EE640 Multivariable Control, Midterm exam 10th Sept 2015

Weightage 25%. Attempt all questions. See note at the end.

Q-1 Design an observer such that the estimation errors decays to zero at rates corresponding to observer poles at -2 & -3 for the system $\dot{x} = Ax$, $y = Cx$ with

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

Q-2: Suggest matrices with given properties

(a) (A, B) controllable with controllability indices 2, 3

(b) (A, B) controllable with controllability indices 1, 4.

Q-3: Suppose $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and let $\mathcal{C}$ = controllability matrix, $\mathcal{O}$ = observability matrix.

Suppose $\mathbb{R}^n$ is decomposed into $\mathbb{R}^n = X_1 \oplus X_2 \oplus X_3 \oplus X_4$ with $X_1 = \operatorname{Im} \mathcal{C} \cap \operatorname{Kernel} \mathcal{O}$

X_2 defined such that $X_1 \oplus X_2 = \operatorname{Im} \mathcal{C}$

X_3 defined such that $X_1 \oplus X_3 = \operatorname{Kernel} \mathcal{O}$

and X_4 defined such that $X_1 \oplus X_2 \oplus X_3 \oplus X_4 = \mathbb{R}^n$.

Suppose A in this new basis has matrix

$$A = \begin{bmatrix} A_{11} & p & q & r \\ s & A_{22} & t & u \\ v & w & A_{33} & z \\ e & f & g & A_{44} \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \end{bmatrix}.$$

For each of p, q, r, ~~s~~, t, ~~u~~, ~~v, w~~, ~~z~~, ~~e, f~~, g, B_1, B_2, ~~B_3, B_4~~, ~~C_1~~, C_2, C_3, ~~C_4~~. give zero/nonzero with a brief reason.

Q-4 Consider $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Suggest $x_0 \in \mathbb{R}^2$ such that (see next page)

$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

for any $u \in$ continuous functions $\mathbb{R}$ to $\mathbb{R}$,

$x(t) \to \infty$ as $t \to \infty$.

($x(t)$ becomes unbounded for that initial condition no matter which input $u(t)$ is given).

Give reason why your choice of x_0 has this property.

A, B

Q-5. Suggest pair (A, B) controllable such that

There does not exist [$b \in \operatorname{Im} B$ such that (A, b) is controllable]

Q-6: Suggest pair (A, B) controllable such that there does not exist $b \in \operatorname{Im} B$ such that (A, b) is controllable, but with A cyclic.

$b_1\ b_2\ b_3$ $\quad Ab\ Ab_1\ Ab$
$b_1 \quad Ab_2 \quad A^2 b_3$

Note: 1. Whenever you are to suggest matrices, give brief reason why your suggested matrix meets requirement.

$b\ Ab\ A^2b$

$B \quad AB \quad A^2B$

2. May be sought answer does not exist. If you are sure about this, give reasons.

$\begin{matrix} 1 & 2 \\ 0 & 3 \end{matrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ 1

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\begin{matrix} 1^2 & 0 \\ 2 & 3 \end{matrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} -2 & 0 \\ 2 & 0 \end{bmatrix}$ $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ 2 $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{matrix} 0 & 0 \\ 2 & 2 \end{matrix}$