Q1 For a matrix A, define:

$$\sin(A) = \frac{1}{2j}\left[e^{jA} - e^{-jA}\right].$$

Then, for $A = \begin{bmatrix} -3 & 1 \\ 0 & -5 \end{bmatrix}$, we have

(a) $\sin(A) = \begin{bmatrix} \sin(-3) & \frac{1}{2}(\sin(-3)-\sin(-5)) \\ 0 & \sin(-5) \end{bmatrix}$

(b) $e^{A} = \begin{bmatrix} e^{-3} & \frac{1}{2}(e^{-3}-e^{-5}) \\ 0 & e^{-5} \end{bmatrix}$

(c) $e^{A} = \begin{bmatrix} e^{-5} & \frac{1}{2}(e^{-3}-e^{-3}) \\ 0 & e^{-3} \end{bmatrix}$

(d) $\sin(A) = \begin{bmatrix} \sin(-5) & \frac{1}{2}(\sin(-3)-\sin(-5)) \\ 0 & \sin(-3) \end{bmatrix}$

EE302

Assignment 4.

Submission date

See moodle post of 9th (April 2021).

Q-2 (Question 2)

Consider the system,

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t),$$

where $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$,

$C = [1 \quad 1]$.

Which of the following is/are true?

(a) The system's transfer fn has two real poles.

(b) The transfer function corresponding to this system represents an overdamped second order system.

(c) The d.c. gain of the transfer function for the system is non-unity.

(d) The eigenvalues of A are real and distinct.

Question 3

Q3

Q3 For the system $\dot{x}(t) = Ax(t)$, suppose $\delta(A) = \det(A)$ & $\tau(A) = \text{trace}(A)$, when $A \in \mathbb{R}^{2\times2}$. Which of the following are true?

(a) If $\delta < 0$, the two eigenvalues of A ~~have~~ are real with opposite signs

(b) If $\delta > 0$, the two eigenvalues of A are real with same sign.

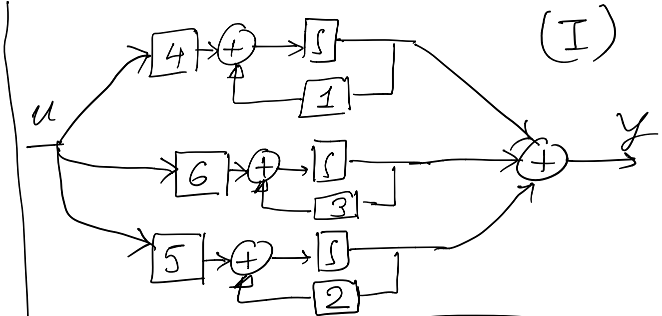
(c) If $\delta > 0$ and $\tau^2 - 4\delta \geq 0$, then the system is stable for $\tau < 0$ and unstable for $\tau > 0$.

(d) If $\delta > 0$ and $\tau = 0$, the response of the system to non-zero initial conditions will be oscillatory.

Q-4 Match the following ckt implementations with the state space eqns:

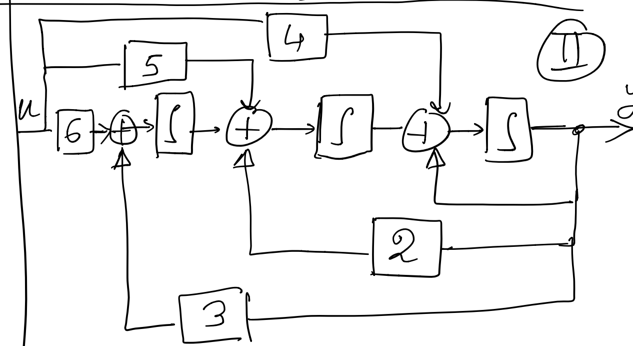
A) $$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 1 \\ -3 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$



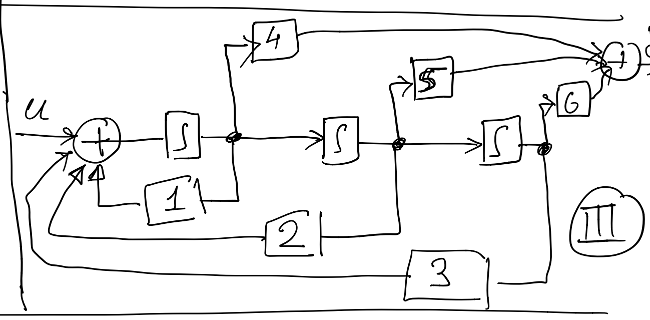
B) $$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} x + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x$$



