1. (a) $$\frac{Y(s)}{R(s)} = \frac{\dfrac{C(s)}{1+C(s)(1-e^{-2s})G_m(s)} \cdot G(s) e^{-2s}}{1+\dfrac{C(s)G(s)e^{-2s}}{1+C(s)\cdot(1-e^{-2s})G_m(s)}}$$

$$= \frac{C(s) \cdot G(s) e^{-2s}}{1+C(s)\cdot(1-e^{-2s})G_m(s) + C(s)G(s)e^{-2s}}$$

Since, $G(s) = G_m(s)$, we have

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)e^{-2s}}{1+C(s)G_m(s)}. \quad (i)$$

Now,

$$\frac{Y(s)}{D(s)} = \frac{G(s)e^{-2s}}{1+\dfrac{C(s)\cdot G(s)e^{-2s}}{1+C(s)(1-e^{-2s})G_m(s)}}$$

$$= \frac{G(s)\left[G_m(s)(1-e^{-2s})C(s)+1\right]e^{-2s}}{1+C(s)\cdot G_m(s)} \quad (ii)$$

Let $R(s) = D(s) = \frac{1}{s}$; ~~~~

Therefore, for the particular system given here, we have

$$Y(s) = \frac{1}{s} \cdot \frac{\left(\frac{2s+1}{s}\right)\frac{1}{s(1+5s)} \cdot e^{-2s}}{1+\frac{(2s+1)}{s}\left(\frac{1}{s(1+5s)}\right)}$$

$$+ \frac{1}{s} \cdot \frac{\frac{1}{s(1+5s)} \cdot \left[\frac{1}{s(1+5s)}(1-e^{-2s}) \cdot \frac{2s+1}{s}+1\right]e^{-2s}}{1+\frac{(2s+1)}{s}\left(\frac{1}{s(1+5s)}\right)}.$$

$$\Rightarrow Y(s) = \frac{1}{s} \cdot \frac{(2s+1)e^{-2s}}{5s^3+s^2+2s+1} + \frac{1}{s} \cdot \frac{5s^3+s^2+(2s+1)e^{-2s}(1-e^{-2s})}{(5s^3+s^2+2s+1)(1+5s)s}$$

If $5s^3+s^2+2s+1$ is Hurwitz, we can apply FVT.

Thus, $\lim_{t\to\infty} y(t) = 1+1 = 2$. Thus, disturbance cannot be rejected.

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(b) To evaluate ~~$y(t)$~~ $e(t) = r(t) - y(t)$, we need to ensure that $5s^3 + s^2 + 2s + 1$ is Hurwitz. We construct Routh array.

$$\begin{array}{c|c|c} s^3 & 5 & 2 \\ s^2 & 1 & 1 \\ s & -3 & 0 \\ 1 & 1 & \end{array}$$

Two sign changes indicate two roots in rhp.
So FVT does not apply, as the designed controller does not stabilize the system.

2. We construct the Routh Array as follows:

$$\begin{array}{cccccc} s^6 & : & 1 & -6 & -1 & 6 \\ s^5 & : & 1 & 0 & -1 & \\ s^4 & : & -6 & 0 & 6 & \\ s^3 & : & \cancel{0}\ -24 & 0 & & \\ s^2 & : & \cancel{0}\ \epsilon & 6 & & \\ s^1 & : & \frac{144}{\epsilon} & & & \\ s^0 & : & 6. & & & \end{array}$$

At s^3 row we have all zeros. So we consider auxiliary ~~eqn.~~ polynomial.

$$P(s) = -6s^4 + 6$$

$$\frac{dP}{ds} = -24s^3$$

At s^2 row, we use ϵ-$\cancel{0}$ method.

From the first row, we deduce that there are two sign changes.

