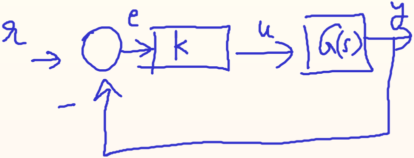
Refining

Root locus sketching

Madhu N. Belur
Control and Computing group, EE, IITB
www.ee.iitb.ac.in/∼belur/

$G(s) = \dfrac{n(s)}{d(s)}$ ← zeros of $G(s)$; ← roots of d ≡ poles of $G(s)$ — open loop poles/zeros



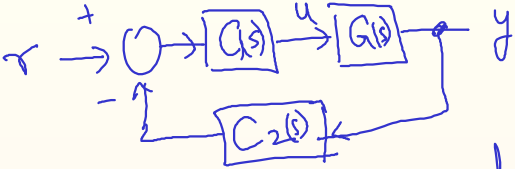
$u = ke$.

$r \xrightarrow{\dfrac{kG}{1+kG}} y$

poles/zeros of closed loop transfer

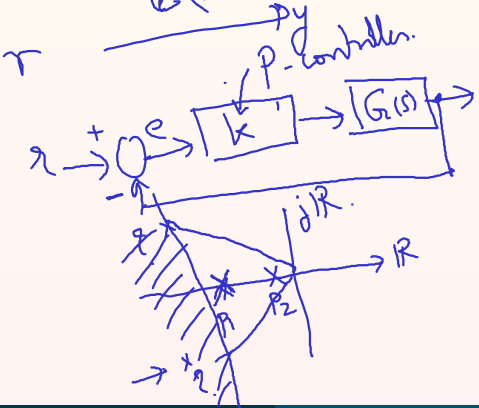
$r \xrightarrow{\dfrac{1}{1+kG}} e$

zeros of $(1+kG)$ ≡ closed loop poles



$$\frac{C_1 G}{(1 + C_1 G C_2)}$$

closed loop poles $=$ zeros of $1 + C_1 G C_2(s)$



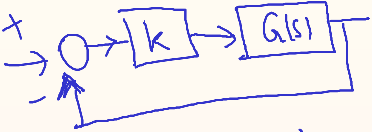
$G(s) = \dfrac{n(s)}{d(s)}$ ← proper

closed loop poles

$=$ roots of $\left(d(s) + k\, n(s)\right)$

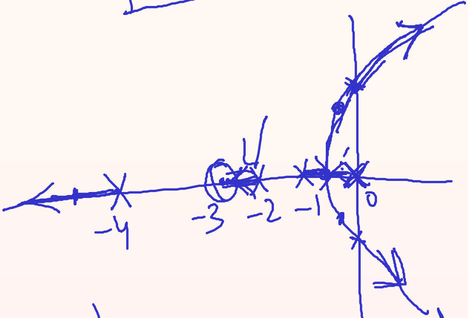
Assume $G(s) = \dfrac{n(s)}{d(s)}$ ← both monic

$k > 0$



$s \neq 4$

$$\frac{s+3}{s(s+1)(s+2)(s+4)}$$



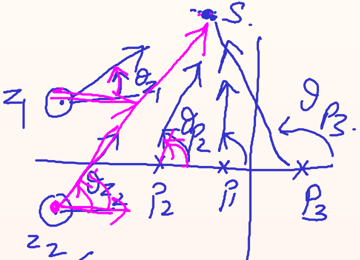
$0 \rightarrow k \rightarrow +\infty$

$\deg \big(d(s) + k\, n(s)\big) = 4$

closed loop poles

What value of real (& +ve k) and $s \in \mathbb{C}$ satisfy $1 + kG(s) = 0$?

then that value of s "lies" on the root locus (point in $\mathbb{C}$)



$$G(s) = \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)(s - p_3)}$$

$$\angle G(s) = \sum_{i=1}^{2} \theta_{z_i} - \sum_{j=1}^{3} \theta_{p_j} = 180^\circ \text{ (or odd multiple of } 180^\circ\text{)}.$$

$$G(s) = -\frac{1}{k} \quad \text{(for } k \text{ real \& +ve)}.$$

$$k := -\frac{1}{G(s)} > 0$$

For $1+kG(s)$

$s \rightarrow \frac{1}{1+kG} \rightarrow e$

$s \rightarrow \frac{kG}{1+kG} \rightarrow y$

$s_0 \in \mathbb{C}$ lies on the root locus.

