EE 640 Multivariable Control Endsem Exam
2015 -Nov-19th

Note: All questions are compulsory. Read more notes at the end and also entire q-paper first.
After first 10 minutes, no clarifications: assume suitably.

Q-1 Consider the minimization problem $\int_0^\infty (x^2+u^2)\,dt = J(x_0,u)$ with $x(0)=10$ for the system $\dot{x} = 3x + 4u$

(a) Find optimal control law that minimizes the above integral using ARE.

(b) Assume $u = fx$ achieves minimization. Find integral as a function of f and find f^* that minimizes J.

(c) Find closed loop A_F (for $\dot{x} = A_F x$) in (a) or (b).

Q-2: Consider the system $\dot{x} = Ax$, $y = Cx$ with C full row rank. Assume $C = [I_p \;\; 0]$, $C \in \mathbb{R}^{p\times n}$, $A \in \mathbb{R}^{n\times n}$.

(a) Derive how a reduced order observer can be designed to have observer poles ($n-p$ of them) wherever desired. (Assume (A,C) is observable and use pole-placement theorem of controllable (A,B) pair.)

(b) Find transfer function from y to $\hat{x}$ in the reduced order observer.

(c) Give reasons why there also exists an "observer" without any poles such that $x = M(\frac{d}{dt})y$.

(d) Suppose $\dot{x} = Ax$, $y = x_1$ with $A = \begin{bmatrix} 0 & 1 \\ 0 & +2 \end{bmatrix}$

Design a reduced order observer with one pole: at -3

(e) For system (d), design a PD observer.

Q-3 Consider the system $\dot{x} = Ax + Bu$ with (A, B) controllable, $B \in \mathbb{R}^{n \times m}$.
Given $Q > 0$, design a feedback law $u = Fx$, $F \in \mathbb{R}^{m \times n}$ such that the closed loop $A_F := A + BF$ satisfies

$$A_F^T P + P A_F \lneq -Q \quad \text{for some } P > 0.$$

- Use a suitable ARE to claim existence of such a P & F
- Prove that this designed F satisfies

$$A_F^T P + P A_F \leq -Q.$$

Q-4 Consider the equation $A^T P + PA = -Q$ with $Q \geq 0$, A Hurwitz.
Prove any one inclusion

(a) $\ker P \subseteq$ Q-unobservable subspace of A

(b) $\ker P \supseteq$ Q-unobservable subspace of A.

State clearly which of a or b you have proved.

Q-5 For system $\dot{x} = ax + bu$, and initial condition $x_0 \in \mathbb{R}$, for each case below, find minimum energy $\int_0^\infty u^2 \, dt$ to take x_0 to 0. (at $t = \infty$)

(i) $a = -1$, $b = 1$, $x_0 = 5$

(ii) $a = +1$, $b = 1$, $x_0 = 5$

(iii) $a = 1$, $b = 0$, $x_0 = 5$

(iv) $a = -1$, $b = 0$, $x_0 = 5$.

(You can use formula: no need to derive formula.)
But show calculations from formula to final answer.

Q-6: It is intended to design a controller $u = K(s) y$ for stabilizing the system $\dot{x} = Ax + Bu$, $y = Cx$ using a dynamic output feedback controller.

(a) State necessary conditions on A, B, C for this to be possible.

(b) Assuming the necessary conditions, show that $K(s)$ of order n can be constructed: prove that static state feedback law and observer can be separately designed and combined to achieve this.

Q-7: For the system $\dot{x} = Ax + Bu$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $m > 1$

The following are <u>claimed</u> to be equivalent. Find flaws in (b) onwards. B is assumed full column rank.

((a) is correct.)

(a) (A, B) is controllable.

(b) For some $d \in \mathbb{R}[s]$, there exists $F \in \mathbb{R}^{n\times m}$ such that

$$\lambda(A+BF) = \text{roots of } d.$$

(c) $[A - \lambda I \quad -B]$ has full row rank for all $\lambda \in \mathbb{C}^+$

(d) $[B \ AB \cdots A^{n-1}B]$ is nonsingular.

(e) The m - controllability indices $k_1, k_2, \ldots k_m$ satisfy

$$\frac{n}{m} \leq k_i \leq n \quad \text{for each } i$$

(f) $$\bigcap_{F \in \mathbb{R}^{m\times n}} \lambda(A+BF) = \lambda(A)$$

($\bigcap$ denotes intersection, $\lambda(\cdot)$ denotes set of eigenvalues.)

(1) $\lambda(\cdot) \equiv$ set of eigenvalues (ignore multiplicity complications in this paper).

(2) Some questions do <u>not</u> have sought answer: in that case ~~Sol.~~ give reasons why sought answer is not possible.

(3) Show intermediate calculations for all problems.

(4) After first 10 minutes, please do <u>not</u> call for clarifications (except handwriting ~~itd~~ illegibility issues.

(5) Marks: 20 marks per question. Total weightage: 40%.