$\therefore$ Two roots on the r-hp

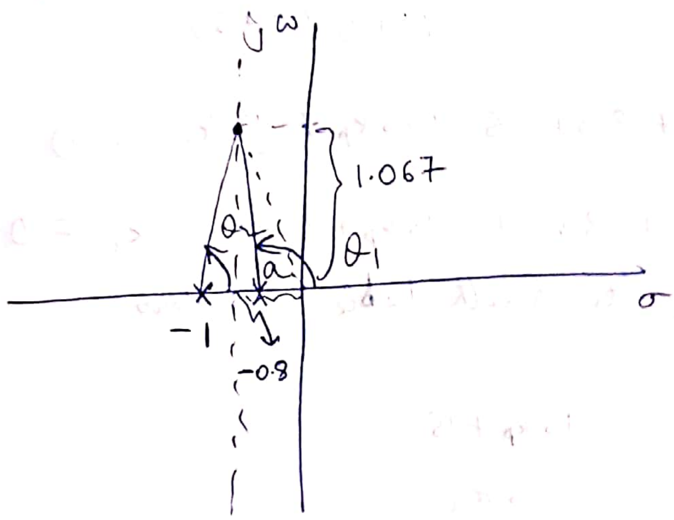
Closer inspection of $P(s)$ reveals (last ~~four~~ five rows of R-table

that $P(s)+P'(s)$ has one root on the r.h.p.
This implies $P(s)$ cannot have all its roots on the imaginary axis. At most two roots can be on the imaginary axis. Now, $P(s)$ and $P'(s)+P(s)$ have the same no. of roots on r.h.p.
$\Rightarrow$ $P(s)$ has one root in r.h.p.
Further, due to symmetry, $P(s)$ has one root in l.h.p.
$\therefore$ $P(s)$ has 2 roots on $j\omega$ axis.

Thus, the overall distribution is

2 in rhp, 2 in lhp, 2 on $j\omega$ axis.

3.



Suppose the characteristic poly. of the CL system is $s^2+2\xi\omega_n s+\omega_n^2$.

From system specs. we have,

$$\frac{4}{\xi\omega_n} = 5 \Rightarrow \xi\omega_n = 0.8$$

$$e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.1$$

$$\xi = 0.59 \approx 0.6$$

$$\omega_n = 1.33.$$

Now, applying angle criterion, we have $\theta_1+\theta_2=180^\circ$.
$\Rightarrow a = +0.6$.

Further, $\left|\frac{k}{(s+a)(s+1)}\right| = 1$

$$\Rightarrow \quad \frac{k}{1.067^2+0.2^2} = 1.$$

$$\Rightarrow \quad k = 1.178$$

Thus, desired controller is

$$\frac{1.178}{s+0.6}$$

4. The closed loop characteristic polynomial is obtained from.

$$1 + \frac{15\left(k_p + \frac{k_i}{s}\right)}{(s+3)(s+5)} = 0$$

$$s^2 + 8s + 15 + 15k_p + \frac{15k_i}{s} = 0$$

$$\Rightarrow s^3 + 8s^2 + (15k_p + 15)s + 15k_i = 0$$

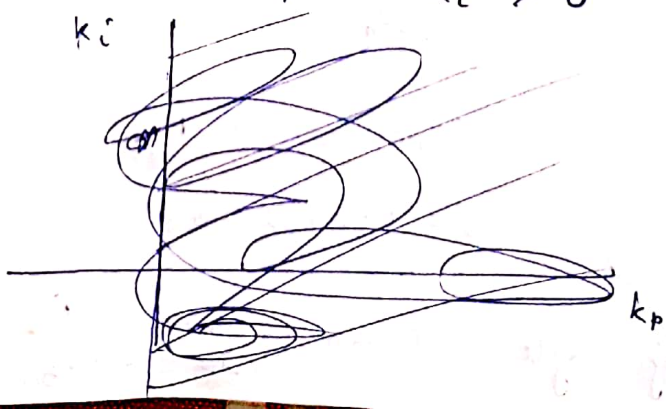
We prepare the Routh table as follows.

