \frac{1}{s}$

(assuming limit exists)

$$e(\infty) = \frac{1}{1 + C(0)\,G(0)}$$

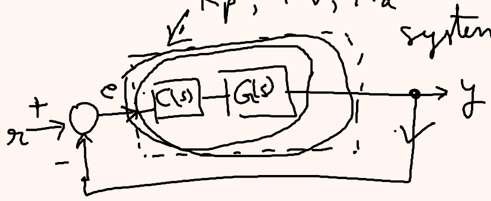
$$K_p = \lim_{s \to 0} C(s)\,G(s) \quad \text{position constant}$$

$$K_v = \lim_{s \to 0} s\,C(s)\,G(s) \quad \text{velocity constant}$$

$$K_a = \lim_{s \to 0} s^2\,C(s)\,G(s) \quad \text{acceleration constant.}$$

K_p, K_v, K_a

system type $\to$ 0, 1, 2



$\leftarrow C(s) = 1$

$$C(s)\,G(s) = \frac{n(s)}{d(s)} = \frac{\tilde{n}(s)}{s^n \tilde{d}(s)}$$

$n = 0 \rightarrow$ system is type 0 $\qquad \tilde{n}(0) \neq 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \tilde{d}(0) \neq 0$

$n = 1 \rightarrow$ system is type 1

$n = 2 \rightarrow$ system is type 2.

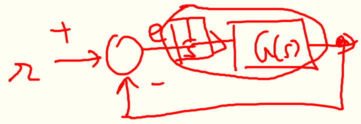
$r(t)$ → unit step

$r(t)$ → ramp

parabolic

for $r(t)$	Type 0	error	Type 1		Type 2	
step input	K_p is (nonzero) constant	$\frac{1}{1+K_p}$	$K_p = \infty$.	~~(PI)~~ 0 $e(\infty)$	∞.	$e(\infty)=0$
ramp input	$K_v = 0$	∞	K_v is constant	$\frac{1}{K_v}$	∞	$e(\infty)=0$
parabolic input.	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{const}$	$\frac{1}{K_a}$

error as $t \to \infty$ assuming F.V.T. applicable.



$$G(s) = \frac{1}{s(s+3)} \rightarrow K_v = \frac{1}{3}. \quad \text{type } 1.$$

$$r = \frac{1}{s} \quad \text{unit ramp.}$$

$$E(s) = \frac{1}{1+G(s)} \quad \lim_{s \to 0} \quad \frac{1}{1+K_p} \rightarrow .$$

zeros are in OLHP

closed loop poles are in OLHP

$e(\infty)$ (for ramp).

$$E(s) = \lim_{s\to 0} \underbrace{\frac{1}{1+G(s)}}\left(s\right) \times \frac{1}{s^2}$$ ← ramp

$$\frac{1}{s\left(1+\frac{1}{s(s+3)}\right)} = \frac{1}{s+\frac{1}{s+3}} = \frac{s+3}{s^2+3s+1}$$

ramp. $3 = \frac{1}{K_v}$

$e(\infty)$ for unit step

$$\lim_{s\to 0} \frac{s}{1+G(s)} \times \frac{1}{s}$$ ← unit step

$$= \lim_{s\to 0} \frac{\cancel{s}(s+3)}{s^2+3s+1} = 0.$$

closed loop stable.