Routh-Hurwitz criteria: Special cases.

Recall : Construct Routh-table.

— check 1^{st} column sign changes in consecutive rows.

— n^{th} degree monic polynomial

$\Rightarrow$ $n+1$ rows $s^n, s^{n-1}, \dots s^0$

So n sign comparisons between consecutive rows.

sign changes = # roots in Open RHP.

— sign of 0 ?? — in first column.

— division by 0 (in next row)

1st column zero

entry only in 1st column (in that row).
(atleast one entry in that row is nonzero)

ε-method

reverse polynomial

"entire row becomes zero"

s^2

s^1

s^0

even polynomial

odd polynomial.

only even powers appear in $p(s)$

only odd powers appear in $p(s)$

$p(s) = p(-s)$

s^{n_2}(even polynomial)

$p(s) = -p(-s)$

n_1 odd, [s^{n_1}(even polynomial)]

$$p(s) = \underline{s}^5 + \underset{2.00001}{2}s^4 + \underline{3}s^3 + \underset{6.0001}{6}s^2 + 5s + 3$$

real coeffint.

Routh table

s^5	1	3	5
s^4	2	6	3
s^3	ϵ	$7/2$	0
s^2	$\frac{6\epsilon - 7}{\epsilon}$	3	0
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	0	
s^0	3	0	

$\epsilon \approx 0$

slightly +ve ←

slightly −ve ←

First column analysis (of Routh table) for
$s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$

	1st column	ϵ: +ve.	ϵ : -ve
s^5	1	+	+
s^4	2	+	+
s^3	ϵ	ϵ: +	ϵ : -ve
s^2	$\frac{6\epsilon-7}{\epsilon}$	$\frac{-7}{\epsilon}$: -ve	$\frac{-7}{\epsilon}$: +ve.
s	$\frac{42\epsilon-49-6\epsilon^2}{12\epsilon-14}$	$\frac{-49}{-14}$: +ve.	+ve.
1	3	+ve.	+ve.

1 sign swap

1 sign swap

1 sign swap

+1 sign swap

2 sign changes $\Rightarrow$ 2 in o RHP

2 sign swaps

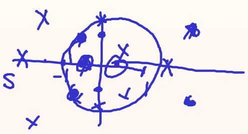
degree 5 polynl.

2 in ORHP

hence 3 in OLHP closed X.

roots on jR

even poly part at 0.



reverse coefficient method

$1\cdot s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$

$s \to \frac{1}{z}$

$3z^5 + 5z^4 + 6z^3 + 3z^2 + 2z + 1$

reverse coeff → construct Routh table & "maybe" no zeros in 1st column (not entire row)

Entire row zero. $(a_n s^n + a_{n-1} s^{n-1} + \cdots (\ a_1 s + a_0)) = p(s)$

Assume (W L O G) $a_n \neq 0$

$a_0 \neq 0$ ← (else extract factor s^{n_1}, n_1 odd/even).

Main idea.

entire row zero ⇒ previous row → $q(s)$ is a even factor (of $p(s)$).

hence remainder (expected odd polynl) $= 0$.

Luckily... (hard part to prove)

Routh table analysis

holds when "continuing" with $\frac{d}{ds} q(s)$!!

So replace row of zeros with $\frac{d}{ds} q(s)$ → odd polynl.

$p(s) = s^5+7s^4+6s^3+42s^2+8s+56.$ → $p(s)$ is divisible by $q(s)$

Routh table

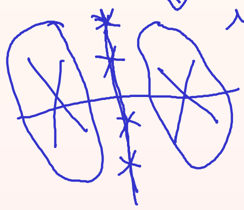
s^5	1	6	8
s^4	7	42	56
s^3	$\cancel{0}\ \cancel{4}\ 1$	$\cancel{0}\ \cancel{12}\ 3$	$\cancel{0}$
s^2	3	8	
s	$1/3$	0	
1	8		

$56 \to 7s^4 + 42s^2 + 56$

$= 7\underbrace{(s^4+6s^2+8)}_{q(s)}$

$\frac{d}{ds}(q(s)) = 4s^3+12s$

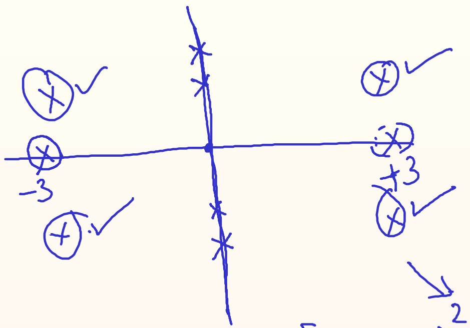
$q(s)$ has no roots in ORHP



$p(s) \to 5$

$q(s) \to 4$ on $j\mathbb{R}$

hence $p(s)$ has 1 in OLHP, 4 on $j\mathbb{R}$



$p(s) = p(-s)$

even poly

$(s^2 - 9)$

$\left[(s-3)^2 + 1\right]\left[(s+3)^2 + 1\right]$

products of even poly

$\Downarrow$

even polynomial.

check we get even poly.

- on $j\mathbb{R}$.
- real & $\pm$ on real axis
- 4 symmetric (about $j\mathbb{R}$) & complex

$$s^{11} + s^{10} + 12 s^9 + 22 s^8 + 39 s^7 + 59 s^6 + 48 s^5 + 38 s^4 + 20 s^3 \quad ?? = 0.$$

$= s^3[$ 8th order polynomial with nonzero constant coeff $]$

$= s^3[\ p(s)\ ]$

$p(s)$ R table:

s^8	1	12	39	48	20
s^7	1	22	59	38	
s^6	$\cancel{-10}$ -1	$\cancel{-20}$ -2	$\cancel{10}$ 1	$\cancel{20}$ 2	
s^5	$\cancel{20}$ 1	$\cancel{60}$ 3	$\cancel{40}$ 2		
s^4	1	3	2		
s^3	$\cancel{0}$ $\cancel{4}$ 2	$\cancel{0}$ $\cancel{6}$ 3	0		
s^2	$3/2 \to 3$	$2 \to 4$			
s	$1/3$				
1	4				

9 rows 8 comparisons

4

4

$s^4 + 3s^2 + 2$

$4s^3 + 6s$

	even	"quotient"	$p(s)$
ORHP	0	2	2
OLHP	0	2	2
$j\mathbb{R}$	4	0	4
			8