Quiz 2, EE 640 Multivariable Control, 28th Oct 2015.

(First read note at the end and read all questions carefully.)
Time 2 hours.

Q-1 Consider the Lyapunov equation $A^TP + PA = Q$ with A and Q given and P unknown. $Q = Q^T$.
Suppose A is Hurwitz. Consider the map $L_A(P)$, $L_A : \mathbb{R}^{n\times n}_{Sym} \to \mathbb{R}^{n\times n}_{sym}$. Assume A has real distinct eigenvalues.

(a) What are the eigenvalues of L_A?

(b) Restricted to $n\times n$ symmetric matrices P, how many eigenvalues does L_A have?

(c) Use a & b to conclude: for each Q, there is a unique P.

Q-2: Suggest a 2×2 A with no eigenvalues on $j\mathbb{R}$ and A is nonsingular such that $A^TP + PA = Q$ has no solution P (for the Q you choose.)

Q-3: Consider $\dot{x} = Ax$, $y = Cx$ with $A = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

(a) Build an observer with poles at $-2, -1$.

(b) Build a reduced order observer with one pole: at -1

(c) Build a differentiator such that $x = P\left(\frac{d}{dt}\right) y$ such that x can be obtained from y.

(d) Find transfer function from y to $\hat{x}$ in (a) & (b).

Q-4: Consider $\dot{x} = 2x + u$ and $x(0) = 2$. $\min_u \int_0^\infty (x^2 + u^2)\,dt$

(a) Find optimal feedback law $u = fx$, with f fixed by differentiating cost w.r.t. f (and verify 2nd order condition).

Q-4b: Write the Algebraic Riccati Equation corresponding to the above problem and find minimum cost and the optimal feedback law (using the ARE solution).

Q-5 Without using ARE arguments, show that solution X to $AX + XA^T = BB^T$ is invertible if and only if (A, B) is controllable. (Assume A is anti-Hurwitz and do not use duality arguments.).

Q-6: For each of $G(s)$ below, find the H_∞ norm. (Show calculations briefly).

(a) $G(s) = \frac{1}{s+10}$ (b) $\frac{1}{s-2}$.

(c) $\frac{1}{s^2+4}$ (d) $\frac{(s+3)(s-2)}{(s+4)}$

(e) Define H_∞ norm of a SISO transfer function.

Note: (a) First read all questions.

(b) ARE $\equiv$ Algebraic Riccati Equation.

(c) Some questions do not have sought answer, in that case give clear reasons why sought answer does not exist.

(d) All questions carry 7 marks each..

(e) For all questions, Show intermediate steps briefly.