Tutorial 3, MVCS.

Q1. Assume (A, B) is controllable, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ show that there exists a feedback matrix $F \in \mathbb{R}^{m\times n}$ such that $A+BF$. is cyclic.

Q2. Assume $A \in \mathbb{R}^{n\times n}$, is cyclic (maybe repeated eigenvalues) show that each of the four companion forms are similar to Jordan Canonical form.

Q3. Any cylic matrix with same eigenvalues are similar (Counted with multiplicities). Prove the statement.

Q4. Given $A \in \mathbb{R}^{n\times n}$, $B = [b_1\ b_2\ \cdots\ b_m] \in \mathbb{R}^{n\times m}$

Construct $M = [b_1\ Ab_1\ \cdots\ A^{n_1-1}b_1,\ b_2\ Ab_2\ \cdots\ A^{n_2-1}b_2, \cdots$

$, b_p\ Ab_p\ \cdots\ A^{n_p-1}b_p]$

where, each column of above matrix is independent of all previous columns. $(p \leq m)$

Let K_i denote the controllability indices for $i=1 \cdots m$.

Is $\max K_i = \max n_j$? for $i = 1 \cdots m$
$j = 1 \cdots p$.

Q5. Design a reduced order observer with two poles at $s=-10$ for

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 11 & -6 \end{bmatrix};\quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\quad c = [1\ 0\ 0]$$

Q6. Consider $\dot{x} = Ax + v$ $\quad A \in \mathbb{R}^{n\times n}$, $v: t \to \mathbb{R}$, $y = Cx$ for $t \geq 0$

Also let $\mathcal{N} = \bigcap_{i=1}^{n} \mathrm{Ker}(CA^{i-1})$. Let $x_1(\cdot)$, $x_2(\cdot)$ be solutions of the above system for the same input $v(\cdot)$, but for possibly different initial states x_{10}, x_{20}. If for some $t \geq 0$; $x_1(t) - x_2(t) \in \mathcal{N}$. then show that $y_1(s) = y_2(s)$; $s \geq 0$.