EE613 Nonlinear dynamical Systems, Tutorial Sheet 1, Jan 2026

Note (a) Both in tutorials & quizzes/exams, some questions could have situation that sought answer not possible; then give reasons for such a question.

(b) Always have a calculator during test; No sharing of calculators. (Mobile $\neq$ calculator).

Q-1:

Give example of a function $f: \mathbb{R} \to \mathbb{R}$ such that

(a) Locally Lipschitz at $x_0 = 3$ but not globally Lipschitz

(b) Locally Lipschitz everywhere but not globally Lipschitz.

(c) Locally Lipschitz everywhere, and globally Lipschitz with $L = 100$.

(d) Globally Lipschitz with $L = 25$, but not for $L = 30$.

(e) — " — $L = 30$, but not for $L = 25$.

(f) Continuous at $x_0 = 5$, but not locally Lipschitz at $x_0 = 5$

(g) ________ , but not ________

(h) Differentiable at $x_0 = 5$, but not locally Lipschitz at $x_0 = 5$

(i) ________ , but not ________

Q-2: Give example of

(a) F.D. normed set which is not a space.

(b) F.D. normed space which is not complete.

(c) Infinite dimensional normed space which is not complete ✗ (w.r.t. that norm).

(d) Infinite dimensional normed space which is complete ✗.

Q-3: For $C^0([0,10], \mathbb{R})$

(a) is this a vector space? Finite or infinite dimensional?

(b) Suggest 2 different norms, & this space is complete w.r.t. one & $\underline{\text{not}}$ complete w.r.t. the other.

Q-4: Give example of f for each of the following cases such that each time f is locally Lipschitz at x_0, for $\dot{x} = f(x)$, $x(0) = x_0$,

(a) Soln does not exist, nor is unique.

(b) Soln exists, but not unique.

(c) Soln does not exist, but is unique.

(d) Soln exists & is unique.

Q-5: Same as Q-4 but without locally Lipschitz.

Q-6: Give an example of $\dot{x} = f(x)$ & $x(0) = x_0$ with each situation.

(a) f is linear & we have solution only for t upto T (finite T)
(b) f is bounded by linear and — " —.
(c) f is locally Lipschitz every where & — " —.
(d) Converges to an equilibrium point in finite time (without starting from an eq. pt).
(e) Same as (d), but with f locally Lipschitz every where
(f) Same as (d), but with f not locally Lipschitz at eq. pt.

Q-7: Plot (qualitatively) solutions of $\dot{x} = 3x^2$

$$\dot{x} = -x^2 - x + 2$$

clearly separating initial conditions into whether converge or diverge.

Q-8: (a) Prove existence & uniqueness of soln for some time $T > 0$ (of $x(t)$ on $t \in [0, T]$) for f locally Lipschitz at x_0 for $\dot{x} = f(x)$, $x(0) = x_0$

(b) Pinpoint where various assumptions on f got used in proof.

(c) Show that soln x (on the interval $[0, T]$) depends continuously on initial condition x_0.

(d)

Q-9 Look-up ϵ, δ definition of

(a) Limit — for $x \to x_0$ & also $x \to \infty$
(b) continuity
(c) Differentiability

Pay close attention within definition about distinction between
(d) given — & there exists —
(e) for some — & for every —.
(f) there exists or some & for any/every/arbitrary.