Routh-Hurwitz criteria & the Routh table

Routh & Hurwitz : 2 persons who worked on stability

- Hurwitz polynomial
-

$a_n \neq 0.$

Roots of ?

$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots \cdots a_1 s + a_0$

open left half complex plane / closed left half complex plane

OLHP   CLHP

$p(s) = a_n s^n + \cdots a_0 \longrightarrow$

$P_{even}(s) + P_{odd}(s) \longrightarrow$

P_{even}: only even powers of s

P_{odd}: only odd powers.

Routh

s^n	a_n	a_{n-2}	a_{n-4}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	$\cdots$	
s^{n-2}	$\cdot$			
s^{n-3}	0	$\cancel{10}$ -20		
$\vdots$				

1^{st} column

number of sign changes $\rightarrow$ sign of 0?

= number of ORHP roots of $p(s)$

Regular case $\rightarrow$ no zero is encountered in 1^{st} column

2 special cases $\rightarrow$ 0 is encountered in first column.

- classify if all roots of $p(s)$ in CLHP Yes/No.
- if some in ORHP — then how many.

are all roots in the OLHP ?

$\cancel{a_n}^{1} s^n + \frac{a_{n-1}}{a_n} s^{n-1} + \cdots \frac{a_0}{a_n}$ all signs are same or not

$a_n \neq 0$. monic polynml $\equiv$ leading coeff $=1$

all coeff >0? necessary condit for Hurwitz poly.

$(s+z_1)(s+z_2)\underline{(s^2+q_{1}s+q_{2})}(s^2+q_1 s+q_2)(s^2+q_3)$

$q_i > 0$, $z_i > 0$

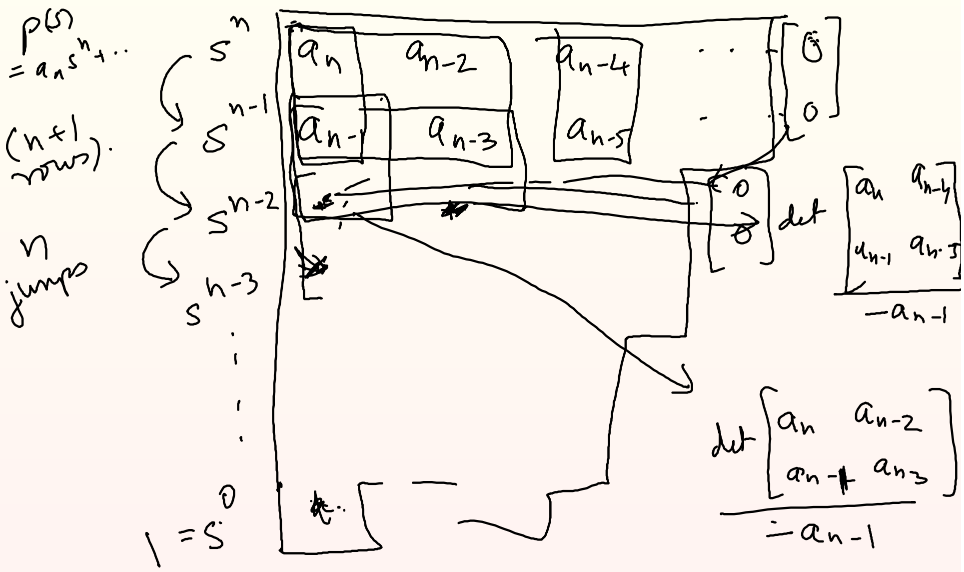
A necessary condit for all roots of
$p(s)$ to be in C-LHP

is all coefficients same sign

if all coeff not same sign $\Rightarrow$ one or more roots in RHP
$\not\Leftarrow$

(monic one or more are $-$ve) $\Rightarrow$ one or more in RHP

$(s^3 + 5s)$ $a_0 = 0$
odd / even polynomial. $a_1 = 0$


p(s) = a_n s^n + ..
(n+1 rows).
n jumps
s^n
s^(n-1)
s^(n-2)
s^(n-3)
1 = s^0
a_n   a_(n-2)   a_(n-4)
a_(n-1)   a_(n-3)   a_(n-5)
det [a_n a_(n-4) ; a_(n-1) a_(n-5)] / -a_(n-1)
det [a_n a_(n-2) ; a_(n-1) a_(n-3)] / -a_(n-1)

$$p(s) = s^3 + 10s^2 + 31s + 1030$$

(s^2+3), (s^2-3), (s^2+8)

s^3	1	31	0
s^2	$\cancel{10}$ 1	$103\cancel{0}$	0
s^1	$\frac{103-31}{-1} = -72$ -1	0	
s^0	103		

Scale by nonzero +ve constant

Scale by +72.

2 sign changes ⇒ 2 roots in ORHP

"rest" = 1 → in closed LHP. open.