

Quiz 2, Nov 2017, EE640 Multivariable Control.

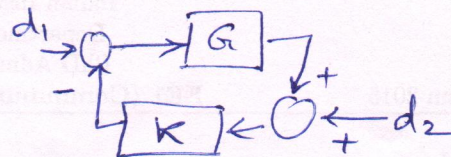
First Read note at the end

Q-1: Define wellposedness of interconnection

and give one example of G & K (SISO)

such that interconnection is not wellposed.

- Verify that the interconnection (for your example) is not wellposed.



Q-2: Suppose $G(s)$ & $K(s)$ of Q1 are SISO with state space realization

$$\dot{x} = Ax + Bu, y = Cx \quad \text{and} \quad \dot{z} = Pz + Qv, w = Rz$$

Prove this will be wellposed.

Q-3: Give an example of (A, B) such that

(a) controllability indices are $(0, 1, 2)$

(b) $(1, 1, 2)$

(c) $(1, 1, 3)$

(d) In (b) we need controllability for (A, b_1) and (A, b_2) & (A, b_3) In addition.

Q-4: Calculate Hankel matrix for $G(s) = \frac{s+1}{s^2+2s-1}$

and use these and demonstrate that 3×3 matrix is singular.

Q-5: Suppose a system has a state space realization of order N .

(b) Show that the $H(m, n)$ has rank at most N for $m \geq N$ $n \geq N$.

(a) Define $H(m, n)$ Hankel matrix with m rows & n columns & relate with Markov parameters.

Q-6: A given $G(s)$ has series expansion $G(s) = 5 + s^{-2} - s^{-3} + s^{-5} - s^{-6} + s^{-8} - s^{-9} + s^{-11} - s^{-12} \dots$ (coefficient of $s^{-1}, s^{-4}, s^{-7}, s^{-10}$ is zero.)

Find ~~the~~ $G(s)$ numerator & denominator using Hankel & Toeplitz matrices.

~~Q-7~~

Q-7: Of the two statements below, one of them is almost equivalent to "A is Hurwitz" while the other is wrong (not equivalent).
 Correct the one which is close to equivalent by adding additional conditions and give a counter-example to the one which is not equivalent. Note: for one, modify to make equivalent, for other, give counterexample to demonstrate non-equivalence.

- (a) for every $Q \geq 0$, $\exists P > 0$ such that $A^T P + P A = -Q$
 (Q & P are symmetric in both a & b)
- (b) for every $P > 0$, $\exists Q \geq 0$ such that $A^T P + P A = -Q$

Q-8: For $\dot{x} = Ax + Bu$, $y = Cx$, show that a similarity transformation on A does not change the eigenvalues of PQ (P & Q are controllability & observability Gramians respectively, A is Hurwitz, (A, B) & (A, C) are controllable & observable).

~~Q-9: Show that~~

- Marks: Each question carries 5 marks & weightage of quiz is 25%.
- Ask questions within first 15 minutes only.
- Time: 2 hours.