Tutorial Sheet, EE640, Multivariable Control, 9th Oct 2015.

Q-1: Consider $\dot{x} = Ax$ and the quadratic form $x^T P x$.

(a) Suppose P & Q satisfy $A^T P + PA = Q$,

Show that $\frac{d}{dt}(x^T P x) = x^T Q x$.

(b) Suppose A is Hurwitz. Show that $P := \int_0^\infty e^{At} \, e^{At} \, dt$

Solves $AP + PA = Q$, with $Q :=$

(c) Show that $L_A : \mathbb{R}^{n \times n} \to \mathbb{R}^{n \times n}$ defined by $L_A(P) := A^T P + PA$

is a linear operator (in P).

(d) Prove that if P is symmetric, then $L_A(P)$ is symmetric.

(e) Show that eigenvalues of L_A (restricted to symm. $\mathbb{R}^{n \times n}$)

are $\lambda_i + \lambda_j$ $1 \leq i \leq j \leq n$, λ_i are eigenvalues of A.

(Prove only for A with real, distinct eigenvalues.)

(d) Show that if A has one or more eigenvalues at 0, then

$A^T P + PA = Q$ is <u>not</u> solvable for arbitrary Q.

(e) Prove that if (Q, A) is <u>unobservable</u> (and $Q = Q^T$ is semidefinite)

then the unobservable subspace is contained in $\ker P$.

Q-2 For the following system (assume observable)

$$\begin{bmatrix} \dot{x}_r \\ \dot{x}_n \end{bmatrix} = \left[\begin{array}{c|c} A_r & b_r \\ \hline c_r & d_r \end{array}\right] \begin{bmatrix} x_r \\ \hline x_n \end{bmatrix} \qquad \text{where } \left[\begin{array}{c|c} A_r & b_r \\ \hline c_r & d_r \end{array}\right] \in \mathbb{R}^{n \times n}$$

$$y = [0 \cdots 0 \; 1] \begin{bmatrix} x_r \\ \hline x_n \end{bmatrix} \qquad A_r \in \mathbb{R}^{(n-1) \times (n-1)}$$

$$\begin{bmatrix} x_r \\ \hline x_n \end{bmatrix} \in \mathbb{R}^{n \times 1}$$

$$x_r \in \mathbb{R}^{(n-1) \times 1}$$

(a) Write down the equations for the reduced order ~~behavior~~ observer which estimates the first $(n-1)$ states, when

$b_r = 0$; $d_r = 0$

(b) Do (a) with $b_r \neq 0$; $d_r \neq 0$.

(c) Evaluate the transfer function for observer system & show that it is biproper for both (a) & (b).

<u>Note</u>:- Let observer be $\frac{d}{dt}\hat{x} = A_r \hat{x}_r$

'without feedback', no reason for $x \to \hat{x}$ as $t \to \infty$. As $\dot{x}_n$ didn't get used so feedback becomes (for part (a))

$$\frac{d}{dt}(\hat{x}_r) = A_s \hat{x}_s + l(\dot{x}_n - \hat{\dot{x}}_n)$$

with $x_n = y$ & $\hat{\dot{x}}_n = c_r \hat{x}_r$

Q2. Repeat Q1 for

$$B = \begin{bmatrix} g_r \\ \hline g_n \end{bmatrix} \in \mathbb{R}^{n \times 1}; \quad g_r \in \mathbb{R}^{(n-1)\times 1}$$

Q3. for the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x; \quad y = [0 \quad 1]\, x.$$

(a) find a PD observer. ~~with~~

(b) Design a reduced order observer with one pole at -3

(c) Design a full order observer with poles at $s = -3$ (both)

Q5 Design a reduced order observer with two poles at $s = -10$ for

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 11 & -6 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad C = [0 \ 0 \ 1]$$

Q6. Suppose we have a realization (c, A)

$c = [0 \cdots 0\ 1]$

A is in right companion matrix with $-[a_n\ a_{n-1} \cdots a_1]^T$ as its last col^m

Find a non singular T so that $\bar{A}_r$ which is the $(n-1)\times(n-1)$ left hand submatrix of $T^{-1}AT$ has characteristic polynomial

$$\det(sI - \bar{A}_r) = s^{n-1} + \alpha_1 s^{n-2} + \cdots + \alpha_{n-1}$$

Also what is the structure of $\bar{c} = cT$

(Assume $\bar{A}_r$ to also be in right companion form with last col^m $-[\alpha_{n-1} \cdots \alpha_1]^T$)