EE636 Matrix Computations, Tutorial Sheet, Jan 2026

Note: ~~Some~~ questions (in every tutorial sheet/quiz/midsem/endsem) do not have sought answers; give reasons why not for such questions. Only scientific calculators are allowed, no mobile, & no sharing

Q-1 (a) Find level curves for $\|x\|_2=3$, $\|x\|_2=5$ in $\mathbb{R}^2$.

(b) Find a norm such that level curves $\|x\|_\circ = a$ & $\|x\|_\circ = b$ intersect (for $a \neq b$).

(c) Find $p \geq 0$ such that $\|x\|_p$ is not a norm.

?? ??

(d) Show that $\|x\|_0$ is <u>not</u> a norm

??

Q-2 For $x \in \mathbb{R}^2$, show that $\|x\|_2, \|x\|_1$ are "equivalent" norms. and for $\|x\|_2$ & $\|x\|_\infty$.

Q-3: ~~Show~~ that induced norm is a matrix norm (i.e. satisfies all 4 norm axioms for a matrix).

Q-4: Argue why $\|x\|_p$, $p\to\infty$ is same as $\|x\|_{max}$ norm.

Q-5: Express AB as a sum of p rank-1 matrices for $A \in \mathbb{R}^{m\times p}$, $B \in \mathbb{R}^{p\times m}$

Q-6: —"— $p+2$ rank-1 matrices —"—.

Q-7 Find ~~SVD~~ of $A := \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 3 & -4 \end{bmatrix}$ & use SVD $A = U_1\Sigma_1 V_1^T$ to obtain A as a sum of 2 rank-1 matrices.

Q-8: For the A in Q-7, modify your SVD to get a different $U_2\Sigma_2V_2^T$: each one different.

Q-9 In Q-7, obtain only U & Σ & then use $A = U\Sigma V^T$ to get V. Can this always be done (when A is nonsquare? square & nonsingular?))

Q-10: Use Q-7's SVD to show that U & V^T <u>cannot</u> be independently obtained (as eigenvectors of AA^T & A^TA due to
- non uniqueness of eigenvectors (for distinct eigenvalues)
- non uniqueness of eigenvectors (for repeated eigenvalues).

Q-11: Use SVD of Q-7 to obtain
- $\kappa_2(A)$
- ΔA matrix such that $\dfrac{\|\Delta A\|_2}{\|A\|_2} = \dfrac{1}{\kappa_2(A)}$

Q-12: Suggest a 2×2 matrix with $\|A\|_F = \|A\|_2$.

Q-13: For $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, suggest a ΔA matrix such that ~~A+ΔA is singular~~ and

(a) $\|\Delta A\|_2 = 1$ and $A+\Delta A$ is singular

(b) $\|\Delta A\|_2 = 3$ and $A+\Delta A$ is singular.

(c) $\|\Delta A\|_2 = 4$ and $(A+\Delta A)$ is nonsingular

(d) $\|\Delta A\|_2 = 2$ and $(A+\Delta A)$ is nonsingular.

(e) — " — — " — singular.

Q-14: Suggest $A, B \in \mathbb{R}^{3\times 3}$ such that

	rank A	rank B	rank AB
(a)	1	2	3
(b)	1	1	1
(c)	2	1	0
(d)	2	2	0
(e)	2	2	1

Q-15: Give an example of a matrix A such that $\ker A = \operatorname{Im} A$.

with $A \in \mathbb{R}^{2\times 3}$

$A \in \mathbb{R}^{1\times 1}$

$A \in \mathbb{R}^{2\times 2}$

$A \in \mathbb{R}^{3\times 3}$

$A \in \mathbb{R}^{4\times 4}$

what can be said of the

– eigenvalues of A?

– singular values of A?

Q-16: Suggest matrices A, B such that

(a) $\|A\|_2 \|B\|_2 > \|AB\|_2$

(b) — " — $= \|AB\|_2$

(c) — " — $< \|AB\|_2$

Q-17: Suggest 2 non coprime polynomials $p(x), q(x)$ such that very small perturbation of p coefficients to get p_ϵ

no change in degree.

– causes coprimeness of p_ϵ, q

– retains non coprimeness of p_ϵ, q