EE302-S2, Control Systems, Midsem Exam (26th Feb 2024)

Notes & instructions:

- Attempt all 7 questions: each question carries 10 marks. Unnecessarily long, convolved, redundant or irrelevant text could attract marks-reduction.
- Root locus sketch means: (a) show real-axis segments, (b) direction of k changing from 0 to $+\infty$, for $k > 0$, (or, if explicitly sought for $k < 0$, from 0 to $-\infty$), (c) mark poles by $\times$ and zeros by O, (d) calculate break-away and break-in point values and (e) classify the break-away/break-in points accordingly, (f) obtain angle of arrival/departure for every nonreal zero/pole, (g) asymptotes (if any): their angles and point of intersection: all these showing brief intermediate calculations. Make use of symmetry if sketching/calculating simplifies.
- Unless otherwise explicitly specified, k is real and positive.
- Some questions might not have the sought answer. In such a case, give reasons why the sought answer is not possible.
- If you feel a question has ambiguity and/or needs clarification, then assume yourself appropriately, state and justify your assumption and then proceed to solve the problem with that assumption.
  Do not call any TA or instructor for your query.

Ques 1: Sketch the root-locus of $G(s) = \dfrac{1}{s(s+2)(s+4)(s+6)}$ for $k > 0$.

Ques 2: Sketch the root-locus of $G(s) = \dfrac{(s+1)}{(s^2+1)(s-1)}$ for $k > 0$.

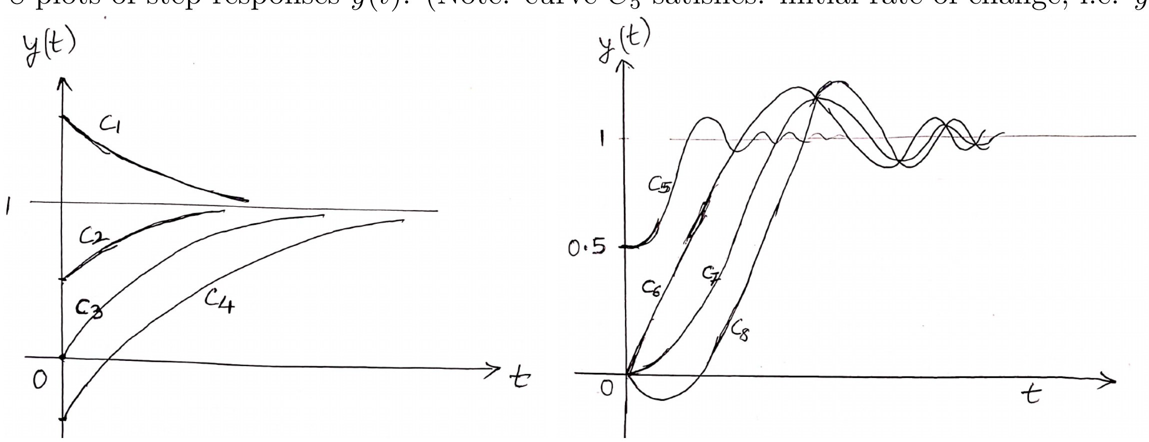
Ques 3: Sketch the root-locus of $G(s) = \dfrac{(s+1)(s-1)(s-3)(s-5)}{s(s-2)(s-4)(s-6)}$ for both $k > 0$ and $k < 0$.

Ques 4: Consider the 8 transfer functions below:

$$\frac{-s+5}{s^2+2s+5},\quad \frac{1}{s+2},\quad \frac{s-1}{s^2+2s+5},\quad \frac{3s+2}{2s+2},\quad \frac{5}{s^2+2s+5},\quad \frac{-s+3}{3s+3},\quad \frac{s+1}{s^2+2s+5},\quad \frac{s+2}{2s+2}.$$

(a) Obtain initial rise rate $\dot{y}(0^+)$ for each of the transfer functions' step-response: all eight of them.

Below are 8 plots of step responses $y(t)$. (Note: curve C_5 satisfies: initial rate of change, i.e. $\dot{y}(0^+)$, is 0.)



(b) Match transfer functions to their step-responses, if its step response is shown.

(c) If a response is unmatched to any of the given transfer functions, then suggest a transfer function having same characteristics (like initial value, initial rise rate, underdamped/overdamped, final value).

Ques 5: Consider the step response of the 2nd order system $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$.

(a) Plot curves C_1, C_2, C_3, C_4 and C_5, within the same figure, with:

C_1: curve along which % Over-Shoot (OS) is constant,
C_2: curve along which 2% settling time T_s is constant,
C_3: curve along which peak time T_p is constant,
C_4: curve along which natural frequency ω_n is constant,
C_5: curve along which damped frequency ω_d is constant.

(Please cross-check that C_i are labelled exactly as above.)

(b) Describe how T_s changes along any three of the curves $C_1, \ldots, C_5$,

(c) Describe how T_p changes along any three of the curves $C_1, \ldots, C_5$,

(d) Describe how % OS changes along any three of the curves $C_1, \ldots, C_5$,

(e) Describe how rise-time T_r changes along any three of the curves $C_1, \ldots, C_5$.

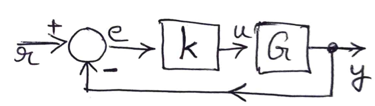
(Describing how a variable 'changes' means specifying whether the variable 'increases/decreases/remains-constant' along the curve.)

Ques 6: Use root-locus method to calculate value of k, if it exists, such that $d(s) + kn(s)$ has roots such that the roots correspond to poles resulting in the step response having:

(a) 2% overshoot, and $T_s \leqslant 2$ s, with $d(s) = s^2 + 3s + 2$, $n(s) = 1$.
(b) 2% overshoot, and $T_s \leqslant 1$ s, with $d(s) = s^2 + 3s + 2$, $n(s) = 1$.
(c) 2% overshoot, $T_s \leqslant 1$ s, and $T_p = 2s$, with $d(s) = 3s + 2$, $n(s) = 1$.

(T_s and T_p as defined in Question-5. Non-use of root-locus method would fetch less marks.)

Ques 7: Consider the standard negative unity feedback configuration as shown beside, with r the unit step input.



Sketch the steady state error $e(\infty)$ (when it exists) as a function of k (with k varying from $-\infty$ to $+\infty$) for $G(s)$ being the following transfer functions.

(a) $G(s) = \dfrac{5}{s+5}$, (b) $G(s) = \dfrac{s-5}{s+5}$, (c) $G(s) = \dfrac{s+5}{s-5}$.