(is a closed loop pole for some value of k)

if & only if

$$\underbrace{G(s_0)}_{\sum \theta_{z_i} - \sum \theta_{p_j}} = \cancel{2k+} (2\ell+1)180^\circ$$

for integer ℓ.

(+ve or -ve integer).

plot ~~roots~~ roots
$d(s) + k\,n(s)$ as k varies
from $0 \longrightarrow +\infty$

$G(s) = \dfrac{n(s)}{d(s)}$

both n, d monic

$\deg d \geq \deg n.$

1. branches of the root locus

\# branches = \# closed loop poles.
$= \deg d(s).$

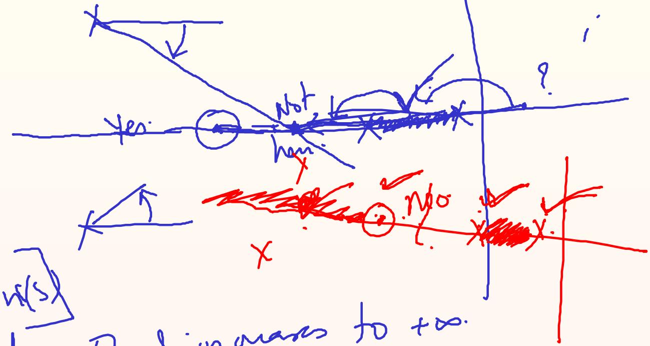
\# closed loop ~~pts~~ poles
$=$ \# open loop poles.
$= \deg d.$

2. d, n real polynomials.

roots $(d + k n)$ are symm about real axis

3. Real-axis segments.

(3) Real axis segments



$\boxed{d(s)+k\,n(s)}$

k start from 0 & increases to $+\infty$.

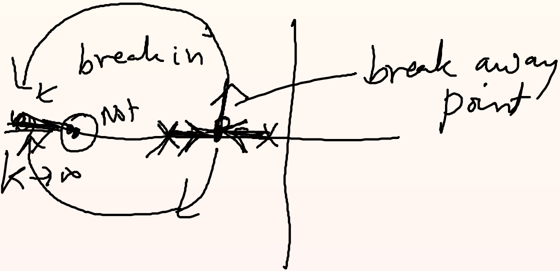
$$\angle G(s) = (2\ell+1)\,180^\circ.$$

then real axis segments with odd # [poles & zeros] to the right of that real points

(4) starting / ending pts.

start at open loop poles $\equiv$ roots of $d(s)$

& end at open loop zeros $\equiv$ roots of $n(s)$



(5) Butterworth pattern

3 asymptotes

$d(s) + k\, n(s)$

$s^3 + k \qquad \sqrt[3]{k}$

as $k \to \infty$ if $\deg n < \deg d$

$\#$ asymptotes $= \deg d - \deg n = 3$

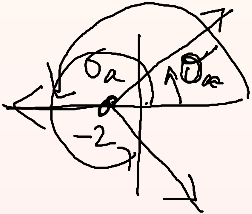
Real axis intersection pt $=$ σ_a $=$ $\dfrac{\Sigma \text{ fin poles} - \Sigma \text{ fin zeros}}{\# \text{ asymptotes}}$

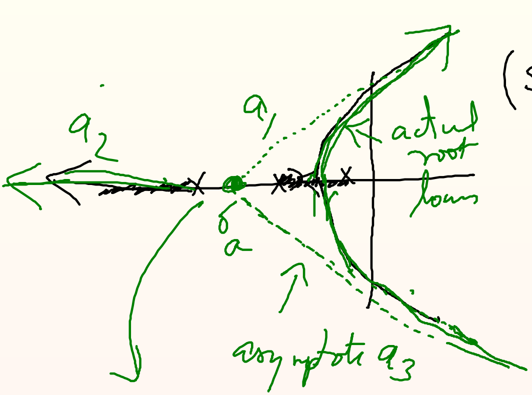
$$\theta_a = \frac{(2k+1)\,180^\circ}{\# \text{ asymptotes}}$$

$k=0$, $k=1$, $k=2$

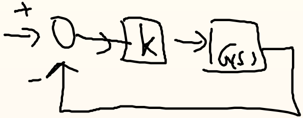
$k=3$

3





$$\frac{1}{(s+1)(s+2)(s+3)} = G(s)$$



$$\frac{1}{(s+3)(s+5)}$$