EE640 Multivariable Control, Quiz 2, 26th Oct 2016.

Weightage 25 %.

Note (a) Marks breakup at the end. Total marks 70.

(b) Some questions do not have sought answer. Give reason why for such a case.

Q-1: Consider $\dot{x} = x + u$ ($x(t) \in \mathbb{R}$) in which the cost $J(x_0, u) := \int_0^\infty (3x^2 + u^2)\,dt$ is to be minimized over u.

(a) Use ARE to find optimal input ~~u~~ (and optimal feedback law $u = Fx$). (Show ARE, obtain soln P & use P to get feedback law.)

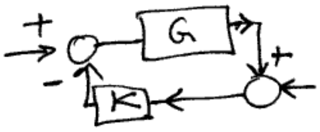
(b) Assume $u = Fx$ is a law and minimize $J(x_0, u)$ over F to get best F. Check that your F minimizes.

Q-2: A certain system of order 1, 2 or 3 or 4 has Markov parameters $h_1 = 1$, $h_2 = 2$, $h_3 = 3$, $h_4 = 5$, $h_5 = 8$, $h_6 = 13$, $h_7 = 21$, $h_8 = 34$, $h_9 = 55$. Find (A, B, C, D) as needed below.

(a) Use Hankel matrix to obtain a state space realization. (Find A using Hankel matrix. B, C by any method.

(b) Use Toeplitz matrix to find A, B, C, D.

Q-3: Consider $G(s) = \dfrac{s+1}{s+4}$ and feedback



(a) Find a K s.t. interconnection is not well-posed.

(b) Find a strictly proper K s.t. interconnection is ~~well posed~~ not well posed.

(c) Define when an interconnection is called well posed.

Q-4: Consider $\dot{x} = Ax$, $y = x_1$, $A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$.

(a) Design a reduced order observer with one pole at -3.

(b) Design a full order observer with poles at $-2, -3$.

Q-5: For each case below, construct an example of A, B, P, Q as required. (Separate examples. No need for them to be the same.) Sometimes example not possible. Then say why.

Q-5 (Contd)

(a) A not cyclic, $B \in \mathbb{R}^n$, (A, B) controllable.

(b) A is not cyclic, $B \in \mathbb{R}^{n\times m}$, (A, B) controllable

$\exists\ b \in \operatorname{Im} B$ s.t. (A, b) controllable.

(c) A is cyclic, $B \in \mathbb{R}^{n\times m}$, $\exists\ b \in \operatorname{Im} B$ s.t. (A, b) controllable

but b is not any single column of B.

(d) A is Hurwitz, $P > 0$ but

$(A^T P + PA)$ has one +ve eigenvalue.

(e) $Q > 0$, $P > 0$, A-anti-Hurwitz s.t. $A^T P + PA = -Q$

(f) $Q > 0$, $P > 0$, A-Hurwitz s.t. $A^T P + PA = -Q$.

(g) (A, B) s.t. the controllability indices $(\mu_1, \mu_2, \cdots \mu_m)$ (perhaps unsorted)

is $(3, 3, 0)$ (A, B) has to be controllable.

(h) Same as (g) but $(\mu_1, \mu_2, \cdots \mu_m) = (2, 1, 1)$

(A, B) has to be controllable.

(i) An example of Hankel matrix H

when 1^{st} principal minor is rank 0,

2^{nd} principal minor is rank 0,

but system $G(s)$ is order 3.

give $\frac{n(s)}{d(s)}$ with strictly proper (& no pole/zero cancellation)

Marks. 10 marks each for Q-1 to 5

2 marks for Q-6 Totally 70 marks.