C) $$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 6 & 5 & 4 \end{bmatrix} x$$



Options (Mark all correct ones)

(i) A $\leftrightarrow$ II

(ii) A $\leftrightarrow$ III

(iii) B $\leftrightarrow$ I & A $\leftrightarrow$ III

(iv) B $\leftrightarrow$ II & A $\leftrightarrow$ II

(v) A $\leftrightarrow$ II & B $\leftrightarrow$ I & C $\leftrightarrow$ III

(vi) A $\leftrightarrow$ III & B $\leftrightarrow$ I & C $\leftrightarrow$ II

Q-5

~~b)~~ Consider the following state space realizations:

A) $\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$
$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$

B) $\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$
$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$

C) $\dot{x} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$
$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$

D) $\dot{x} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$
$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$

Mark true or false

i) A, B, C, D are NOT asymptotically stable (A.S.)

(ii) A, B are NOT A.S. but C, D are A.S.

(iii) A, B, C are NOT A.S. but D is A.S.

(iv) A, B are diagonalizable (by real change of coordinates)

(v) C, D are diagonalizable (by real change of coordinates)

(vi) $\lim_{t \to \infty} \|x(t)\|_2$ is same for all initial conditions for (C) & (D) (with $x(t) = e^{At} x_0$)

(vii) A state feedback controller can be designed for (A) to place the closed loop poles at $(-2, -3)$

(viii) A state feedback controller can be designed for (B) to place the closed loop poles at $(-2 \pm j2)$

(ix) A state feedback controller can be designed for (C) to place the closed loop poles at $(-1, -1)$

(x) A state feedback controller can be designed for (D) to place the closed loop poles at $(-1, -1)$

(xi) (A) & (B) are similar (A similarity transform exists between them)

(xii) (A) & (C) are similar

6. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ and consider the state feedback law $u = Fx$ in order to get desired closed loop poles, i.e. desired eigenvalues of $(A + BF)$. Choose the right option(s).

   (a) There exists a unique feedback law $u = Fx$ to place closed loop poles at -2,-3.

   (b) There does **not** exist any feedback law $u = Fx$ that places closed loop poles at -3,-1.

   (c) There exist non-unique feedback laws $u = Fx$ that place closed loop poles at -2,3.

   (d) There exist a unique feedback law $u = Fx$ that places closed loop poles at -3,3.

7. In continuation with the A and B defined in the previous question, pick the right statement(s) from below.

   (a) (A, B) is controllable and poles can be placed anywhere.

   (b) (A, B) is uncontrollable ~~and 3 is an uncontrollable pole.~~

   (c) (A, B) is uncontrollable and pole/zero cancellation would happen in $C(sI - A)^{-1}B$ for any C (which is 1 row and 2 columns).

   ~~(d) (A, B) is controllable but pole/zero cancellation can be commented only after C is specified.~~

   ~~(e) 3 is a controllable eigenvalue and the transfer function $C(sI - A)^{-1}B$ would not have a common pole/zero.~~

8. Suppose $G(s) = \frac{1}{s^2}$ and consider a state space realization (A, B, C, D) of $G(s)$ in which A is 2×2. Pick all the right statement(s).

   ~~(a) Poles of $G(s)$ need not be eigenvalues of A in this case because $G(s)$ has repeated poles.~~

   (b) Any A matrix of the state space realization would be of rank 1 and both eigenvalues would be zero.

   (c) Any A matrix of the state space realization would be non-diagonalizable.

   (d) ~~G has no zeros, and possibility of pole/zero cancellation is ruled out, and hence~~ any state space realization (with eigenvalues of A as poles of $G(s)$) would be controllable.

9. Consider A and B in a so-called canonical form as follows.

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and $u = Fx$ in order to achieve desired closed loop poles, i.e. desired eigenvalues of $(A + BF)$. Pick the statement(s) from below that are right for any values of a_i above.

(a) A is always diagonalizable.

(b) (A, B) is always controllable.

(c) Eigenvalues of A are roots of the polynomial $s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$.

(d) If the desired closed loop poles are specified as the roots of the polynomial: $s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0$, then solving for entries of F needs mere 'differencing' of corresponding coefficients.

~~(e) $C = [1\ 0\ \cdots\ 0]$ would give $C(sI - A)^{-1}B$ to never have pole/zero cancellation.~~