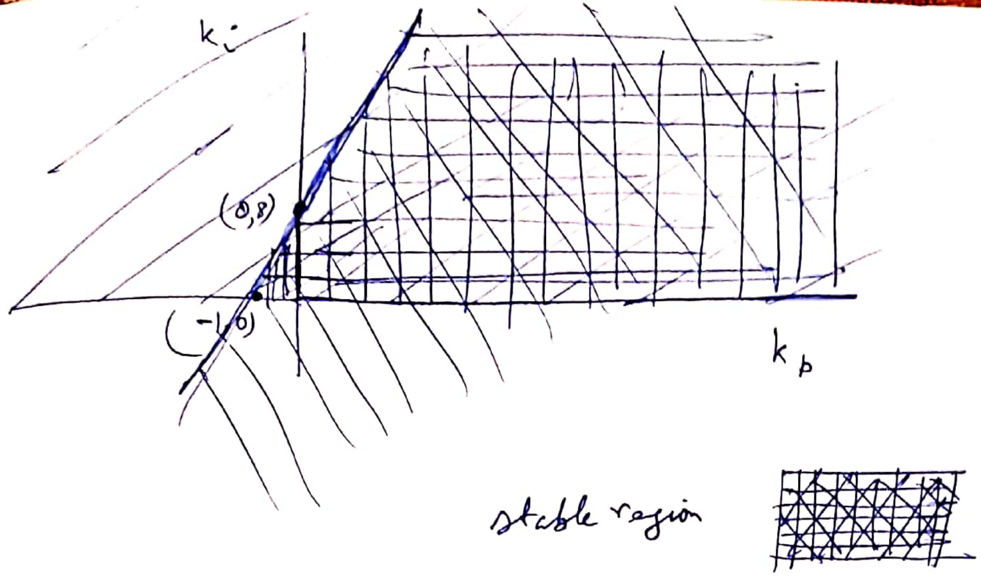
s^3	1	$15k_p + 15$
s^2	8	$15k_i$
s^1	$\frac{60k_p + 60 - 7.5k_i}{4}$	
s^0	$15k_i$	

For stability we need first row to have all its entries positive.

Thus, $k_i > 0$,

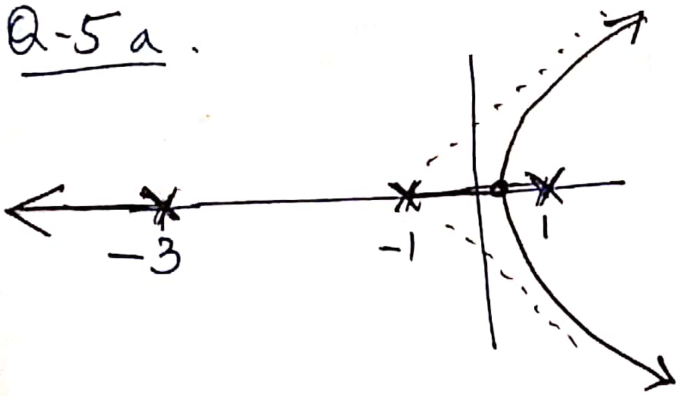
and. $8k_p + 8 - k_i > 0$





Clearly, for $k_p < 0$, $\exists$ a region in the $k_p - k_i$ plane that corresponds to stable region. Hence, it is possible.

Q-5 a.



for k varying from 0 to ∞,
1 breakaway point.
Breakaway point in either RHP or LHP
(reveals in Routh Hurwitz ~~ot~~ and breakaway pt. calculation).

Closed loop

$$(s^2-1)(s+3)+k = s^3+3s^2-s+(k-3)$$

Routh Table

s^3	1	-1
s^2	3	$k-3$
s	$-k$	0
1	$\frac{(k-3)\times k}{k} \to (k-3)$	

for $k \in (0,3)$ one root in RHP

for $k>0$, always has atleast one root in RHP

hence breakaway pt in RHP.

