EE 640 Multi-variable Control, Endsem, 12^{th} Nov 2024.

Note: Read full question paper first, including the "Note" at the end.

Q-1: Suppose $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ and it is intended to place the closed loop poles using a feedback controller $u = F\hat{x}$, with closed loop poles as roots of $d_c(s) \cdot d_o(s)$.

(a) State conditions on $d_c(s)$ & $d_o(s)$: polynomials of appropriate (and same) degrees

(b) State conditions on A, B, C (for $\dot{x} = Ax + Bu$, $y = Cx$) such that this design is possible.

(c) Prove that the design procedure can be separated into appropriate observer design & state feedback controller design.

Q-2: For $A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, obtain (showing intermediate calculations)

(a) $\int_{-\infty}^{0} e^{A\tau} BB^T e^{A^T\tau} d\tau$ (b) $\int_{0}^{\infty} e^{A\tau} BB^T e^{A^T\tau} d\tau$

Q-3: (a) Define well-posedness of interconnection of plant & controller (both SISO) for the appropriate feedback configuration.

(b) for SISO plant, controller, both proper, give necessary & sufficient conditions on transfer functions $G_p(s)$ & $G_c(s)$ for wellposedness.

(c) For $G_p(s) = \dfrac{s+10}{s+3}$, suggest a controller such that the first condition alone is violated.

(d) For same $G_p(s)$, suggest a controller such that only the second condition for well-posedness is violated.

(e) For same $G_p(s)$, suggest a controller such that both first & second conditions (for well posedness) are violated.

Q-4: Consider transfer function $G(s) = \dfrac{-3s^2 - 7s}{s^2 + s + 2}$.

(a) Obtain (by any method, show calculations) the Markov parameters (until h_8).

(b) Construct Hankel matrix (upto 4×4 matrix). (with the purpose of finding a state space realization)

(c) Use this Hankel matrix to obtain a state space realization.

Q-5: Find H_2 norm of following transfer functions using four different methods: (a) $\dfrac{s+1}{s+2}$, (b) $\dfrac{1}{s^2+3}$, (c) $\dfrac{3}{s+2}$

Q-6: For each transfer function below, obtain H_∞ norms (show intermediate calculations)

(a) $\dfrac{(s+3)(s+5)(s+7)}{(s+4)(s+6)(s+8)}$. (b) $\dfrac{s-3}{s^2+9}$ (c) $\dfrac{3}{s^2+s+3}$

Q-7: Let $A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $C = [2 \quad 3]$, $D = 0$.

(a) Use Gilbert test for following: hence bring (A, B, C) to an appropriate form using a similarity transformation.

(b) Use Gilbert test for identifying uncontrollable pole, if any.

(c) Use Gilbert test for identifying unobservable pole, if any.

Q-8: For each of the supposedly equivalent statements (b), (c), ... (g). there is a flaw. Correct the flaw to make each statement equivalent to (a). ((a) has no flaw). $A \in \mathbb{R}^{n \times n}$, $P = P^T$, $Q = Q^T$

(a) A is Hurwitz. (b) All eigenvalues of A are in closed left half complex plane.

(c) $\|e^{At} x_0\|_2 \to 0$ for some $x_0 \in \mathbb{R}^n$ as $t \to \infty$

(d) For every $P > 0$, $AP + PA = Q$ and $Q < 0$

(e) For every $Q > 0$, $A^T P + PA^T = Q$ has a solution $P > 0$.

(f) For every $P \geq 0$, $AP + PA = Q$ and $Q \leq 0$

(g) For every $Q \geq 0$, $A^T P + PA = Q$ has a solution $P \geq 0$.

Q-9: For $A \in \mathbb{R}^{3 \times 3}$, $B \in \mathbb{R}^{3 \times 2}$, suggest (A, B) satisfying each condition.

1 (a) (A, B) controllable and A diagonalizable.

2 (b) (A, B) controllable, A repeated eigenvalues & diagonalizable,

2 (c) — " — — " — — " — and rank(B) =.

1 (d) A non-cyclic, (A, B) controllable & rank $B = 1$

2 (e) (A, B) stabilizable but not controllable.

1 (f) $AP + PA^T = -BB^T$ does not have a solution P. (no condition on rank B)

1 (g) $AP + PA^T = -BB^T$ has a +ve definite solution $P = P^T$, but $A^T P + PA = -BB^T$ does not have a +ve definite solution!

Note: Each question has 10 marks. All questions are compulsory. Call us only for handwriting query. Question query in 1st 10 minutes only

Some questions do not have sought answer: provide reason.