A CRITERION FOR A-INVARIANCE.

19th Aug, '15

(Essential for some topics in EE 640)

Suppose S is A-invariant

Let columns of $G \in \mathbb{R}^{n \times m}$ span S, i.e. $S = \mathrm{Im}(G)$

$= \mathrm{span}\{g_1, \ldots, g_m\}$.

Since $Ag_1 \in S$, we can write it as:

$$Ag_1 = t_{11} g_1 + \cdots + t_{n1} g_m .$$

Similar expressions can be written for $Ag_2, \ldots, Ag_m$. Then we have

$$A[g_1 \cdots g_m] = [g_1 \cdots g_m] \begin{bmatrix} t_{11} & \cdots & t_{1m} \\ \vdots & & \\ t_{n1} & \cdots & t_{nm} \end{bmatrix} .$$

i.e. $AG = GT$.

Hence if $S (= \mathrm{Im}(G))$ is A-invariant, then there exists a matrix T such that $AG = GT$ //.

Now assume G has rank $= m \Rightarrow \{g_1, \ldots, g_m\}$ is basis for S. Then every eigenvalue of T will be an eigenvalue of A. The eigenvectors corresponding to the eigenvalue will be in S.

Consider $x \neq 0$ s.t. $Tx = \lambda x$. Then $A(\underline{Gx}) = G(Tx) = \lambda \underline{Gx}$

$\Rightarrow$ eigenvalues of T are a subset of eigenvalues of A. //.

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