EE302, Control Systems, Tutorial Sheet 4, March 16th, 2020.

Q-1 Draw root locus for (a) $\frac{1}{(s+1)(s+2)(s+3)}$ for $k<0$

(b) $\frac{2-s}{(s+3)(s+4)}$ for $k>0$ (c) $\frac{(s-z)}{(s+10)(s^2+2)}$ for $k \in (-\infty, \infty)$

First take $z = -15$

For (c), Find angle of arrival/departure at all poles/zeros. (for $z=-15$)

For (c) Find a value of $z \in \mathbb{R}$ such that breakaway/breakin points coincide.

Q-2: Plot root locus for $\frac{1}{[(s-1)^2+5^2]^2}$ by first plotting for $\frac{1}{(s^2+25)^2}$

(Check using root locus plotter).

Q-3: Consider polynomial $2s^3 + 3ps^2 - 6ps - 3p + 6s^2 + 6s + 2$.
Plot roots of this polynomial in s as parameter p varies from $-\infty$ to ∞ (using root locus methods).

Q-4: Consider $G(s) = \frac{n(s)}{d(s)}$ with n, d having real, distinct roots.

(a) Obtain formula for breakaway/breakin points using maxima/minima property of k on real axis

$$\sum_{p_i} \frac{1}{s-p_i} = \sum_{z_i} \frac{1}{s-z_i}$$

(b) Use "repeated root" property of $d+kn$ at that value of k to get same formula for s value at breakaway/breakin pt.

(c) Look up Sylvester resultant, (of two polynomials), discriminant (of a polynomial) & relate with discriminant of ax^2+bx+c. $(a \neq 0)$.

Q-5 Consider the plant given by $G(s) = \frac{1}{s(s^2+\omega^2)}$, $\omega > 0$, real.

Suppose one wants to design a controller by first cancelling the $j\mathbb{R}$ poles. Consider (a) $C(s) = s^2 + (\omega+\epsilon)^2$, $\epsilon > 0$.

(b) $C(s) = s^2 + (\omega - \epsilon)^2$

Use root locus to establish which mismatch (in a or b) is acceptable & justify.

Q-6: (a) Consider system $\dot{x} = 3x + u$, in which $x(0) = 7$ and using $u = -5x$, check that $x(t) \to 0$, $u(t) \to 0$ as $t \to \infty$.

(b) Now find $u(t)$ explicitly as a function of time (using $x(0)=7$)

Q-~~6~~ e: Use Laplace transform to get output for system

$u(t) \rightarrow \boxed{\frac{1}{s-3}} \rightarrow x(t)$, with $u(t)$ as in Q-6b, and $x(0)=7$.

Q-6 d: Now, solve Q-6c, but with system $\dot{x} = (3+\epsilon)x$

Q-6 e: Solve Q-6c, but with initial condition $7+\epsilon = x(0)$

Q-7: (a) Consider $G(s) = \frac{1}{(s+1)(s+2)}$. Consider 15% OS for closed loop.

(b) Design a PD controller that gives 2% settling time to half of that in (a) (& same % OS).

(c) Design 2 different lead compensators (meaning diff pole/zero locations). (instead of b).

(d) Design a PI controller to achieve steady state error = 0.

(e) Design a lag compensator to get steady state error of Q-7c to one-tenth. (Two diff lag-compensators)

Q-8 Sketch (on the same figure) Bode magnitude/phase plots

$\frac{s+1}{s+8}$, $\frac{s-1}{s+8}$, e^{-s20}, $\frac{s-1}{s+1}$, $\frac{(s-1)(s-3)}{(s+3)(s-1)}$.

For each of above, correct by 3dB at corner ~~corner~~ frequency

Q-9 Sketch for $\zeta = 0.1$ & 0.9 for $\frac{5}{s^2+20\zeta s+100}$, Find value of ζ (range) for which there is a peak in magnitude plot. Find frequency at which peak happens (as a function of ζ).

Q-10. Classify into lead, lag, PD, PID and plot Bode plot.
Also classify from Bode plot, and get transfer function from Bode plot.

$\frac{s+8}{s+20}$, $3s+9$, $6+\frac{s}{3}+14s$, $\left(\frac{s+3}{s+0.5}\right)25$

Q-11 Look up. lead, lag & lag-lead compensator realizations
- using RC components (Read from Norman Nise).
- using op amp.

Consider the following set-up shown in the figure below, consisting of an input source, a linear time-invariant system, and an oscilloscope. Suppose the input signals generated by the source are of the form $u(t) = A \sin \omega t$, with adjustable $\omega \in \mathbb{R}$. (i) Show that for a given ω, the oscilloscope will display an ellipse, or a straight line. (ii) How can you compute the phase $\phi(\omega)$ and the magnitude $M(\omega)$ of the LTI system by inspecting the aforesaid ellipse / straight line?

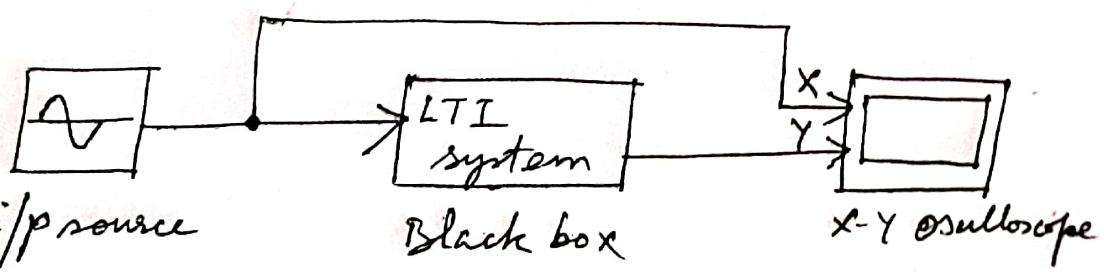

Fig.: Experimental set-up to compute $\phi(\omega)$ and $M(\omega)$.