Breakaway pt

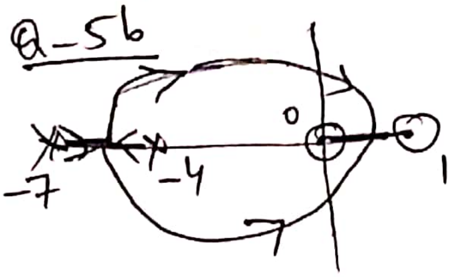
$$\frac{1}{\sigma-1}+\frac{1}{\sigma+1}+\frac{1}{\sigma+3}=0$$

$$(\sigma+1)(\sigma+3)+(\sigma-1)(\sigma+3)+(\sigma-1)(\sigma+1)=0 = 3\sigma^2+6\sigma-1$$

$$\sigma = \frac{-6\pm\sqrt{6^2+12}}{6} = \frac{-3\pm\sqrt{12}}{3} \approx +0.1, -2.1$$

Asymptotes at $\pm 60°$ and $180°$, and intersect at $\frac{-3+1-1}{3} = -1$

Q-5b



closed loop

$$(s^2+11s+28)+k(s^2-s)$$

$$(k+1)s^2+s(11-k)+28.$$

s^2	$k+1$	28
s	$11-k$	
1	28.	

$j\omega$ crossing at $k=11$

and ω_0 = roots of $(12s^2+28)$

$$= \pm j\sqrt{\frac{7}{3}}$$

Q 5(b) Contd.

Breakaway / breakin pt: $\frac{1}{(\sigma+4)} + \frac{1}{(\sigma+7)} - \frac{1}{\sigma} - \frac{1}{\sigma-1} = 0.$

$$(\sigma+7)\sigma(\sigma-1) + (\sigma+4)(\sigma)(\sigma-1)$$
$$- (\sigma+4)(\sigma+7)(\sigma-1) - \sigma(\sigma+4)(\sigma+7)$$
$$= \sigma^3 + 6\sigma^2 - 7\sigma + \sigma^3 + 3\sigma^2 - 4\sigma - (\sigma^3 + 10\sigma^2 + 17\sigma - 28)$$
$$- (\sigma^3 + 11\sigma^2 + 28\sigma)$$
$$= \sigma^2(-12) + \sigma(-56) + 28 = -4(3\sigma^2 + 14\sigma - 7)$$

Roots $\frac{-14 \pm \sqrt{14^2 + 21 \times 4}}{6} = \frac{-7 \pm \sqrt{70}}{3} = 0.455$ and -5.122

Break-in point 0.455 (between the two zeros).

Break away pt: -5.122 (between the two poles).

Q-6: By linearity to get transfer function from T_1 to θ_1 & T_1 to θ_2, put $T_2 = 0$.
Same as in past tutorial sheet.

Q-7: $G_1(s)$ has zero in LHP. Hence Plot 1.

$G_2(s)$ has no zero. Hence Plot 4.
(Plots 1, 4, 3 intersect at a point, and that point is maxima for plot 4.)

$G_3(s)$ – should have steady state value $= 0$ ≡ none of the given plots.

$G_4(s)$ – "Non minimum phase" zero: initial part opposite sign of final value ⇒ plot 3.

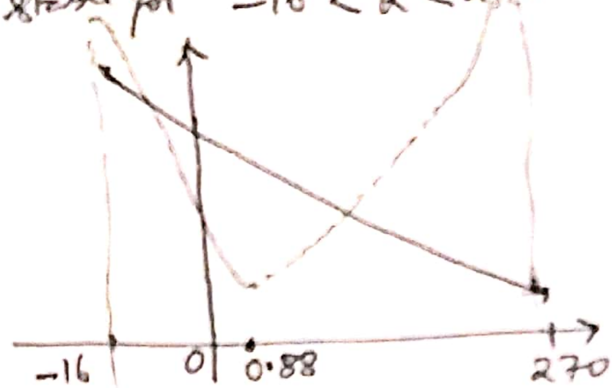
Plot 2 – None of the given options.

Q-8 For closed loop, $s^3 + 11s^2 + 26s + (16+k)$

Routh Hurwitz table

s^3	1	26
s^2	11	16+k
s	270−k	0
1	16+k	

Stable for $-16 < k < 270$.



(More elaborate than needed in exam.)

steady state error = $\dfrac{1}{1+k/16}$

closed loop r to y: $\dfrac{k}{s^3+11s^2+26+(16+k)}$